Bayesian Networks for Modeling and Assessment of Credit Concentration Risks

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Outline

- Problem statement: The destructive power of credit risk concentration.
- How to capture and model uncertain relationships between the risk related variables?
- BN probabilistic graphs as a tool for risk modeling and assessment.
- Typical BN structures and decomposition of the probability distribution.
- Empirical framework: Related party disclosure and BN structure.
- Measure of mutual information for model assessment.
- Updating algorithms and stress testing.
- Conclusions and scope for futute.

Bank Concentration Risk

BY JONATHAN YORK

What's the chance of everything going wrong at the same time in your credit portfolio? That's really the question that keeps bankers—and their regulators—awake at night.

Bayesian networks

- **Bayesian networks** are a special case of multivariate (discrete) probability distributions embodying a collection marginal and conditional independencies which may be represented by means of a directed acyclic graph.
- Consider a set of random variables, $\mathcal{X} = \{X_1, \ldots, X_d\}$. Two components of a BN model induced over $\mathcal{X}$ are $\langle \mathcal{G}, P \rangle$.
- $\mathcal{G}$ is the directed acyclic graph representing the independence assumption: each X_i is conditionally independent of its non-descendants given its parent nodes $\Pi_{[i]}$ in $\mathcal{G}$ and
- $P = \{P(x_1|\Pi_{[1]}), \ldots, P(x_d|\Pi_{[d]})\}$ which represents the set of d conditional probability distributions given the set of parent nodes $\Pi_{[i]}$ for each X_i, $i = 1, \ldots, d$.
- Convention: We are going to identify nodes of a graph with random variables.
- Probability factorization $P(x_1, \ldots, x_d) = \Pi_{i=1}^{d} P(x_i|\Pi_{[i]})$.

BN structures and probability factorization

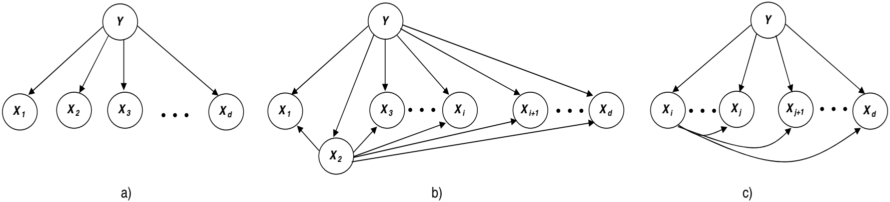


Figure: Three typical Bayesian Network structures: a) N-BN b) TAN c) k-BN.

- We specify the simultaneous distribution by a set of simpler conditional distributions (modularity)

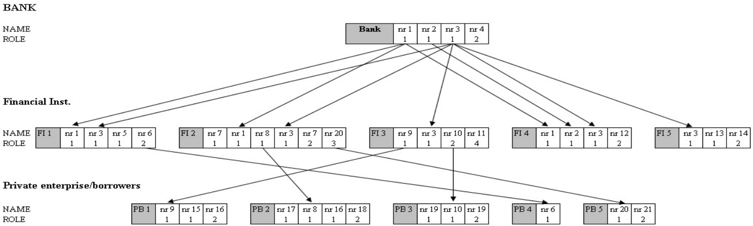
a) $P_{Y,X_1,\dots,X_d}(y, x_1, \dots, x_d) = P(y) \cdot \Pi_{i=1}^{d} P(x_i|y),$

b) $P_{Y,X_1,\dots,X_d}(y, x_1, \dots, x_d) = P(y) \cdot P(x_i|y) \cdot \Pi_{j=1,j\neq i}^{d} P(x_j|x_i, y),$

c) $P_{Y,X_1,\dots,X_d}(y, x_1, \dots, x_d)$
$= P(y) \cdot P(x_1|\Pi_{[1]}) \cdot \dots \cdot P(x_{d-1}|\Pi_{[d-1]}) \cdot P(x_d|\Pi_{[d]})$

Related party disclosure and risk related chracteristics

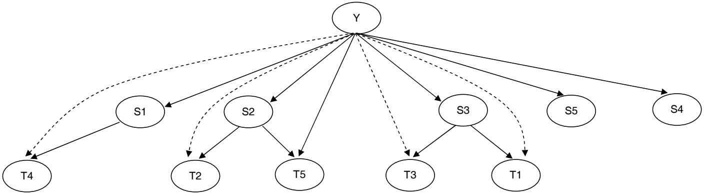
- Direct relationships in a group of related borrowers according to Related Party Disclosures requirements IAS 24 (2008)



- Decoding codes for the related party disclosures.

Role	Characteristic
1	Founder of bank or business partner
2	Director of a business partner
3	Depositor/Guarantor
4	Connected persons (e.g family members)

TAN structure and probability distributions



$P(Y = b)$	$P(Y = nb)$
0.5	0.5

	$y = nb$	$y = b$
$P_{S_1 \mid Y}(s \mid y)$	0.7	0.2
$P_{S_1 \mid Y}(ns \mid y)$	0.3	0.8
$P_{S_2 \mid Y}(s \mid y)$	0.7	0.2
$P_{S_2 \mid Y}(ns \mid y)$	0.3	0.8
$P_{S_3 \mid Y}(s \mid y)$	0.6	0.3
$P_{S_3 \mid Y}(ns \mid y)$	0.4	0.7
$P_{S_4 \mid Y}(s \mid y)$	0.8	0.1
$P_{S_4 \mid Y}(ns \mid y)$	0.2	0.9
$P_{S_5 \mid Y}(s \mid y)$	0.6	0.3
$P_{S_5 \mid Y}(ns \mid y)$	0.4	0.7

	$x = s$	$x = ns$
$P_{T_1 \mid S_3}(s \mid x)$	0.6	0.3
$P_{T_1 \mid S_3}(ns \mid x)$	0.4	0.7
$P_{T_2 \mid S_2}(s \mid x)$	0.6	0.3
$P_{T_2 \mid S_2}(ns \mid x)$	0.4	0.7
$P_{T_3 \mid S_3}(s \mid x)$	0.55	0.25
$P_{T_3 \mid S_3}(ns \mid x)$	0.45	0.75
$P_{T_4 \mid S_1}(s \mid x)$	0.65	0.3
$P_{T_4 \mid S_1}(ns \mid x)$	0.35	0.7
$P_{T_5 \mid S_2}(s \mid x)$	0.52	0.45
$P_{T_5 \mid S_2}(ns \mid x)$	0.48	0.55

	$y = nb$	$y = b$
$P_{T_1 \mid Y}(s \mid y)$	0.5	0.3
$P_{T_1 \mid Y}(ns \mid y)$	0.5	0.7
$P_{T_2 \mid Y}(s \mid y)$	0.5	0.4
$P_{T_2 \mid Y}(ns \mid y)$	0.5	0.6
$P_{T_3 \mid Y}(s \mid y)$	0.5	0.35
$P_{T_3 \mid Y}(ns \mid y)$	0.5	0.65
$P_{T_4 \mid Y}(s \mid y)$	0.6	0.5
$P_{T_4 \mid Y}(ns \mid y)$	0.4	0.5
$P_{T_5 \mid Y}(s \mid y)$	0.6	0.3
$P_{T_5 \mid Y}(ns \mid y)$	0.4	0.7

Model assessment

- Mutual information measure across X_is and $\Pi_{[i]}$, $i = 1, \dots, d$.

$$MI_{X_i,\Pi_{[i]}} = \sum_{x,y} P_{X_i,\Pi_{[i]}}(x,y) \log \left(\frac{P_{X_i,\Pi_{[i]}}(x,y)}{P_{X_i}(x) \cdot P_{\Pi_{[i]}}(y)} \right)$$

- Conditional mutual information

$$MI_{X,Y|Z} = \sum_{x,y,z} P(x,y,z) \log \frac{P(x,y|z)}{P(x|z)P(y|z)}$$

i	1	2	3	4	5
$MI_{S_i,Y}$	0.1325	0.1325	0.0462	0.2753	0.0462
$MI_{T_i,Y}$	0.0211	0.0051	0.0116	0.0051	0.0463

Table 4. Mutual information $MI_{S_i,Y}$ and $MI_{T_i,Y}$.

$MI_{S_i,T_j\|Y}$	T_1	T_2	T_3	T_4	T_5
S_1	0	0	0	0.0670	0
S_2	0	0.0344	0	0	0.0419
S_3	0.0515	0	0.0484	0	0
S_4	0	0	0	0	0
S_5	0	0	0	0	0

Table 5. Conditional mutual information between T_i and S_j given Y.

T_i	$\Pi[i]$	$MI_{T_i,\Pi[i]}$
T_1	(S_3, Y)	0.0726
	$(S_i, Y), \quad i \neq 3$	0.021
T_2	(S_2, Y)	0.0395
	$(S_i, Y), \quad i \neq 2$	0.0051
T_3	(S_3, Y)	0.060
	$(S_i, Y) \quad i \neq 3$	0.0116
T_4	(S_1, Y)	0.0721
	$(S_i, Y), \quad i \neq 1$	0.0051
T_5	(S_2, Y)	0.0882
	$(S_i, Y), \quad i \neq 2$	0.0463

Table 6. Joint mutual information $MI_{T_i,\Pi[i]}$ for the whole TAN.

Model assessment

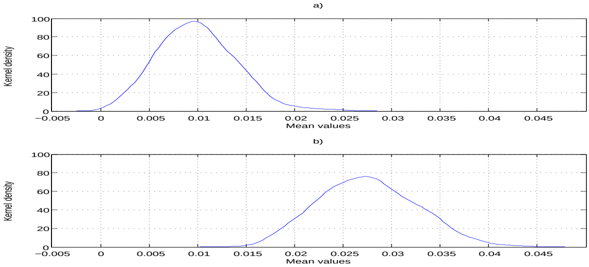


Figure: Kernel density approximation of the average values of $MI_{S,T|Y}$ and $MI_{S,T,Y}$ computed by a normal kernel smoother based on 1000 bootstrap replicates from Table 5 (top panel) and from Table 6 (bottom panel), respectively. the kernel-smoothing window. The bandwidth of the kernel-smoothing window was chosen to be optimal for estimating normal densities. The number of equally spaced points in both bootstrap samples were equal to 100.

"What if" stress scenario

- Posterior bankruptcy risk given S_2 and S_4

$$P_{Y|S_2,S_4}(b|ns,ns) = \frac{P_Y(b)P_{S_2|Y}(ns|b)P_{S_4|Y}(ns|b)}{\sum_{y\in\{b,nb\}} P_Y(y)P_{S_2|Y}(ns|y)P_{S_4|Y}(ns|y)} = 0.923$$

- Posterior bankruptcy risk given T_2 and T_3

$$P_{Y|T_2,T_3}(b|ns,ns) = \frac{P_{T_2|Y}(ns|b)P_{T_3|Y}(ns|b)P_Y(b)}{\sum_{y\in\{b,nb\}} P_{T_2|Y}(ns|y)P_{T_3|Y}(ns|y)P_Y(y)} = 0.6094$$

- Local posterior probability updating

$$P_{S_2|T_2,T_5}(ns|ns,ns) = \frac{P_{T_2|S_2}(ns|ns)P_{T_5|S_2}(ns|ns)P_{S_2}(ns)}{\sum_{x\in\{s,ns\}} P_{T_2|S_2}(ns|x)P_{T_5|S_2}(ns|x)P_{S_2}(x)} = 0.7233.$$

Conclusions and scope for future

- **Advantages of BN in modeling credit concentration risk**
  - Ability to integrate uncertain expert knowledge with data
  - Visual representation
  - Updating prior expert knowledge with new information as it is learned, thereby building a solution of increasing scope and complexity
  - TAN and *k*-BN as classifiers provide a support tool in credit decision making
- **Further steps**
  - Extention to a multinomial graph and more complicated graph structures
  - Extention continuos variables
  - Structure learning/optimization in combination with experts beliefs