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Dear Readers,

Statistika: Statistics and Economy Journal of the Czech Statistical Office celebrates its 98th anniversary this year. Upon this occasion we would like to express our thanks for your continuous interest in this scientific peer-reviewed quarterly.

The journal of Statistika, in its current form, has been issued since 1964 following the tradition of the *Československý statistický věstník* (Czechoslovak Statistical Bulletin), established back in 1920. In the period from 1931 to 1961 the journal was called *Statistický obzor* (Horizons of Statistics) and then in a short period (1962–1963) it was named *Statistika a kontrola* (Statistics and Control). Since 2011, it has been published quarterly in English only (both in print and online versions).

Over the years the journal of Statistika has went through numerous changes following requirements of its readers, visions of the journal board members, and professional interests of its authors. It has become a truly valuable heritage of the Czech Statistical Office, which is carefully looked after by the Editorial and Executive Boards with the objective to strive for its quality and development. We are therefore really proud of the fact that the journal has been indexed in Scopus (since 2015) and in the *ESCI* database of the Web of Science (WoS; since 2016). This year we also expect a full evaluation in the core WoS database. Starting from 2018, the journal of Statistika also newly introduces continuous pagination of the whole volume throughout the entire year.

In 2018, we celebrate the 100th anniversary of foundation of Czechoslovakia. The 100th anniversary of the state official statistics will follow next year (the State Statistical Office was founded in 1919). These events will also be commemorated on the pages of the journal: throughout 2018 and 2019 special papers reminding previous developments in the fields of official statistics as well as essential achievements will be published.

On behalf of the Executive Board of the journal we are looking forward to further cooperation with all the authors bringing results of analyses in the fields of economy, the environment, or social sciences and reflecting the role of official statistics in supporting decision making processes at all levels.

We also believe that the papers published in our journal will continue to be of great value both for the everyday work and professional growth of our readers. We wish all our partners plenty of creative thoughts and professional success.

Marek Rojíček
President of the Czech Statistical Office

Stanislava Hronová
Editor-in-Chief

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About Statistika

The journal of Statistika has been published by the Czech Statistical Office since 1964. Its aim is to create a platform enabling national statistical and research institutions to present the progress and results of complex analyses in the economic, environmental, and social spheres. Its mission is to promote the official statistics as a tool supporting the decision making at the level of international organizations, central and local authorities, as well as businesses. We contribute to the world debate and efforts in strengthening the bridge between theory and practice of the official statistics. Statistika is professional double-blind peer reviewed open access journal included in the citation database of peer-reviewed literature **Scopus** (since 2015), in the **Web of Science** *Emerging Sources Citation Index* (since 2016), and also in other international databases of scientific journals. Since 2011, Statistika has been published quarterly in English only.

Publisher

The Czech Statistical Office is an official national statistical institution of the Czech Republic. The Office's main goal, as the coordinator of the State Statistical Service, consists in the acquisition of data and the subsequent production of statistical information on social, economic, demographic, and environmental development of the state. Based on the data acquired, the Czech Statistical Office produces a reliable and consistent image of the current society and its developments satisfying various needs of potential users.

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Coffee Index as Quick and Simple Indicator of Purchasing Power Parity

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Abstract

Purchasing Power Parity is the corner stone of all international comparisons. Various approaches to Purchasing Power Parity, such as the Big Mac Index, KFC Index, iPad Index, Tall Latte Index or Ikea Index were popularized to the broader audience besides the OECD Purchasing Power Parity indices. The aim of this paper is to develop a new ready-to-use quick and simple index based on the prices of Nespresso coffee's capsules and show the main challenges of such indices as well as of the PPP concept. For the purpose of our research the data on the Nespresso capsules prices were collected. Taking into account also the popularity (demand side) of the capsules types, the Espresso line was chosen as the basis for which all further calculations are made. The Nespresso Index provides us with clear evidence that the Law of One Price cannot work in recent world because of three key features. Firstly, differences in taxes make the perfect parity impossible. Secondly, the price discrimination prevents the rational subjects from arbitrage. Finally, the changes in the exchange rate make such indices highly volatile. Despite these weaknesses, the Nespresso Index could be used as the useful supplement of the OECD PPP as it is low cost, easy and fast to compute and digestible for the lay public.

Keywords

Price index, purchasing power parity, Big Mac Index, Nespresso Index

JEL code

E31, F31

INTRODUCTION

Macroeconomic aggregates are subjects of two contradicting pressures. The first one is based on the competition between countries and regions. Its main aim is to compare and contrast the economic level, economic power, wages or productivity among different economies. Question "Which state will catch up and overtake the USA?" has been a permanent subject of economic discussion since the post-war period. The second pressure, rooted especially in the European Union, seeks to close the gap between member states as well as between individual regions to reach the convergence and achieve homogenous economic area.

For performing such comparisons as well as for responsible decision making on the cohesion and regional policy the comparable macroeconomic data are needed. If the European Commission

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distinguishes i. Less Developed regions, ii. Transition regions and iii. More Developed regions (McCann, 2015) it should be able to compare the Gross Domestic Product (GDP) in all 276 NUTS 2 regions precisely.

Similarly, there is an extensive discussion on the wage differences between EU member states (e.g. Pereira and Galego, 2016), especially between former Western and Eastern Europe. The easiest way – using the nominal exchange rate – is, however, widely seen as inappropriate. As Rojíček et al. (2016) mentions, the exchange rate can convert the indicators to the same currency, but not to the same pricing level. Less developed countries tend to have lower price level due to the lower prices in non-tradable goods (Balassa-Samuelson effect, compare Balassa, 1964; and Samuelson, 1964). According to Lombardo and Ravenna (2012) the non-tradable sector ranges from 21.4% of consumption in Slovakia and 29.75% of investments in Sweden to 78.53% of consumption in the U.S.A. and 69.6% of investments in the Japan. Such part of economy cannot be neglected easily. As Berend noticed "Lower wages have greater purchasing power because of lower general price levels in less-developed countries (Berend, 2009:122)."

The simultaneous problem of currency *and* price level conversion has been, at least since 1918, solved by using the purchasing power parity as introduced by the prominent Swedish economist Gustav Cassel. Cassel developed the original contribution of the 16th century Salamanca School (Rogoff, 1996) and included it into the concept that he summarized as follows: "At every moment the real parity between two countries is represented by this quotient between the purchasing power of money in the one country and the other. I propose to call this parity '*the purchasing power parity.*' As long as anything like free movement of merchandise and a somewhat comprehensive trade between the two countries take place; the actual rate of exchange cannot deviate very much from this purchasing power parity." (Casel, 1918:413).

Purchasing Power Parity (PPP) promptly influenced modern macroeconomics. The Purchasing Power Parity theory became part of the Czechoslovak statistics already in 1929 when it was used as one of the pricing indices principle (see Kohn, 1929). The Czechoslovak government referred to the Cassel's theory as soon as in the middle of the Great Depression (Lidové Noviny, 1936). It lost its importance during the Central Planning as its main condition – the free trade and unbounded flow of goods and service, were not met. Nowadays, the Purchasing Power Parity is the corner stone of all international comparisons. There are numerous conversion rate used by the different institutions, e.g. PPP calculated by the World Bank as local currency unit (LCU) per international US dollar or PPP calculated by OECD for GDP and related indicators.

Various approaches to Purchasing Power Parity were popularized to the broader public by The Economist suggesting trivial Big Mac Index (BMI). The Big Mac Index follows the Law of One Price, examining if the currencies are overvalued or undervalued to the US dollar.

The aim of this paper is to develop a new ready-to-use index based on the prices of *Nespresso* coffee's capsules and show some of the challenges of such indices as well as of the PPP concept. It could be used as a useful supplement of the OECD PPP as it is low cost, easy and fast to compute and digestible for the lay public. Nespresso capsules were chosen because of their absolute homogeneity and tradability. Furthermore, they are fully standardized, being identical in all countries. Coffee market is also in the current decade highly competitive and growing. Nespresso brand was preferred as its data are easily available on-line via national e-shops for 116 territories in all five continents. Nespresso is also the oldest seller of coffee capsules. Together with other large capsules-seller *Dolce Gusto* is brand of *Nestlé*. In comparison to cheaper *Dolce Gusto* Nespresso represents a premium brand with a flavor of luxury. Apart from coffee capsules the Nestle market has been competing also on the market of instant coffee (Nescafé, since 1938) or Coffee Machines. Nestlé therefore stays for 22.3% of the world coffee retail market ahead of the second Jacobs Dough Egberts (16%), and with a huge lead ahead Tchibo (2.3%) or Starbucks (1.4%, compare Statista, 2017).

The rest of the paper is organized as follows. Firstly the Law of One Price, Absolute Purchasing Power Parity as well as its relative version are briefly introduced. In the second chapter OECD PPP and five commercial indices are discussed. Then, data and methodology are described and, finally, the results are presented.

1 PURCHASING POWER PARITY THEORY

The corner stone of the Purchasing Power Parity Theory is the Law of One Price. In this chapter the Law of One Price is defined and its weaknesses are shown. Then the Purchasing Power Parity is introduced.

1.1 Law of One Price

Law of one price assumes, that identical goods must be sold in different countries for the same price (Krugman, Obstfeld and Melitz, 2011).

To apply the Law of One Price, following assumptions must be held:

- goods must be transportable;
- there must not be any barriers of trade such as tariffs and quotas, or they must be included in the price.

The Law of One Price is than formalized as follows:

$$p_i = e \times p_i^*, \tag{1}$$

where p_i is price of i-th goods in the country, e is the exchange rate in direct quotation and p_i^* is the price of i-th goods in the foreign country.

However, Law of One Price does not apply perfectly in the real economy. Miljkovic (1999:126) states that "Although called a law, it has probably been violated more than any other economic law (on the basis of the results of numerous empirical studies)."

As Choi, Laibson and Madrian showed, even prices of highly homogenous products such as index mutual funds are often subject to white noise. It is caused by non-transparent system of management fees (Choi, Laibson and Madrian, 2006). Other obstacle of Law of one price might be information frictions connected to the insufficient technical level (Steinwender, 2014) or differences in language and religion (Fielding, Hajzler and Macgee, 2015). Yet Debreu (1959:29) showed that locations (country borders) as well as dates matter. Furthermore, there is pricing-to-market (Miljkovic, 1999:134), which is based on different elasticities among various countries.

1.2 Purchasing Power Parity

Law of One Price is strong version of Purchasing Power Parity theory (Pakko and Pollard, 2003). According to the Absolute Purchasing Power Parity (APPP) theory the nominal exchange rate is determined by the ratio of the overall price levels between local economy and foreign country (MacDonald, 2007:42). It is based on the bundle of goods and services which makes APPP possible to hold even if Law of One Price had not held for some of the products.

Let's P be the vector of domestic price index, P^* vector of foreign price index and e the direct exchange rate. The APPP then holds, if:

$$e = \frac{P}{P^*}, \tag{2}$$

(compare Dornbusch, 1985:3).

Softer version of APPP, the Relative Purchasing Power Parity (RPPP) assumes that currency in countries with higher inflation tend to depreciate while currency in countries with lower inflation tend to appreciate (Mac Donald, 2007:43).

2 PURCHASING POWER PARITY INDICATORS

Purchasing Power Parity is calculated by the international organizations for the aim of serious statistical comparison as well as by the commercial subjects wishing to present problem of undervaluation and overvaluation in the comprehensible form. In this chapter six various attitudes to the Purchasing Power Parity are introduced.

2.1 OECD PPP

Widely used Purchasing Power Parities are calculated by the Organization for Economic Co-operation and Development (OECD) in line with the EUROSTAT-OECD Methodological manual on Purchasing Power Parities (OECD/Eurostat, 2012). OECD PPPs are constructed in three years cycles and are based on market basket of goods and services. Such basket contains

- 3 000 consumer goods and services – e.g. long-grain rice, Ladies' haircut and coloring;
- 30 occupations in government – e.g. Statistician, Police officer;
- 200 types of equipment goods – e.g. Mercedes Benz;
- about 15 construction projects – e.g. Masonry, ground floor double-skin external wall (OECD/Eurostat, 2012).

Collecting data, their processing and evaluation is naturally lengthy and costly. Moreover, it is not manageable to collect data on prices throughout the whole territory. For example, the Regional Office of the Czech Statistical Office in Brno collects data on the consumer prices just in two South Moravian towns, Brno and Znojmo. It is therefore attractive to have simple, even imperfect, indicator which is always at hand, cheap and easy to use. According to the OECD/Eurostat PPP Manual, purchasing prices for consumer products are being collected in the capital city. If there is data also from other cities and towns, the national prices are calculated as an average. Complex Purchasing Power Parity is therefore challenged by numerous simple commodity indices.

Besides the Big Mac Index there are further indices used for various regions and purposes. In this chapter we briefly introduce the KFC Index, iPad Index, Starbuck's Tall Latte Index and Ikea Index. At the end of the chapter there is a brief comparison of their strengths and weaknesses.

2.2 Big Mac Index

Since 1980' difficulty of PPP estimating had led to numerous attempts to simplify the whole process. The most remarkable indicator, the Big Mac Index, was introduced by magazine *The Economist* in 1986 (Reinert, 2011:235). The Big Mac Index is Exchange Rate Deviation Index (ERDI) in kind, which calculates overvaluation or undervaluation of currency C to the basic currency C^* (often the US dollar). Firstly the Big Mac PPP is calculated as:

$$PPP = \frac{P}{P*}, \tag{3}$$

and then the Big Mac Index is calculated as:

$$BMI = \frac{PPP}{E} \times 100\% \cdot \tag{4}$$

The Big Mac Index is therefore based on the Law of One Price as introduced in section 1.1. Advantage of the Big Mac Index is that it can be easily computed even for the regions within one country. E. g. Fischer and Lipovská (2015) constructed regional Big Mac Index estimating inter-area price levels in the Czech Republic. On the other hand, its price is heavily influenced by the location of McDonald Restaurants, which are usually situated on the busy tourist crossroads (airports, main streets, large department stores). Those locations are characterized by high rent which is translated into the price of final product and service. Moreover, Big Mac is no more a homogenous commodity. For example, in Hindu India the beef is replaced by chicken, in Islamic countries it is halal, while in Israel it is prepared as kosher (Pakko and Pollard, 2003). Finally, just a small count of Big Macs is sold at the menu board price, that is at the price listed on the "menu-board" screens in every McDonald restaurant. Fischer and Lipovská (2015) remind, that most of the Big Macs create part of special bundles together with other goods (such as fizzy drinks or chips) or are subject to special discounts (for example discount for students, quantity discounts or discounts coupons).

2.3 KFC Index

The KFC Index is calculated by pan-African research and market intelligence company *Sagaci*. It is based on the prices of Chicken Buckets which are being sold in KFC (Faramawy, 2016). The KFC Index is inspired by the Big Mac Index, however, McDonald is present just in two African countries, while KFC operated in 18 states. The KFC Index shows how many KFC Original 15 pc. buckets can be purchased in individual African states at 100 USD. If there could have been bought 4 buckets in the USA ($P_\$$ = 25 USD apiece) and 9 buckets in Lesotho (P = 11.33 USD apiece), the Lesotho Loti is undervalued by 55%:

$$\frac{P - P_\$}{P_\$} = \frac{11.33 - 25}{25} \times 100\% = 55\% \cdot \qquad (5)$$

KFC Chicken Buckets also differ across the countries. For example, in Israel they must respect the Jewish kosher rule forbidding combination of meat and dairy (Brady, 2014).

2.4 iPad Index

The iPad Index has been calculated annually since 2007 by Australian broker company *CommSec*. It was designed as a new way of looking at the Purchasing Power Parity theory (Craig, 2016). The main weakness of iPad Index includes shipping costs, taxes as well as fact that iPad is luxury goods in some countries (Thao and Tsanthaiwo, 2017). Moreover, the iPad Index cannot be easily comparable during longer time period, as the new models are launched regularly which makes iPad heterogeneous in time.

2.5 Starbuck's Tall Latte Index

Thanks to the popularity and digestibility of the Big Mac Index *The Economist* started to compute Starbuck's Tall Latte Index in 2004. Then the Starbucks operated in 32 countries, while in 2017 it can be found in 72 countries. At the beginning there were significant differences between the Big Mac Index and Starbuck's Tall Latte Index in the Asian countries. For example yen was undervalued to dollar by 12% according to BMI while it was overvalued by 13% according to STLI (The Economist, 2004). This difference is caused by different composition of both goods.

Similarly to Big Mac or KFC, Starbucks is not the goods, but rather the service. For example between October 2014 and July 2015 the coffee bean prices decreased by 44% while Starbucks prices increased (Sommer, 2015).

Somewhat surprisingly, Starbucks coffee is not homogeneous, but heterogeneous goods, as different amount of caffeine is used for its production in individual countries. For example in USA or Australia the Tall Latte contains 75mg caffeine while in United Kingdom 150mg (Caffeine Informer, 2017).

2.6 Ikea Index

Ikea is the biggest multinational retailer of furniture (Baxter and Landry, 2017). Ikea Index constructed by the Swedish statistician Gabriel Thulin (2004) is the only index presented in this paper, which consists of more than one goods and represents not the law of one price, but also the Absolute Purchasing Power

Parity. It was constructed in 2003 for the basket consisting of 26 Ikea goods in 22 countries (20 European and 2 North-American). Nowadays, Ikea sells its furniture in 38 territories largely in the OECD countries and rest of Europe.

2.7 Comparison of Indices

All indices mentioned in subchapters 2.2–2.6 have several common features: they are based on highly standardized goods being sold by global brand. Theoretically, there should not be any difference between KFC Chicken Buckets sold in Cape Town and Prague or between Starbuck's Tall Latte in Kazakh Almaty and Czech Ostrava. Nevertheless, neither the chicken nor coffee make arbitrage possible. Even if there was free shipping from Almaty to Ostrava, hardly anybody would order his cup of coffee from Kazakh just to save a few pennies. On the contrary, Ikea furniture as well as iPods should be transportable all over the world despite the difficulties with shipping, transport costs, tariffs and quotas of warranty.

Moreover, Starbuck's, McDonald's of KFC's goods would be demanded quiet often by tourists. For example in the Czech Republic the higher prices (and higher turnover) are in the McDonald restaurant placed in the Prague International Airport (see Fischer and Lipovská, 2015). Big Mac Prices therefore mirror the purchasing power not of the local citizens, but of the foreigners.

Table 1 Comparison of indices

	Big Mac	KFC	iPod	Starbucks	IKEA
Data availability	✓	✓	✓	✓	✓
Homogeneity	×	×	✓	×	✓
Transportability	×	×	✓	×	✓
Countries	120	125	worldwide	72	38
Product/Service	Service	Service	Product	Service	Product
Luxury	✓	✓	✓	✓	×

Note: Number of countries, in which the goods is sold, is valid for August 2017 and be subject to continuous changes.
Source: Own elaboration

3 DATA AND METHODOLOGY

In this chapter data and methodology for our Nespresso index calculation are introduced. The first part is devoted to the capsules prices. Further, other data sources are briefly introduced. Finally, the construction of Nespresso Index is explained.

3.1 Nespresso capsules prices

For the aim of our research data on the Nespresso capsules prices were collected. Nespresso produces six basic types of capsules (Nespresso, 2017): Espresso, Pure Origin, Lungo, Intenso, Decaffeinato capsules and Variation. Those capsules differ in intensity, serving size, origin as well as the key flavors and, primarily, in price. In all 30 countries for which we have data available, Espresso is the cheapest line of capsules, together with three brands of Intenso. On the other hand, the most expensive in most of the OECD countries are capsules of Variation line (with caramel, chocolate or vanilla flavor). Figure 1 shows the price categories among the capsules line in most of the OECD countries.

Figure 1 Price classes of Nespresso capsules types

Espresso				Intenso					Pure Origin				Decaffeinato				Lungo				Variations		
I	I	I	I	I	I	I	II	II	II	II	II	II	II	III	III	III	III	III	III	III	IV	IV	IV

Source: Own construction

This structure, however, differs in Canada, USA, Chile, Mexico, New Zealand, Greece, Turkey and Japan (where are only three price classes), Sweden (with five price classes) and Finland and Switzerland where capsules from the typical price class II are being sold in price class IV.

Relations between price classes in individual countries are not same. For example, in Canada class II and I as well as III and II differ identically by 16 US cents, while in Mexico class II differs from I by 12 US cents and class III from II by 23 US cents. Taking into the account also the popularity (demand side) of the capsules types, the Espresso line was chosen as the basic one for which all the further calculations are done.

Nespresso capsules are produced in three Nespresso factories in Switzerland. Romont factory was built up for the need of the North American market in 2015 (Duperret, 2015), older factories were founded in Avenches and Orbe. Those capsules are then exported to the local branches in individual countries. However, capsules are not delivered across the borders, which rules out the possibility of arbitrage. This enables Nespresso to apply the price discrimination among countries depending on local demand. In this point Nespresso Index suffers from the same pain as the Big Mac Index. While Big Mac cannot be transportable as it is rather service than goods, Nespresso is not transportable because of the artificial barriers of trade.

3.2 Exchange rates and VAT

Data on the Value Added Tax (VAT) for the coffee capsules were collected from the websites of the individual responsible ministries.

3.3 Index construction

The Nespresso Index is calculated in the same way as the Big Mac Index. Firstly, the price P of capsules in the i-th country is compared to the price P^* in the basic country (usually USA) converted to the local currency ($P^* \times E$). The direct quotation is used. Secondly, the rate of overvaluation or undervaluation is calculated relatively to the price $P^* \times E$:

$$NI = \frac{P_i - P^* \times E}{P_i^* \times E} \times 100\% \, . \qquad (6)$$

Let's the price of Nespresso capsule in the Czech Republic be $P = 9.9\ CZK$, price in the USA $P^* = 0.70\ USD$ and the exchange rate $E = 22.86\ CZK/USD$. According to the Law of one price the real exchange rate between the Czech crown and the US dollar should be 9.9/0.7 CZK/USD = 14.14 CZK/USD. However, the nominal exchange rate is 22.86 CZK/USD, which means that the Czech crown is undervalued by:

$$\left(\frac{14.14}{22.86} - 1\right) \times 100\% = -38.13\% . \qquad (7)$$

In other words, the Czech crown should, according to the Nespresso Index, appreciate by 61.67% towards the exchange rate for which was the US dollar sold in July 2017.

In the same way also the Nespresso index which takes into account the Value Added Tax was calculated:

$$NI_{VAT} = \frac{(P_i - VAT)_i - (P^* - VAT^*) \times E}{(P_i^* - VAT^*) \times E} \times 100\% \, . \tag{8}$$

The Value Added Tax is considered to verify the validity of the Law of one price. It is valid just for the basic prices, the purchaser prices should be therefore calculated in term of the basic prices as well. For the aim of our paper the results for the Big Mac Index published on the 13th July 2017 were chosen (The Economist, 2017). Full data set published by The Economist contains also the exchange rates to US dollar which we used for the calculation of the Nespresso Index.

4 RESULTS

In this section the results are presented. Firstly, we will compare and contrast the Nespresso Index with the Big Mac Index. Then, we will focus on the role of the Value Added Tax. In the third section the high variability of the Nespresso Index in time is shown. Finally, the cross-country comparison in Eurozone is discussed.

4.1 Nespresso Index vs. Big Mac Index

Nespresso Index for OECD countries is closely correlated to the Big Mac Index as well as to the Purchasing Power Parity calculated by OECD (see Table 2). Nevertheless, for most of the countries Nespresso Index shows greater undervaluation of the domestic currency toward US dollar than the Big Mac Index. There is significant exception of Chile, Turkey and New Zealand. Currencies in these countries are undervalued according to the Big Mac Index but overvalued according to the Nespresso Index.

Differences between the Big Mac Index and the Nespresso Index in Japan and Mexico are remarkable. Both indices consistently suggest that Japanese yen as well as Mexican peso are undervalued. This undervaluation is, however, somewhat deeper according to the Big Mac Index.

On the other hand, the Swiss franc is being undervalued according to the Nespresso Index and overvalued according to the Big Mac Index. It might be explained by the historic development of the Nespresso brand. Nespresso was founded in Switzerland. To date, it has its headquarter in Swiss Lausanne. Switzerland also traditionally belongs (together with France, Spain and Germany) to the most important Nespresso's markets (Daneshkhu, 2013).

Table 2 Correlation matrix

NI	BMI	PPP OECD	
1.00	0.98	0.96	**NI**
	1.00	**0.99**	**BMI**
		1.00	**PPP OECD**

Source: Own calculation

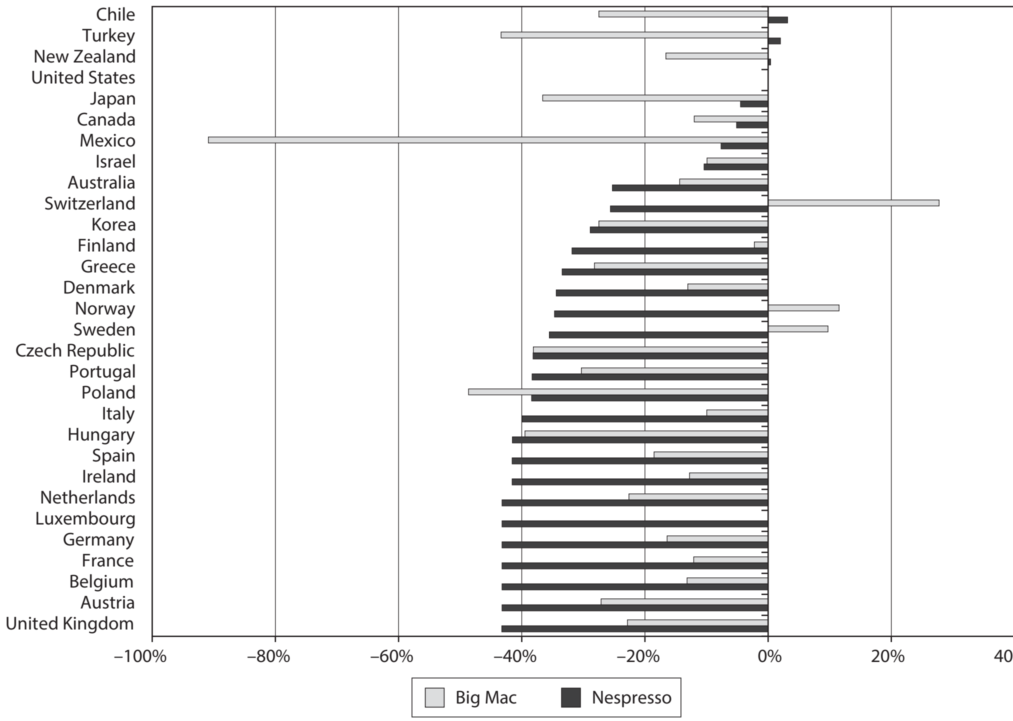
4.2 Nespresso Index regarding Value Added Tax

Value Added Tax (VAT) represents other obstacle of perfect Law of One Price. Even if the capsules were sold from factory in Switzerland to two countries at the very same price, higher VAT in one country would result in lowering the purchasing power and therefore to the greater undervaluation of the currency (compare Baxter and Landry, 2017).

Let's take Austria, Luxembourg and Germany as an example. All those countries use euro currency and the price, at which the capsules are sold on their market, is the same. However, the VAT on the Nespresso capsules in Austria is 20% while in Luxembourg it is only 3% and in Germany 7%. Therefore, while the Law of One Price holds perfectly taking VAT into account, the situation largely differs if we disregard VAT: Austrian euro is according to the Nespresso Index without VAT undervalued towards German euro by 11% while Luxembourgian euro is overvalued towards German euro by 4%.

With the Value Added Tax taken into account, the Nespresso Index suggests even deeper undevaluation of individual currencies towards US dollar. This is therefore the Value Added Tax what narrows the gap in the purchasing power between certain countries (compare Figure 3).

Figure 2 The Big Mac Index and the Nespresso Index comparison (currency undervaluation/overvaluation in %)



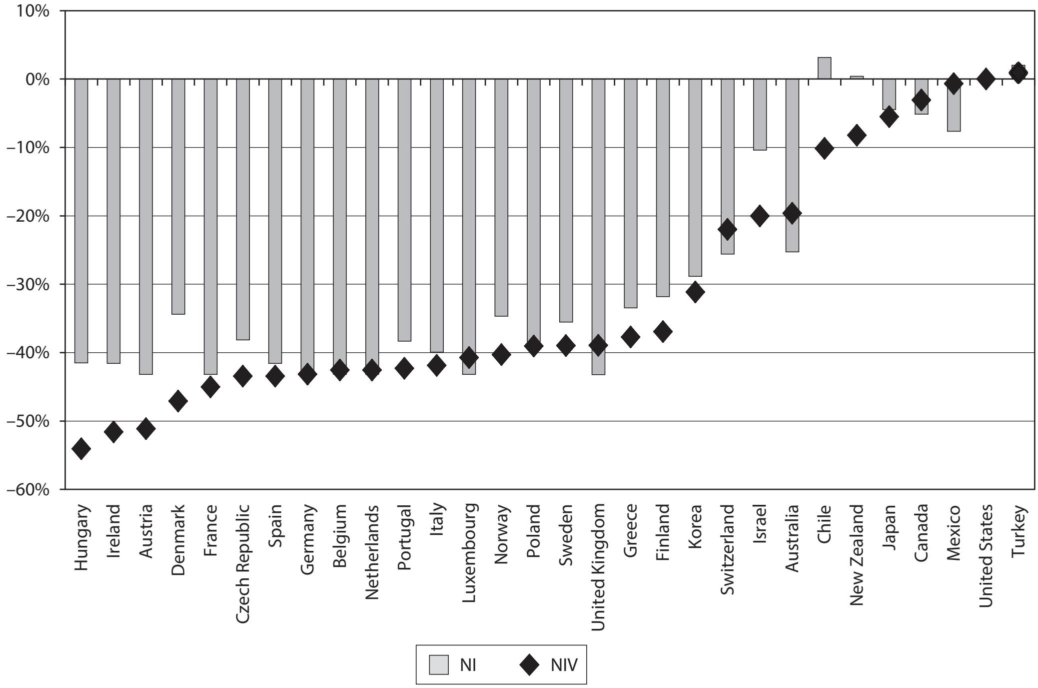
Source: Own construction

Table 3 The Nespresso Index in respect to the Value Added Tax

	P [€]	PPP	NI [%]	VAT [%]	P´ [€]	PPP´	NI´ [%]
Austria	0.35	1	0	20	0.29	0.89	–11
France	0.35	1	0	6	0.32	0.97	–3
Germany	0.35	1	0	10	0.33	1.00	0
Belgium	0.35	1	0	7	0.33	1.01	1
Netherlands	0.35	1	0	3	0.33	1.01	1
Luxembourg	0.35	1	0	6	0.34	1.04	4

Source: Own construction

Figure 3 The Nespresso Index and the Nespresso Index with VAT



Source: Own construction

4.3 Nespresso Index in the flow of time

Prices of goods as Big Mac or Nespresso tend not to be a subject of frequent changes. In the Czech Republic the price of Nespresso capsules (line Espresso) was fixed for the whole reporting period (from January 2017 to July 2017). However, the exchange rate towards euro changed significantly because of the exchange regime change. The Czech central bank intervened on the currency market till 6th April 2017 as it aimed at the exchange rate 27 CZK/EUR. After the end of interventions, the Czech crown started to fluctuate relatively to the euro. At a fixed price the Nespresso Index fluctuated in the opposite direction than the exchange rate. If the Czech crown appreciated, the Nespresso Index decreased as the crown shrunk its pace of undervaluation towards euro. Literally overnight the Czech crown changed according to the Nespresso Index from the undervaluated to the slightly overvaluated currency without the change of real prices between Czech Republic and Eurozone (representented by Germany) in terms of Nespresso capsules.

4.4 Cross-country comparison in Eurozone

Comparison of the cross-country Nespresso Index for Eurozone countries (Table 3) reveals that the member states are clustered into two different groups. For the first one, represented by Austria, Belgium, France, Germany, Luxembourg and Netherlands, the common currency is too weak. On the contrary, for the second group, represented by PIIGS countries (Portugal, Italy, Ireland, Greece, Spain) and Finland is the euro too strong which slows down their international trade and makes worse the balance of trade. All of them except for the Ireland had deficit of the balance of trade. On the other hand, the Irish Republic is a specific small open economy with Gross Domestic Product being 31% higher in comparison to the Gross National Product.

Nespresso capsules are totally standardized as produced on the same place for the whole world. Moreover, the demand is not influenced by the tourism. Nespresso prices does not depend on the "bundling". While Big Mac prices relatively differs according to the type of McDonald's menu or system of individual discounts (e.g. for students), the Nespresso prices tend to be fix. Finally, the capsules can be ordered via e-shop which prevents the price differentiation within one country (there are also Nespresso Boutiques, however, they are situated just in a few big cities. For example in the Czech Republic there are just three boutiques – in Prague and Brno).

Table 4 Eurozone cross-country Nespresso Index [€]

		Austria 0.35	Belgium 0.35	Finland 0.42	France 0.35	Germany 0.35	Greece 0.41	Ireland 0.36	Italy 0.37	Luxembourg 0.35	Netherlands 0.35	Portugal 0.38	Spain 0.36
Austria	0.35		0	+20	0	0	+17	+3	+6	0	0	+9	+3
Belgium	0.35	0		+20	0	0	+17	+3	+6	0	0	+9	+3
Finland	0.42	–17	–17		–17	–17	–2	–14	–12	–17	–17	–10	–14
France	0.35	0	0	+20		0	+17	+3	+6	0	0	+9	+3
Germany	0.35	0	0	+20	0		+17	+3	+6	0	0	+9	+3
Greece	0.41	–15	–15	+2	–15	–15		–12	–10	–15	–15	–7	–12
Ireland	0.36	–3	–3	+17	–3	–3	+14		+3	–3	–3	+6	0
Italy	0.37	–5	–5	+14	–5	–5	+11	–3		–5	–5	+3	–3
Luxembourg	0.35	0	0	+20	0	0	+17	+3	+6		0	+9	+3
Netherlands	0.35	0	0	+20	0	0	+17	+3	+6	0		+9	+3
Portugal	0.38	–8	–8	+11	–8	–8	+8	–5	–3	–8	–8		–5
Spain	0.36	–3	–3	+17	–3	–3	+14	0	+3	–3	–3	+6	

Source: Own construction

CONCLUSION

The Nespresso Index provides us with a clear evidence, that the Law of one Price cannot work in recent world and that the theory of the Parity of Purchasing Power is connected to serious weaknesses. First of all, the differences in taxes make the perfect parity impossible. In this paper only the Value Added Tax was taken in account, however, the total bundle of all taxes might just confirm this tendency.

Furthermore, even if the goods is perfectly tradable in theory (and even if there were no tariffs and quotas) the multinational producers might create their own barriers of trade to nobble customers ordering goods from different localities. Such strategy enable companies performing price discrimination, however, it prevents the rational subjects from arbitrage.

Thirdly, there is great disproportion between individual price changes among as well as within the markets. While the goods prices are quite stable, changing just in the jumps, the price of currency – exchange rates – is a subject of the monetary policy as well as of the demand and supply on the currency markets. Therefore, the same price index might indicate both the overvaluation as well as undervaluation of currency in the very short time period.

If the Law of One Price holds for every commodity, even the absolute Purchasing Power Parity holds. However, we have seen that Law of One Price does not hold for Nespresso capsules. From the similar reasons as listed above, it would be very difficult to ensure, that Purchasing Power Parity holds for the basket of goods and services. The lower level of prices might be caused not only by the lower purchasing power, but also by the pricing strategies of the multinational corporations which maximize profit due to the price discrimination.

All weaknesses of the one-commodity price indices apply also on the composite indices (e.g. OECD PPP). However, the large bundle of goods and services diminishes the amount of errors and may produce more precious results. However, despite the problems connected to the Nespresso Index, it is sufficiently precious to uncover structural problems of the Finnish economy and to highlight other special features. As it is low cost, easy to compute and digestible for the lay public, and as it is not burdened by the Big Mac's weaknesses, it could be used as the useful supplement of the OECD PPP.

Our ambition was not to create new, ideal, competitive Purchasing Power Parity Index which would replace formal OECD PPP. Instead, we wished to perform a kind of mental experiment which would help to reveal further weaknesses of the standard approach to PPP estimates. Purchasing Power Parity is of vital importance for the international comparison; however, our ability to achieve accurate estimates is still seriously limited. Our Coffee Index is therefore not the solution of the problem, but rather a challenge for further methodological work.

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Work Intensity of Households: Multinomial Logit Analysis and Correspondence Analysis for Slovak Republic

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Abstract

Exclusion from the labour market is a serious social problem that is also addressed by the Europe 2020 strategy. While in the past the attention of statisticians and sociologists in the fight against poverty and social exclusion has concentrated mainly on income poverty and material deprivation, in recent times many studies and analyses are much more focused on work intensity as well. Households that use their work potential to less than 20%, have a very low work intensity, and members of such households are included into the population of people who are at risk of poverty or social exclusion. Moreover, the low use of labour potential of households significantly increases the risk of income poverty and the threat of material deprivation. This article provides an analysis of work intensity levels of Slovak households depending on the factors that are monitored by the EU-SILC 2015. The impact of relevant factors is quantified by correspondence analysis and by multinomial logistic regression model.

Keywords	**JEL code**
Work intensity, poverty and social exclusion, EU-SILC – European Union statistics on income and living condition, correspondence analysis, multinomial logistic regression	*I31, J21, C31*

INTRODUCTION

One of the fundamental objectives of the Europe 2020 strategy is to reduce poverty and social exclusion in the EU and its member states. In order to monitor the achievement of this goal, an aggregated indicator measuring the risk of poverty or social exclusion (AROPE) was created. The key objective in the social field

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is to pull 20 million people out of the risk of poverty or social exclusion by 2020 compared to the year 2008, while people are considered to be at risk of poverty or social exclusion if they are at risk of income poverty and / or are materially deprived and / or living in households with very low work intensity. Therefore, the methodology for measuring poverty and social exclusion used in the Europe 2020 strategy, is based on a 3-dimensional concept with the following dimensions: income poverty, material deprivation and exclusion from the labour market.

The article focuses on the third dimension, which is mapped through the work intensity. Work intensity reflects the extent to which the working potential is used in the household, in other words, how much of the theoretically available work time (set in the legislation of the country) the household members in productive age actually work. The household work intensity can have value from 0 (or 0% – no one works) up to 1 (or 100% – all members work). On this basis, the indicator "Share of people living in very low work intensity households" was created, it refers to people aged 0–59 living in households where adults aged 18–59 work less than 20% of their overall work potential. The Europe 2020 strategy mainly tracks very low work intensity and, therefore, the professional and scientific community monitors and analyses the intensity of work of people and households, mainly through measuring very low work intensity. Despite the fact that several scientific publications point out the weaknesses of this indicator, e.g. Ward and Ozdemir (2013) and the delayed disclosure of its estimated values in connection with the complicated nature of EU-SILC micro-data collection and processing (Rastrigina et al., 2015), the very low work intensity rate and the measurement methodology[4] of personal or household work intensity provide important internationally comparable information about exclusion from the labour market. Several authors include unemployment rate and very low work intensity rate among relevant indicators of economic welfare in the countries. For example, Monte et al. (2017) applied very low work intensity rate for monitoring the development of welfare in Europe between 2007 and 2014, while using the cluster analysis. On the basis of the analyses, the Slovak Republic entered together with the Czech Republic, Hungary and Slovenia in a joined cluster in all the years under review, and in 2014 the cluster of these countries reached a relatively low rate of very low work intensity.

This article does not only focus on very low work intensity but our goal is to provide a comprehensive analysis of work intensity of Slovak households, not only in terms of the incidence and risk of very low work intensity, but with regard to all degrees of work intensity.[5] To achieve this goal, we use correspondence analysis and logistic regression in this article. Logistic regression is a popular statistical tool in work intensity analysis and was also used by Mysíková et al. (2015) who demonstrated that work intensity in the SR and the Czech Republic during the period 2006–2013 significantly influenced the risk of income poverty. Through the logistical regression Kis and Gábos (2016) confirmed the significant impact of work intensity on consistent poverty in the EU, Hick and Lanau (2017) quantified the impact of selected factors on in-work poverty and examined the impact of risk factors on very low work intensity in Ireland. We would like to note that Ireland has a long-term history of the highest incidence of very low work intensity in the EU (see: Šoltés and Šoltésová, 2016; Monte et al., 2017). Considering this fact, it is not surprising that a higher number of scientific publications focus on this problem in this country, for example, (Redmond, 2016), (Whelan and Maître, 2014) or (Logue and Callan, 2016). Several of the above-mentioned scientific publications show that many studies focus on analysing mutual relationship between the different dimensions of the concept of measuring poverty and social exclusion.

The use of labour potential in many EU countries significantly affects the occurrence and the risk of poverty as well as material deprivation. Ayllón and Gábos (2015) confirmed the relationship between severe material deprivation and low work intensity in Central and Eastern Europe, whereas in other parts of Europe

[4] See: <http://ec.europa.eu/eurostat/statistics-explained/index.php/EU_statistics_on_income_and_living_conditions_ EU-SILC)_methodology_%E2%80%93_concepts_and_contents#Work_intensity_.28WI.29>.

[5] See: <http://ec.europa.eu/eurostat/statistics-explained/index.php/EU_statistics_on_income_and_living_conditions_ (EU-SILC)_methodology_-_definition_of_dimensions>.

they did not demonstrate such dependence. The strong positive relationship between very low work intensity and poverty has been quantified by the authors in all analysed countries. Guagnano et al. (2013) has again revealed through correspondence analysis that work intensity is one of the major socio-economic factors influencing the perception of subjective poverty in Europe.

1 METHODS OF ANALYSIS

The article focuses on the degree of work intensity of Slovak households, which is recorded in the form of a multinomial categorical variable for individual statistical units (households). In regard to the character of the target variable and the goal of this article stated in the introduction, the results presented in this article are achieved by means of correspondence analysis and analysis of the multinomial logistic model. This part of the article provides a brief description of the methodology of these sophisticated mathematical and statistical methods.

1.1 Correspondence analysis

Correspondence analysis is a method that is based on the analysis of the structure of mutual dependencies of two or more variables. Because it focuses on examining the dependence of predominantly nominal or ordinal variables, in the case of a continuous variable, it is necessary to categorize its values. It solves this problem in a similar way as factor analysis or the principal component method, while hidden or latent variables can be represented as axes of the reduced coordinate system (correspondence maps), in which the individual categories of variables will eventually be displayed. This is a method that in its essence belongs to exploration methods, and can be a good instruction for deciding which categories of variable should be merged and which can be kept separate. It is mainly used in marketing, but its interesting applications are also found in other areas.

In the case of a simple correspondence analysis (Řezanková, 2007; Meloun and Militký, 2012), we deal with a two-dimensional contingency table. From the values of this table (n_{ij}) we can deduce the correspondence matrix P with the elements p_{ij}

$$p_{ij} = \frac{n_{ij}}{n}, \tag{1}$$

where $i = 1, 2, ..., r$ and $j = 1, 2, ...$ s.

Row marginal relative frequencies p_{i+} are called row loads (r_i), with their line percentages being referred to as row profiles. Similarly, column marginal relative frequencies p_{+j} are called column loads (c_j), with their column percentages being referred to as column profiles. The whole correspondence matrix can be schematically expressed as follows:

$$\begin{bmatrix} \mathbf{P} & \mathbf{r} \\ \mathbf{c}^{\mathrm{T}} & 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1s} & r_1 \\ p_{21} & p_{22} & \cdots & p_{2s} & r_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ p_{r1} & p_{r2} & \cdots & p_{rs} & r_r \\ c_1 & c_2 & \cdots & c_s & 1 \end{bmatrix}, \tag{2}$$

where **c** is the s-elements vector of the column loads and **r** is the r-elements vector of the row loads.

Each row (column) of the correspondence matrix can be represented as a point in a s-dimensional (r-dimensional) space with coordinates corresponding to the values of the respective profiles. We can then calculate the distances between individual points, while the most commonly used is the chi-squared distance between the i-th and the i'–th line produced by the relation:

$$\chi^2 = \sqrt{\sum_{j=1}^{s} \frac{\left(r_{ij} - r_{i'j}\right)^2}{c_j}}\ , \tag{3}$$

where r_{ij} are the elements of the row profiles matrix $\mathbf{R}$ and c_j weights correspond to the elements of the column load vector $\mathbf{c}^T$. Similarly, we proceed in computing the differences (dissimilarities) between column categories.

The goal of the method **i**s to reduce the multidimensional space of vectors of row and column profiles, while maximally preserving the information contained in the original data. Usually, a two-dimensional space is used, i.e. plane. The point that lies directly in the plane and is closest to the corresponding point in space is called projection. The solution comes from a matrix **Z** of standardized residuals with elements:

$$z_{ij} = \frac{p_{ij} - p_{i+}p_{+j}}{\sqrt{p_{i+}p_{+j}}} \tag{4}$$

and its singular decomposition according to relationship:

$$\mathbf{Z} = \mathbf{U} \cdot \mathbf{\Gamma} \cdot \mathbf{V}^T, \tag{5}$$

where $\mathbf{\Gamma}$ is the diagonal matrix and where the relationship $\mathbf{U}^T\mathbf{U} = \mathbf{V}^T\mathbf{V} = \mathbf{I}$ applies.

Prior to the estimation of the co-ordinates of each category, the choice of the normalization method should be made, i.e. the way to show points in the correspondence map. The so-called symmetric normalization is most commonly used in which we are interested in the mutual comparison of both row and column categories. In interpreting the results, the points are considered closer when there is a higher similarity between the corresponding categories.

1.2 Multinomial Logit Analysis

The logistic regression model is a special case of the general linear model (see: Ramon et al., 2010) and serves to model the categorical dependent variable depending on the explanatory variables of the continuous or categorical type. The binary logistic regression uses the logarithmic transformation of the odds of probability p for the desired event to occur ($y_i = 1$; the event that is being examined) to the probability $1-p$ of occurrence of the undesired event ($y_i = 0$). The natural logarithm of the odds is called logit and, unlike probability p, acquires any real values and can be modelled by a linear regression model (Stankovičová and Vojtková, 2007):

$$\text{logit}\left(p_i\right) = \ln \frac{p_i}{1-p_i} = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_k x_{ik}\ , \tag{6}$$

where p_i is the probability, so that $y_i = 1$ ($i = 1, 2, \ldots, n$), then $\beta_0, \beta_1, \ldots, \beta_k$ are the parameters of the logit model and $x_{i1}, x_{i2}, \ldots, x_{ik}$ are the values of the explanatory variables $X_1, X_2, \ldots, X_k$ which are observed for the i-th statistical unit. To obtain maximum likelihood estimators of parameters of the logistic regression model the Newton-Raphson algorithm is generally used (see: Allison, 2012).

After estimating the logistic model, it is important to verify its statistical significance and also verify whether the influence of the individual explanatory variables on probability p is significant. The significance of a logistic regression model is performed by a zero hypothesis test $\boldsymbol{\beta}^T = (\beta_1, \beta_2, \ldots, \beta_k) = \mathbf{0}^T$ against an alternative hypothesis – at least one regression coefficient is non-zero, while three different test chi-square statistics are mostly used (Likelihood ratio, Score statistics, Wald statistics). Allison (2012) discusses the differences between these statistical methods and at the same time notes that in large samples, there is no reason to prefer any of these statistics and they will generally be quite close in value.

In order to validate the significance of the explanatory variable influence, a Wald test is used. It tests the zero hypothesis showing that the respective explanatory variable does not affect the probability of occurrence of the explored event. To verify the hypothesis, Wald statistic:

$$Wald = \hat{\boldsymbol{\beta}}^{\mathrm{T}} \cdot \mathbf{S}_{\mathbf{b}}^{-1} \cdot \hat{\boldsymbol{\beta}}, \tag{7}$$

is used, where $\hat{\boldsymbol{\beta}}$ is the vector of regression coefficients estimates that stand at dummy variables for the respective factor (categorical explanatory variable) and $\mathbf{S}_{\mathbf{b}}$ is the variance-covariance matrix of $\hat{\boldsymbol{\beta}}$. Wald statistic has asymptotically χ^2 distribution with degrees of freedom equal to the number of parameters estimated for a given effect. A special case of the above test is the Wald test, which verifies the statistical significance of one regression coefficient. In this case Wald statistics is asymptotically distributed as χ^2 with 1 degree of freedom. The test statistic has an equation:

$$Wald = \frac{\hat{\beta}_i}{s_{\hat{\beta}_i}}, \tag{8}$$

where $s_{\hat{\beta}_i}$ is an estimated standard error of the i-th estimated coefficient.

The quality of the logistic model can be evaluated by different measures. Among criteria that measure a relative quality of statistical models belong AIC – Akaike Information Criterion and SC – Schwarz-Criterion, which are based on the logarithmic transformation of the likelihood function, i.e. –2ln$L$.

Binary logistic regression is used, if the explanatory variable is binomial. If the dependent variable has more than 2 categories (generally these are s categories), we can use a multinomial logit model that is created by (s – 1) logit functions:

$$\begin{aligned}
\ln\left[\frac{P(Y=1|\mathbf{x})}{P(Y=0|\mathbf{x})}\right] &= \beta_{10} + \beta_{11}x_{i1} + \beta_{12}x_{i2} + \cdots + \beta_{1k}x_{ik} \\
\ln\left[\frac{P(Y=2|\mathbf{x})}{P(Y=0|\mathbf{x})}\right] &= \beta_{20} + \beta_{21}x_{i1} + \beta_{22}x_{i2} + \cdots + \beta_{2k}x_{ik} \\
&\vdots \\
\ln\left[\frac{P(Y=(s-1)|\mathbf{x})}{P(Y=0|\mathbf{x})}\right] &= \beta_{(s-1)0} + \beta_{(s-1)1}x_{i1} + \beta_{(s-1)2}x_{i2} + \cdots + \beta_{(s-1)k}x_{ik}
\end{aligned} \tag{9}$$

The effect of the explanatory variable X_j on the dependent variable Y is quantified in logistic regression by the odds ratio (OR – odds ratio) estimated by the formula:

$$OR_j = e^{\hat{\beta}_j}. \tag{10}$$

The odds ratio in binary logistic regression expresses how the odds will change: Y = 1 compared to the odds that Y = 0, in unit growth of the explanatory variable in conditions ceteris paribus. If the explanatory variable is a dummy variable, the odds ratio compares the odds of occurrence of an event at two different levels of the predictor. In the case of multinomial logistic regression, the odds ratio interpretation is analogous to that of binomial logistic regression, we only have to consider which logit equation from Formula (9) we should take into account, and, therefore, which pair of categories of the multinomial explanatory variable we should compare (most often it is l vs. 0, where l = 1, 2, ..., (s – 1)).

2 DATABASE

The analyses presented in this article are based on the EU-SILC 2015 database provided by the Statistical Office of the Slovak Republic which covers the 2014 reference period. According to the methodology used by Eurostat to monitor labour market exclusion as one of the dimensions of poverty and social exclusion, work intensity is divided into 5 categories (Table 1). For the purpose of analysing the work intensity of Slovak households, we created a categorical variable *WI* (Work Intensity) whose variations (0 to 4) express the severity of the reduced use of households' work potential, as shown in Table 1.

Table 1 Levels of household work intensity

Level of work intensity	Value ranges of work intensity index	Category designation	Abbreviation
Very low	⟨0; 0.2)	4	VLWI
Low	⟨0.2; 0.45)	3	LWI
Medium	⟨0.45; 0.55⟩	2	MWI
High	(0.55; 0.85⟩	1	HWI
Very high	(0.85; 1⟩	0	VHWI

Source: Eurostat, own processing

According to the EU-SILC 2015 surveys in the reference year 2014, households with very high labour intensity dominated in Slovakia. There were 56.2% of such households (households that use more than 85% of their work potential) in the selected sample and 93% of them used their work potential up to 100%. 15.4% of households in this sample showed a medium degree of work intensity. The lowest number of households (3.9%) had a low level of work intensity (we included households that use their work potential to at least 20% but not more than 45% into this group). Almost every thirteenth household had to face a very low work intensity, respectively in 7.5% of households we report the use of work potential to less than 20%. Although Slovakia was below the average EU-28 very low work intensity rate within EU-28 in 2015 (while according to the EU-SILC 2015, 7.1% of the population aged 0–59 lived in very low intensity households in Slovakia (Vlačuha and Kováčová, 2016) and in the EU-28 according to Eurostat[6] there were 10.6% of such households), it should be noted that the large majority of households with very low work intensity in Slovakia did not demonstrate any work activity. In our sample, up to 92% of households with very low work intensity had zero use of labour potential throughout the whole reference period. This situation was caused by a high rate of unemployment of 11.5% in the SR in 2015 which was the 7th highest unemployment rate in the EU-28 (after Greece, Spain, Croatia, Cyprus, Portugal and Italy) while the EU-28 unemployment rate was 9.4%. We would like to note that in 2016 the unemployment rate in Europe declined substantially (by 0.9 pp in EU-28), and one of the largest drops in unemployment rate, by 1.9 pp., was recorded in Slovakia (see: Eurostat, 2017).

On the basis of a number of scientific publications (in particular those listed in the introduction to the article) and on the basis of our own experience, we have assumed that the level of household work intensity is affected by these variables observed in the EU-SILC survey: status of economic activity, the highest level of education achieved, the marital status and age of the head of household as well as the type of household, region and degree of urbanization, respectively. The population density on the territory where the household resides. For better clarity, we used custom names for variables and their variations (categories) in the analyses. Because the numbers of households in some categories were low, we have

6 See: <http://ec.europa.eu/eurostat/tgm/refreshTableAction.do?tab=table&plugin=1&pcode=t2020_51&language=en>.

merged them with similar categories of the relevant factor. The description of the input variables[7] and the above mentioned changes in the names and in the definition of the categories of these variables are captured in Table 2.

Table 2 Description of input explanatory variables

Original variables (EU-SILC) – categories and description		Names of new dummy variables
	RB210 – Status of basic economic activity	**EAS**
1	employed	at work
2	unemployed	unemployed
3	old-age pensioner, early retirement pensioner	retired
4	other inactive person	inactive person
	PE040 – The highest level of education achieved (ISCED)	**EDUCATION**
0	less than primary	Less_than_Secondary
1	primary	
2	lower secondary	
3	upper secondary	Upper_Secondary
4	post secondary (not tertiary)	Post_Secondary
5	short cycle of tertiary education	Tertiary_1
6	bachelor education	
7	master's degree or equivalent	Tertiary_2_3
8	doctoral education or its equivalent	
	HT – Type of household	**HT**
5	single-person household	1adult
6	Household 2 adults, both aged 65 years	2a_0ch
7	Household of 2 adults, at least 1 at age 65+	2a_1r
8	Other households without dependent children	other_0Ch
9	Household 1 parent with at least 1 dependent child	1a_at_least_1ch
10	Household of 2 adults with 1 dependent child	2a_1ch
11	Household of 2 adults with 2 dependent children	2a_2ch
12	Household of 2 adults with 3+ dependent children	2a_at_least_3ch
13	Other households with dependent children	other_with_ch
	PB190 – Marital Status	**MARITAL STATUS**
1	Single	single
2	Married	married
4	Widowed	widowed
5	Divorced	divorced
	DB100 – Degree of urbanisation	
1	region with dense population	dense
2	region with overall dense population	intermediate
3	region with sparse population	sparse

Source: EU-SILC 2015, own processing

[7] For a correct interpretation it is necessary to take into account the description of relevant variables that is stated on the website: <http://ec.europa.eu/eurostat/web/income-and-living-conditions/methodology/list-variables>.

Table 2 Description of input explanatory variables (continuation)

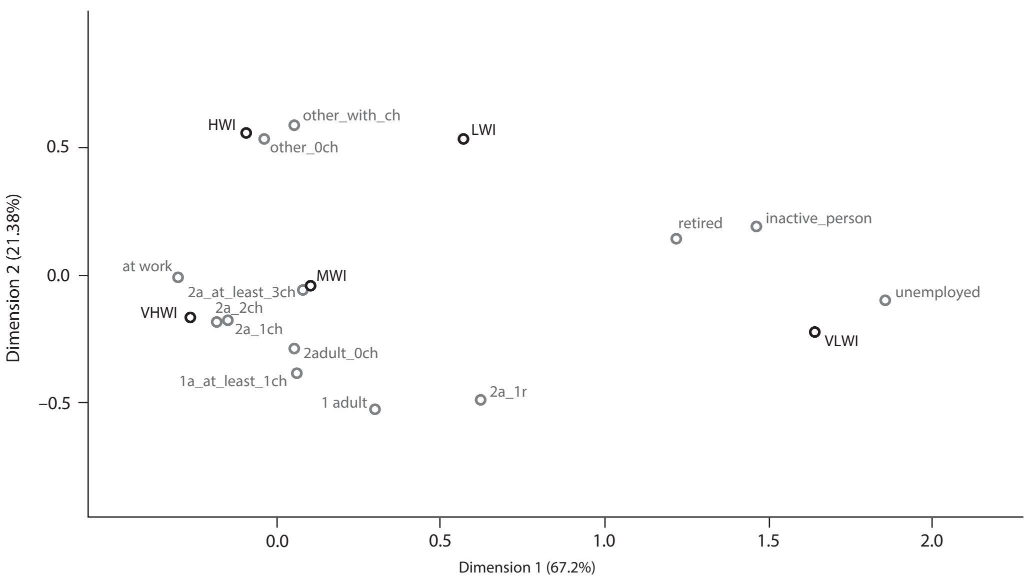
Original variables (EU-SILC) – categories and description		Names of new dummy variables
	Region	**REGION**
1	Bratislavský	BA
2	Trnavský	TT
3	Trenčiansky	TN
4	Nitriansky	NR
5	Žilinský	ZI
6	Banskobystrický	BB
7	Prešovský	PE
8	Košický	KE

Source: EU-SILC 2015, own processing

3 ASSESSMENT OF WORK INTENSITY OF SLOVAK HOUSEHOLDS WITH THE USE OF CORRESPONDENCE ANALYSIS

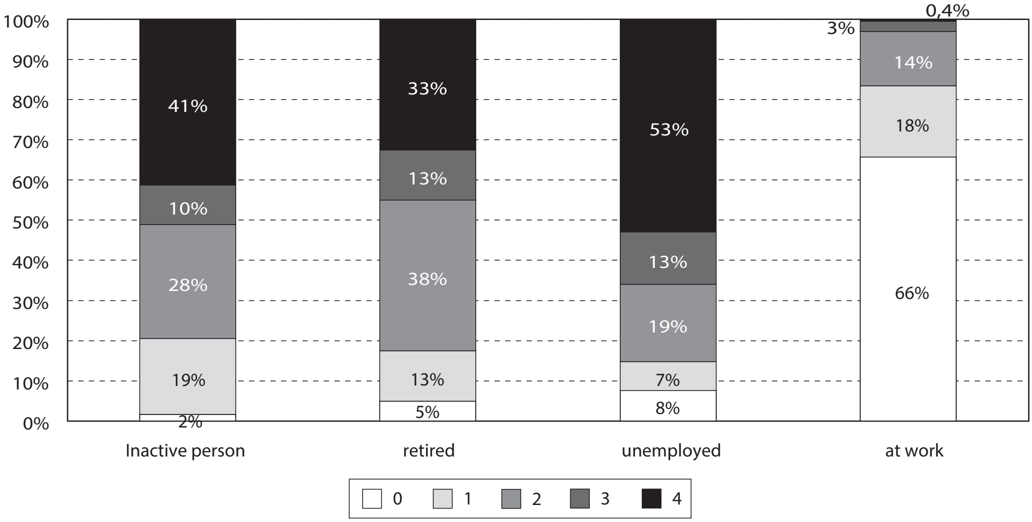
On the basis of correspondence analysis (Figure 1) and the occurrence of individual levels of work intensity of Slovak households in the sample (Figure 2), we can assert that very high labour intensity is associated especially with the households headed by the employed person. On the other hand, very low work intensity is typical for households with unemployed head of household. Correspondence analysis results confirm that "retired" and "inactive person" households in the reference year 2014 were slightly better off than households with the unemployed head. For households headed by the otherwise inactive person and for the households headed by a retired person, the very low work intensity is not as typical, and in particular, the "retired" household group is more strongly associated with low and medium work intensity compared to households with unemployed head of household.

Figure 1 Correspondence analysis of work intensity of Slovak households for factors, such as the economic activity of the household head and the household type



Source: EU-SILC 2015, customized in SAS BASE

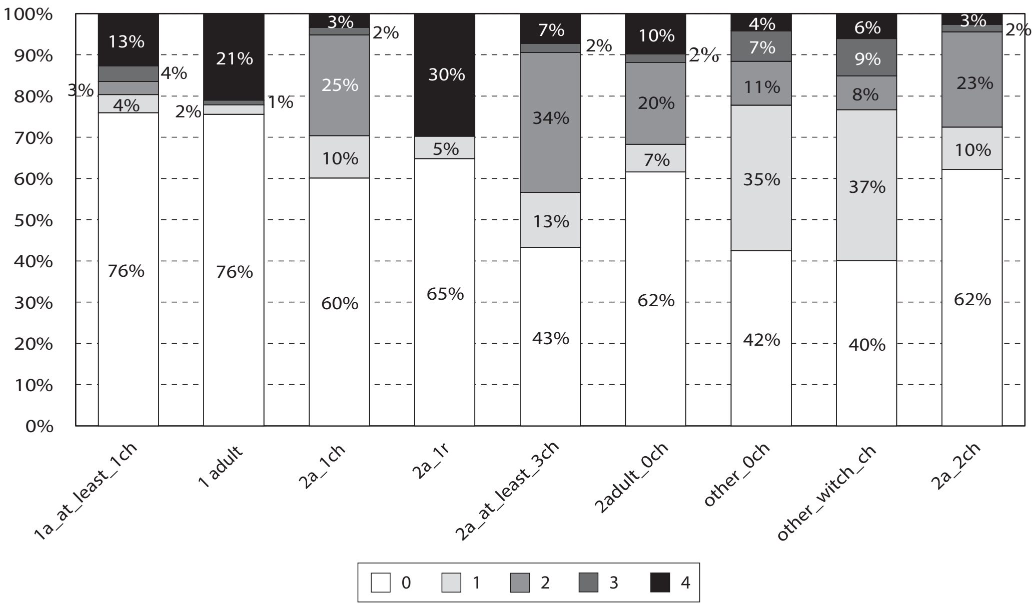
Figure 2 Household work intensity depending on the economic activity of the household head



Source: EU-SILC 2015, own processing

Correspondence analysis showed that the occurrence of very low work intensity is determined significantly more by the economic activity of the person at the head of the household as by the type of household. Very low work intensity is most typical for households where there are no more than 1 person of working age, especially for households of 2 adults, of which at least one is aged 65+ as well as single member households. A generally high representation of very low work intensity

Figure 3 Work intensity of Slovak households depending on the type of household

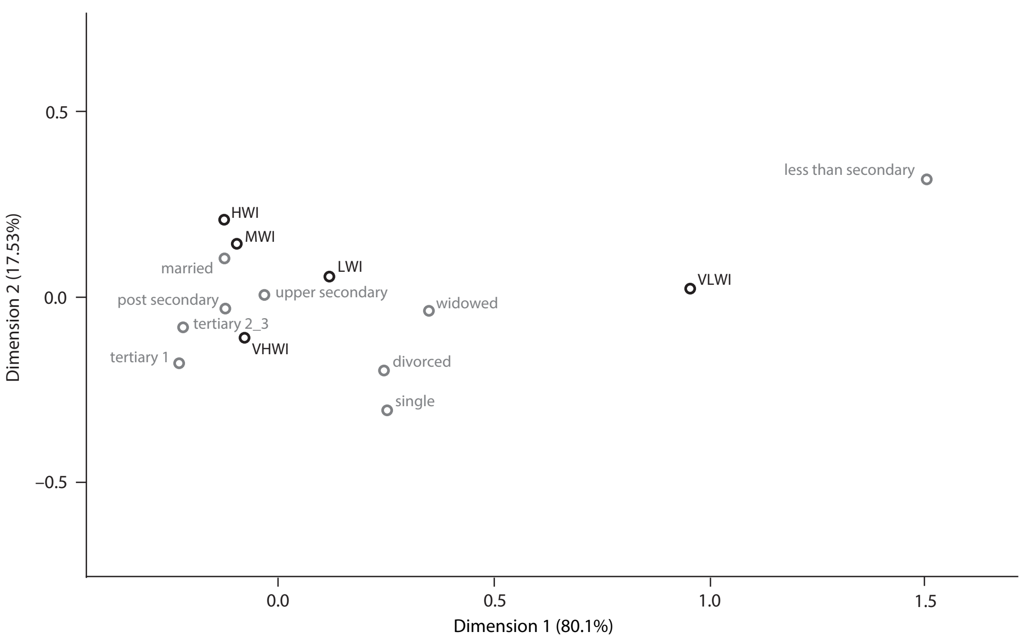


Source: EU-SILC 2015, own processing

(Figure 3) was seen, except in households of 2 adults, of which at least 1 was aged 65+ (30%) and in single member households (21%), in 1 adult household with at least 1 dependent child (13%). For "other" households, whether with or without dependent children, a high work intensity (HWI) is much more typical compared with other types of households (but not very high work intensity). Households of 2 adults and at least 3 dependent children are associated with a medium work intensity (MWI). This finding in the correspondence analysis is also confirmed by Figure 3. Generally, 2-adult households with two dependent children or one dependent child use their work potential to its best. These types of households are associated with very high work intensity (VHWI) the most and, together with households of the "other" type, are the least associated with very low work intensity (Figure 1). Moreover, in 2014, only they had the very lowest incidence of very low work intensity (under 5%) and at the same time they had a relatively high incidence of very high work intensity (Figure 3 at 60%).

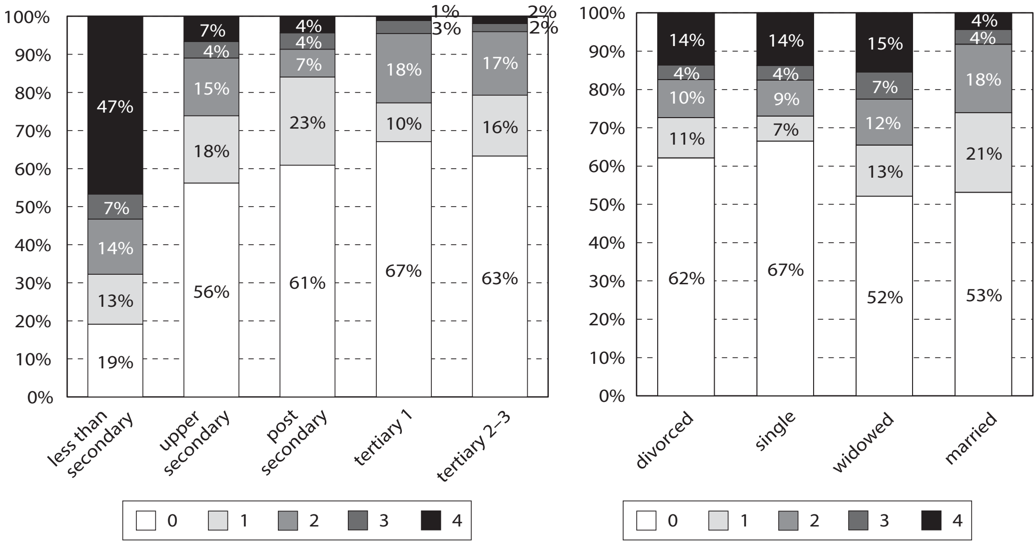
In regard to education of the person at the head of household, the results of correspondence analysis in relation to the household work intensity are very clear (Figure 4). Households headed by a person with lower than secondary education are the least of all households associated with very high work intensity, and a very low use of labour potential is typical for them. While in this group of households we recorded (Figure 5) less than ⅓ of households with high and very high work intensity, households with a person at the head with higher level of education were represented by a high and very high utilization of work potential at approximately ¾. This group of households significantly differs from other household groups, and there are no significant differences between the other groups in the use of labour potential. However, we can observe that the very low work intensity is not typical for households headed by a university graduate with either a doctorate, a master's or a doctoral degree.

Figure 4 Correspondence analysis of work intensity of Slovak households for factors, such as the highest level of education achieved and the marital status of the person at the head of the household



Source: EU-SILC 2015, customized in SAS BASE

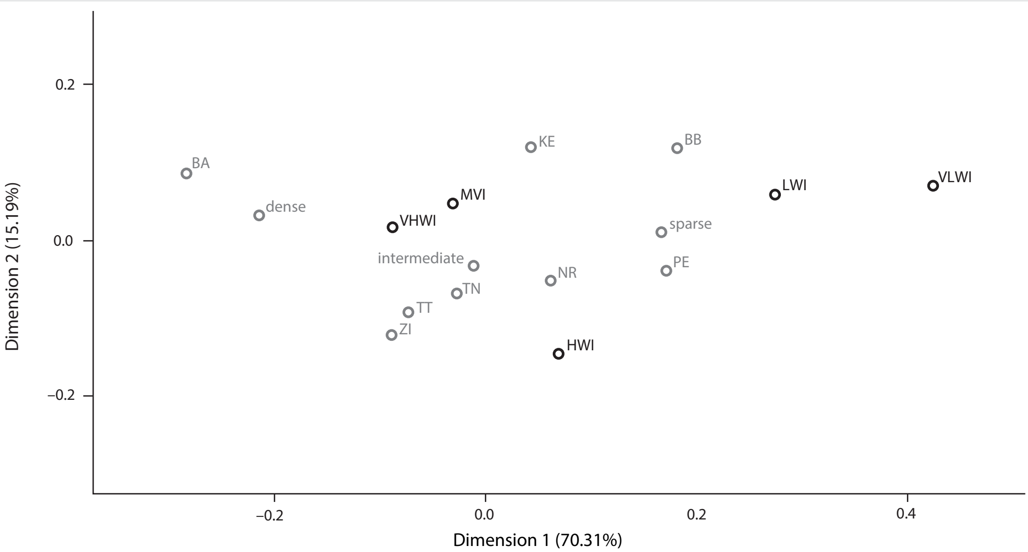
Figure 5 Work intensity of Slovak households in dependence on the highest level of education achieved and the marital status of the person at the head of the household



Source: EU-SILC 2015, own processing

The marital status of a person at the head of the household does not determine such disparities in the use of work potential as education. Very low work intensity is the least typical for households headed by a married couple. These households are more strongly associated with high and medium work intensity than other household groups. Very low and low work intensity are most typical for households with a widowed head of household.

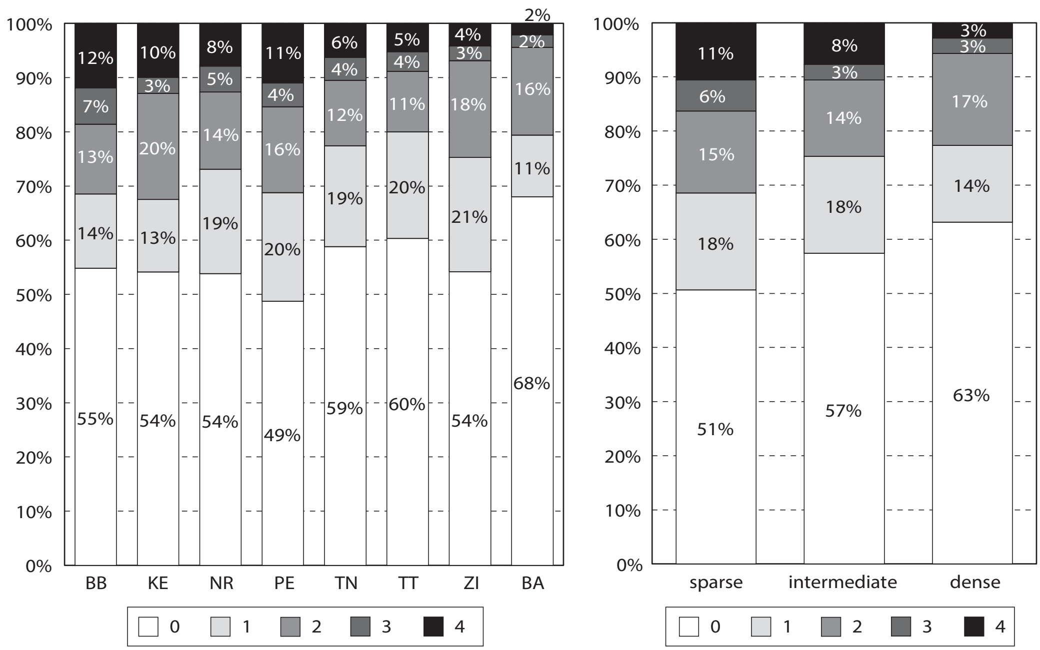
Figure 6 Correspondence analysis of work intensity of Slovak households for factors, such as region and population density on the territory where the household



Source: EU-SILC 2015, customized in SAS BASE

From a geographical point of view and in terms of population density, very low and low use of employment potential was associated with the Banská Bystrica and Prešov regions and with sparsely populated areas (Figures 6 and 7) in 2014. According to Beňuš et al. (2016), these two regions recorded one of the smallest advances in reducing the number of inhabitants living in very low work intensity households from 2010 to 2016, while the most significant decline in the very low work intensity rate was recorded in the Nitra region. Based on our analysis, the smallest threat of very low work intensity in 2014 was clearly in the Bratislava region and in households living in densely populated areas. Based on the correspondence analysis (Figure 6) and estimates of the representation of individual degrees of labour potential reduced use (Figure 7), regional disparities and discrepancies in terms of degree of urbanization are significantly higher in case of very low work intensity than in the case of very high work intensity. Of course, if we talk about relative differences.

Figure 7 Work intensity of Slovak households depending on the region and population density on the territory where the household lives



Source: EU-SILC 2015, own processing

4 ASSESSEMENT OF THE WORK INTENSITY OF SLOVAK HOUSEHOLDS USING MULTINOMIAL LOGISTIC REGRESSION

In this part of the article we will use multinomial logistic regression for the assessment of the statistical significance of the influence of the considered explanatory variables (shown in Table 2) on the degree of Slovak households work intensity. The impact of each relevant factor will be quantified by odds ratio, while the effect of other significant factors being fixed. Based on Table 3, we find that the strongest impact on work intensity is expected to be the economic activity of the person at the head of the household. This is followed by factors such as the type of household, education of the person at the head of the household and the region where the household lives. However, we quantified the smaller impact, which is still significant at the significance level of 0.05, in the case of the variable marital status of the person at the head of the household and the density of population on the territory in which the household lives.

We would like to remind that the influence of all these qualitative variables on the work intensity of Slovak households was assessed in the previous part of the article by the means of correspondence analysis. In the model of multinomial logistic regression, we also considered one continuous numeric variable – the variable age of the person at the head of the household, whose influence on work intensity is also proved to be relevant (*p-value* is less than 0.0001).

Table 3 Assessment of the multinomial logistic regression model quality and verification of the significance of the influence of considered factors on the work intensity degree of Slovak households

Model Fit Statistics

Criterion	Intercept Only	Intercept and Covariates
AIC	7 863.1	5 700.7
SC	7 887.4	6 428.5
-2 Log L	7 855.1	5 460.7

Testing Global Null Hypothesis: BETA=0

Test	Chi-Square	DF	Pr > ChiSq
Likelihood Ratio	2 394.5	116	<.0001
Score	2 625.9	116	<.0001
Wald	1 032.5	116	<.0001

R-Square	0.5288	**Max-rescaled R-Square**	0.5778

Type 3 Analysis of Effects

Effect	DF	Wald Chi-Square	Pr > ChiSq
AGE	4	38.39	<.0001
EAS	12	423.68	<.0001
HT	32	421.68	<.0001
EDUCATION	20	68.78	<.0001
REGION	28	65.12	<.0001
MARITAL_STATUS	12	24.13	0.0195
URBANISATION	8	15.61	0.0483

Source: EU-SILC 2015, customized in SAS EG

In interpreting the results of the estimated logistical model, we must be aware of several facts:

- The multinomial model could be divided into several separate models, depending on the number of categories of the explanatory variable that are being examined. In our case, the dependent variable has 5 categories and the multinomial model has been divided into 4 separate models for clarity and better comparability: 1 vs. 0, 2 vs. 0, 3 vs. 0 and 4 vs. 0 (see Table 4). The values of the explanatory variables 0 to 4 represent the degree of severity of the work potential reduced use, with the value 1 representing the lowest degree of severity, that is high work intensity (HWI) and value 4 representing the highest degree of severity, that is very low work intensity (VLWI). In the above partial models 1 vs. 0 to 4 vs. 0 we compare the corresponding degree of the work potential reduced use to grade 0, which represents a very high work intensity (VHWI).
- In columns in Table 4 we present the values of the odds ratios and *p*-values for testing the significance of the relevant parameter for the logistic model. In Table 4, those *p*-values are highlighted, by which we can assume, at the significance level of 0.05, that the odds of the corresponding degree of the labour potential reduced use compared to zero degree, will be different, in the corresponding household

Table 4 Estimates of the odds ratio of a multinomial logistic regression model

Factor			1 vs. 0		2 vs. 0		3 vs. 0		4 vs. 0	
			Odds Ratio	p-value	Odds Ratio	p-value	Odds Ratio	p-value	Odds Ratio	p-value
AGE			0.983	0.0430	0.952	<.0001	0.974	0.0769	0.975	0.1206
EAS	**inactive_person**	**vs at work**	48.289	<.0001	**137.932**	<.0001	**175.831**	<.0001	**4 553.27**	<.0001
EAS	**retired**	**vs at work**	10.591	0.0077	**89.981**	<.0001	**113.047**	<.0001	**1 378.57**	<.0001
EAS	**unemployed**	**vs at work**	3.733	0.0006	**17.299**	<.0001	**42.322**	<.0001	**1 000.95**	<.0001
HT	**1a_at_least_1ch**	**vs 2a_2ch**	0.305	0.0076	**0.079**	<.0001	0.794	0.7018	2.781	0.1362
HT	**1adult**	**vs 2a_2ch**	0.168	<.0001	<0.001	0.7433	**0.159**	0.0087	**4.422**	0.0196
HT	**2a_1ch**	**vs 2a_2ch**	1.023	0.9151	1.026	0.8744	0.751	0.5584	1.343	0.5716
HT	**2a_1r**	**vs 2a_2ch**	0.591	0.4990	<0.001	0.9372	<0.001	0.9829	**11.065**	0.0287
HT	**2a_at_least_3ch**	**vs 2a_2ch**	1.893	0.0196	**2.061**	0.0006	1.799	0.3388	2.409	0.1477
HT	**2adult_0ch**	**vs 2a_2ch**	0.631	0.0849	0.814	0.3071	0.522	0.2153	1.660	0.3217
HT	**other_0ch**	**vs 2a_2ch**	5.291	<.0001	0.741	0.1897	**3.738**	0.0017	0.728	0.5471
HT	**other_with_ch**	**vs 2a_2ch**	6.038	<.0001	**0.640**	0.0337	**6.713**	<.0001	1.524	0.3801
Education	**less than secondary**	**vs tertiary 2_3**	1.530	0.2355	**2.323**	0.0212	**3.080**	0.0368	**39.726**	<.0001
Education	**post secondary**	**vs tertiary 2_3**	1.409	0.3432	0.453	0.1313	1.749	0.4321	2.770	0.2823
Education	**tertiary 1**	**vs tertiary 2_3**	0.540	0.1250	0.809	0.5335	1.105	0.8883	0.580	0.7257
Education	**upper secondary**	**vs tertiary 2_3**	0.802	0.1310	0.907	0.5043	1.187	0.6008	2.455	0.0545
REGION	**BB**	**vs BA**	1.374	0.2092	1.151	0.5713	**3.046**	0.0154	**3.824**	0.0380
REGION	**KE**	**vs BA**	1.162	0.5175	1.400	0.1113	1.095	0.8486	3.214	0.0592
REGION	**NR**	**vs BA**	1.858	0.0108	1.044	0.8640	2.073	0.1256	1.673	0.4346
REGION	**PE**	**vs BA**	1.739	0.0204	1.312	0.2563	1.689	0.2772	**4.483**	0.0201
REGION	**TN**	**vs BA**	1.633	0.0406	0.877	0.5952	2.178	0.1016	1.688	0.4544
REGION	**TT**	**vs BA**	1.896	0.0115	0.750	0.2894	1.798	0.2581	0.641	0.5491
REGION	**ZI**	**vs BA**	**2.091**	0.0017	1.524	0.0660	1.268	0.6422	1.254	0.7458
Marital_Status	**divorced**	**vs married**	1.053	0.7953	1.288	0.2258	**2.008**	0.0355	**2.480**	0.0134
Marital_Status	**single**	**vs married**	0.857	0.5448	0.739	0.2004	1.944	0.0837	0.619	0.2608
Marital_Status	**widowed**	**vs married**	1.243	0.4881	1.326	0.4153	2.295	0.0612	0.729	0.5319
URBANISATION	**intermediate**	**vs dense**	0.824	0.2288	0.866	0.3837	**0.520**	0.0472	1.637	0.2369
URBANISATION	**sparse**	**vs dense**	0.821	0.2299	1.056	0.7423	1.032	0.9188	**2.223**	0.0493

Source: EU-SILC 2015, customized in SAS EG group, from the odds in the reference household group. The household reference group is listed for the appropriate categorical variable in the name of the row – behind the word "versus" (abbreviated as "vs").

- We consider a partial model that compares households with the degree of the work potential reduced use "s" versus very high work intensity households (VHWI – level 0). If, on the basis of such a model, we find that the odds ratio for the category (households) *A* compared to level 0 is higher than for category *B* of that variable (again with respect to level 0), this does not mean that the probability of a reduced use of the work potential at the level "s" is in category *A* higher than in category *B*. We have to realize that the basis, that is the occurrence of very high work intensity, to which the comparison is made in calculation of the odds ratio, can be significantly different in categories *A* and *B*. We warn of this fact in order to avoid misinterpretation of the results.
- All estimates of regression coefficients and odds ratios calculated from them, are interpreted, assuming ceteris paribus, that is assuming that the other explanatory variables remain fixed.

According to the odds ratio for the economic activity factor, the households headed by an unemployed or otherwise inactive or retired person have at the level of significance of 0.05 significantly higher odds of high, medium, low and very low work intensity in proportion to the very high work intensity compared to households where the head of the household is employed. The probability that a household headed by an unemployed person will have a high, medium, low, or very low degree of the labour potential use compared to the probability of having a very high work intensity is approximately 4 times, 17 times, 42 times or even up to 1001 times higher (see Table 4) than in households headed by an employed person. In the case of households headed by a retired or otherwise inactive person, these odds ratios are even considerably higher in regard to households with an employed head. However, this is not caused by the fact that in these household groups there is a higher incidence of very low and low levels of labour potential use than in the households with the unemployed head, but by the fact that in these households the number of households with very high work intensity (2% and 5%) is several times lower than in households with the unemployed head (8%) (see Figure 1).

The multinomial logistic model has confirmed our previous findings that the largest threat of reduced labour potential was faced, in 2014, by two adult households, of which at least one person was 65+, as well as one-person households. Households of 2 adults, of which at least one was aged 65+, had the odds ratio of very low work intensity compared to very high work intensity, up to 11 times higher, than households of 2 adults with 2 dependent children (reference group) and even more than 15 times higher than households of the "other" type without dependent children (according to Table 4, this is the type of household with the lowest odds ratio of 4 vs. 0). Compared to the reference category (households of 2 adults with 2 dependent children), we also found statistically significantly higher odds of 4 vs. 0 in single-person households where we estimated a 4.4-fold higher chance of this unfavourable phenomenon. According to estimated shares of the very low work intensity in Figure 3, the odds ratio of 1 adult households with at least 1 dependent child have the highest risk of very low work intensity between the two mentioned household types. In this type of households, we account for approximately 13% of households with very low work intensity, which is significantly less than in the households of 2 adults, of which at least one is aged 65+ (30%) and one-person households (21%) (see Figure 3). The odds of very low utilization of the labour potential relative to the odds of very high labour potential utilization (odds ratio 4 vs. 0) was estimated to be in households of 1 adult with at least 1 dependent child 2.8 times than in households of 2 adults with 2 dependent children. However, this difference in regard to the calculated p-value (0.1362). is not statistically significant at the level of significance of 0.05. It should be emphasized that this result was significantly affected by the fact that both the occurrence of very low work intensity and the occurrence of very high work intensity were in the households of 1 adult with at least one dependent child higher than in the households of 2 adults with 2 dependent children.

As the group of households headed by a person with lower than secondary education has a significantly higher incidence of very low work intensity compared to other groups of households (Figure 5), it is not surprising that the analysis of the logistics model (Table 3 and Table 4) confirmed the significant impact of education on the risk of very low work intensity. Statistically significant odds ratios are mainly observed for a group of households headed by a person with lower than secondary education. These households have a nearly 40 times higher risk of very low work intensity relative to the odds of very high work intensity than households headed by a 2nd or 3rd educational degree graduate.

From the geographical point of view, most of the households with very high work intensity are located in the Bratislava region, which we also chose as a reference region. While in the Bratislava region, we, according to Figure 7, estimated the share of very labour intensive households at 68%, in the Prešov region it is only at 49%. The Prešov region together with Banskobystrický and Košický belongs among the regions with the highest risk of very low work intensity (see Figures 6 and 7). On the basis of the above, it is not surprising that the ratio of the probability of very low work intensity to the probability of very high work intensity is the highest in the three regions. In the Prešovský, Banskobystrický and Košický region, we determined by logistic regression that the given odds ratio is 4.5, 3.8 and 3.2, respectively times higher than in the Bratislava region. Moreover, these differences are statistically significant at the level of significance of 0.1 and, in the case of Banskobystrický and Prešovský regions, also at the significance level of 0.05. According to the odds ratio 4 vs. 0, it seems that the worst situation in terms of work potential utilization is in the households of Prešov region, which is also affected by the lowest occurrence of very high work intensity. According to Figure 7, however, the highest frequency of households with very low work intensity is in Banskobystrický region. In addition, the highest share of low work intensity households (7%) is registered in this region. The ratio of the probability of low work intensity to the probability of very high work intensity is in the region of Banská Bystrica, as in one region significantly higher than the given odds ratio (3 vs. 0) in Bratislava region, and this is approximately 3 times.

Finally, we will look at how the degree of work potential reduced use is influenced by the marital status of the head of the household and the degree of urbanization of the territory in which the household lives. While odds ratios of 1 vs. 0 – 4 vs. 0 for households with single or widowed head are not, at the significance level of 0.05, significantly different compared to the odds ratios for households headed by married person, so the households headed by the divorced person have the risk of low and the risk of very low work intensity significantly higher. Divorced households have the odds of very low work intensity in proportion to the chance of a very high work intensity almost 2.5 times higher than households headed by a married couple. Compared to single-member households and households with a widowed head, the odds ratio (4 vs. 0) for households with a divorced head is 4.0 times and 3.4 times higher, respectively.

Although the degree of urbanization does not have such a large impact on the threat of labour potential reduced use, in the case of this factor a category arose for which an increased risk of very low work intensity is typical. These are households living in a sparsely populated area that have a 2.2-fold higher probability of occurrence of very low work intensity compared to the likelihood of very high work intensity as households living in densely populated areas where the threat of very low work intensity is the lowest.

CONCLUSION

In Slovakia in 2014, their work potential was best used by 2 adult households with 2 dependent children or 1 dependent child, confirming the results of the correspondence analysis as well as the logistic regression presented in this article. This is due to the fact that these types of households experienced the lowest incidence of very low work intensity and a relatively high proportion of households with very high work intensity. It is not surprising that, in terms of economic activity and the highest level of education, a very high work intensity was typical for those households headed by an employed person, a person with a master's or a doctoral degree of education. Although other factors did not cause such large disparities in the use of labour potential as the above-mentioned factors, they also significantly determined the level of work intensity of Slovak households. The best use of their working potential was in the households headed by a married couple, households from the Bratislava region and households living in densely populated areas. In these three groups of households we observed a very low work intensity below 5%.

Very low work intensity in 2014 was most typical for households with a maximum of 1 person of working age, especially for households of 2 adults, of which at least 1 is aged 65+ and single households. Due to the high risk of very low work intensity, we assess these two types of households as households with the worst use of labour potential, despite the fact that, besides the high incidence of very low work intensity, there was a high incidence of very high work intensity (with negligible representation of other levels of work intensity). All applied statistical methods have clearly demonstrated that the type of household is a relevant factor determining the intensity of work of Slovak households. But we have to say that economic activity and education have had a greater impact on the use of labour potential. For households headed by an unemployed person, we have quantified that the odds ratio of the very low work intensity in regard to the very high work intensity is up to 1000 times higher than in the households with an employed head. Our analyses have confirmed that with increasing level of education, the use of households' work potential is improving and that the greatest threat of very low work intensity is in households headed by a person with lower than secondary education. The odds ratio of very low work intensity in regard to very high work intensity we estimated for these households is almost 40 times the odds ratio quantified for households that are the least threatened by exclusion from the labour market (households headed by a university graduate with the second or third degree of higher education). Other factors did not determine such large differences in the odds ratio of the very low work intensity and the very high work intensity between individual household groups. Correspondence analysis and analysis of the multinomial logistics model, however, confirmed that even in terms of the marital status of the person at the head of the household and in terms of the region and the density of settlement of the territory in which the household lives, there were significant differences in the use of labour potential in Slovakia in 2014. In regard to these three factors, the households which were using their work potential at its lowest, were the households headed by a divorced person, households from Prešov and Banská Bystrica regions and households living in sparsely populated areas.

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Long Seasonal Cycle Modeling: the Case of Realized Volatility

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Abstract

Time series with long seasonal periods are very common. Several methods have been proposed for modeling of long seasonal cycles, the most commonly used ones being those based on basis expansion. In this paper, we present and discuss these methods. We also use them to model seasonality in realized volatility of several major stock market indices and find evidence for the existence of yearly as well as weekly seasonality. The presented approaches can potentially be used for modeling of any seasonal time series with a long seasonal period.

Keywords	***JEL code***
Time series, seasonality, basis expansion, volatility	*C58, C22*

INTRODUCTION

While seasonality of returns has been studied thoroughly, seasonality in volatility of stock markets has received much less attention in the literature. Back et al. (2013) showed that including a seasonal component in volatility into an option pricing model greatly improves the model performance. This was further studied by Arismendi et al. (2016) who proposed a seasonally varying long-run mean variance process. Day-of-the-week effect in volatility of stock markets during the period from 1988 through 2002 was studied in Kiymaz and Berument (2003) who found that the highest volatility occurs on Mondays for Germany and Japan, on Fridays for Canada and the United States, and on Thursdays for the United Kingdom. Furthermore, Giovanis (2009) studied the calendar effects (turn-of-the-month, day-of-the-week, month-of-the-year and semi-month effect) in the volatility of 55 stock market indices and

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concluded that there is a day-of-the-week and month-of-the-year effect in volatility. Another seasonal anomaly in volatility was reported by Seyyed et al. (2005) who studied data from Saudi Arabian stock market during Ramadan and found a significant and predictable decrease of volatility during this period.

In our paper, we model the *yearly* seasonal cycle in *daily* time series of realized volatility of 19 stock market indices in the period from 2000 till 2017. Such a seasonal cycle is an example of seasonality with a long seasonal period, namely 365 days. We investigate various approaches to modeling of such a seasonal cycle that are based on basis expansion. We demonstrate that the choice of the basis functions is a crucial step in the modeling process. The results we obtain may play an important role in volatility modeling. Moreover, the approaches we present are more general since they can potentially be used for modeling of any time series with a long seasonal period.

The paper is organized as follows. Section 1 introduces the approaches to seasonality modeling based on basis expansion methods. Section 2 performs the analysis of the 19 realized volatility time series. Last section concludes.

1 BASIS EXPANSION METHODS FOR MODELING SEASONALITY

In the following text a daily time series of realized volatility of length N will be denoted as $\{X_t : t = 1, ...N\}$, where time t corresponds to *trading* days. We will assume the following model for $\{X_t\}$:

$$X_t = S_{1,m(t)} + S_{2,n(t)} + E_t \qquad t = 1,\ldots,N, \tag{1}$$

where $\{S_{1,m(t)}\}$ is a yearly deterministic seasonal component *including an intercept* and $\{S_{2,n(t)}\}$ a weekly deterministic seasonal component. $m(t)$, being a function of t, is the *calendar day of the year* and can take any value from 1 to $L = 365$. Leap days have been removed from the analysis for simplicity. Analogously, $n(t)$ is the *day of the week*, where 1 = Monday, 2 = Tuesday, 3 = Wednesday, 4 = Thursday and 5 = Friday. $\{\mathrm{E}_t : t = 1, ... N\}$ is a stationary ARMA(p, q) term given as:

$$E_t = \phi_1 E_{t-1} + \ldots + \phi_p E_{t-p} + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q} + \epsilon_t, \qquad t = 1,\ldots,N\ , \tag{2}$$

where $\{\phi_i : i = 1,\ldots, p\}$ and $\{\theta_i : i = 1,\ldots,q\}$ are parameters and $\{\epsilon_t : t = 1,\ldots,N\}$ is Gaussian white noise.

In the paragraphs below, we introduce several approaches to modeling $\{S_{1,m(t)}\}$ based on basis expansion. By basis expansion we mean rewriting $\{S_{1,m(t)}\}$ as a linear combination of "basis functions".

1.1 Dummy variables

The most common approach to modeling yearly seasonal cycle in daily time series is rewriting $S_{1,m(t)}$ as a sum of an intercept term and 11 dummy variables. Specifically, we can write:[5]

$$S_{1,m} = c + \sum_{k=2}^{12} \alpha_k \psi_k, \qquad m = 1,\ldots,L, \tag{3}$$

where c is an intercept and α_k, for $k = 2, ...12$, are parameters and ψ_k, for $k = 2, ..., 12$, is a dummy variable which is equal to 1 if calendar day m is part of month k, and to 0 if day m is not part of month k.[6]

[5] For simplicity, we write $S_{1,m}$ instead of $S_{1,m(t)}$.

[6] k = 2 is February, k = 3 is March, … , k = 12 is December.

1.2 Fourier basis

Now, we introduce the approach of representing $\{S_{1,m(t)}\}$ by means of Fourier basis functions, which amounts to rewriting $\{S_{1,m(t)}\}$ as a sum of sines and cosines of different frequencies and amplitudes. More specifically, we can write:[7]

$$S_{1,m} = c + \sum_{k=1}^{K} \alpha_k \sin\left(2\pi \frac{k}{L} m\right) + \sum_{k=1}^{K} \beta_k \cos\left(2\pi \frac{k}{L} m\right), \qquad m = 1, \ldots, L, \tag{4}$$

where c is an intercept and α_k and β_k, for $k = 1, \ldots, K$, are parameters. K can potentially take any value from 1 to 182, assuming that $L = 365$. The number of parameters used in the expansion (including the intercept) is $1 + 2K$.

The choice of K has a direct impact on the smoothness of the seasonal cycle. A large value of K results in low as well as high-frequency features being present in the seasonal cycle. On the other hand, a small value of K results only in low-frequency features being present in the cycle.

1.3 Periodic linear spline

A potential disadvantage of using the decomposition of $\{S_{1,m(t)}\}$ into sines and cosines (see Section 1.2) is the fact that sines and cosines are *not* localized in time, which can make *local* features in the seasonal component difficult to model. Consequently, we consider another approach to modeling the seasonal component. Specifically, we represent it using a regression *linear* spline with K knots.

Specifically, let the linear spline function $c(m)$, now considered as a function of a *continuous* variable m, be defined on the interval $[0, L]$ as follows (see Friedman, Hastie and Tibshirani, 2001; Ramsay and Silverman, 2002; or Ramsay and Silverman, 2005):

$$c(m) = \beta_0 + \beta_1 m + \sum_{k=1}^{K} \delta_k \left(m - \xi_k\right)_+, \quad 0 \le m \le L, \tag{5}$$

where β_0, β_1, and δ_k, for $k = 1, \ldots, K$, are parameters and $(.)_+$ denotes the positive part of the expression inside the brackets. ξ_k, for $k = 1, \ldots, K$, are (real or whole) numbers called *knots* which satisfy $0 \le \xi_1 < \xi_2 < \ldots < \xi_K < L$. From the definition, it follows that the linear spline function is continuous on the interval $[0, L]$. Further, it is piecewise linear since it is a linear function on each of the intervals $[0, \xi_1)$, $[\xi_1, \xi_2)$, …, $[\xi_L, L]$ separately. The first derivative does not exist at the knots.

One potential disadvantage of using the linear spline function (compared to sines and cosines of Section 1.2) is the fact that the linear spline function is not "periodic" with period equal to L. We suggest that periodicity be imposed by assuming appropriate constrains. Specifically, we require that:

$$c(0) = c\,(L), \tag{6}$$

$$c(0)^{(1)} = c\,(L)^{(1)}, \tag{7}$$

where $c(0)^{(1)}$ denotes the first right derivative at point 0 and $c\,(L)^{(1)}$ the first left derivate at point L. These constraints ensure that $c(m)$ returns to the same value after period L (the first constraint) and that the "connection of the two ends of the cycle" is smooth (the second constraint).[8] The two constraints reduce the number of parameters of the linear spline function by 2. Specifically, a linear spline function satisfying the constraints will be called a periodic linear spline, has $2 + K - 2 = K$ parameters and can be written as a linear combination of K basis functions. Figure 1 presents an example of a periodic linear spline.

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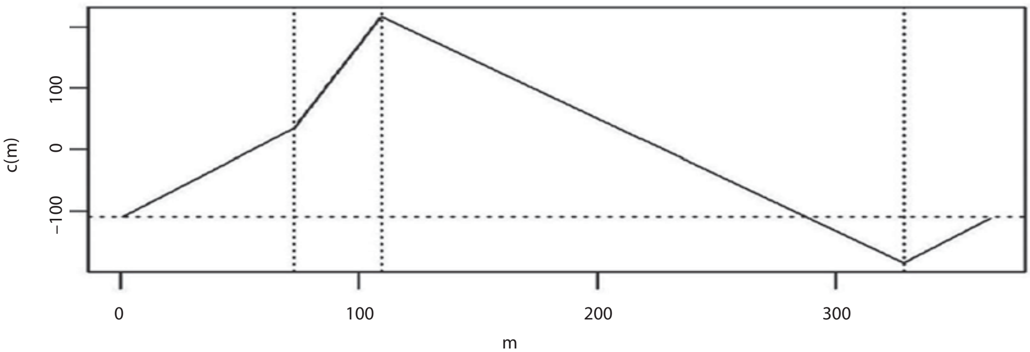
7 For simplicity, we write $S_{1,m}$ instead of $S_{1,m(t)}$.

8 If the first knot ξ_1 is placed at 0, then the first derivative of c(m) does not exist at 0.

If the periodic linear spline is sampled at discrete values of $m = 1, 2, \ldots, L$, it can be used as a representation of the seasonal component $\{S_{1,m(t)}\}$ of Formula (1). An appropriate choice of the position of the knots can help us to capture local features in the seasonal component.

Figure 1 An example of a periodic linear spline (solid line) with L = 365 and three knots placed at 73, 110, 329 (dashed vertical lines)



Source: Own construction

1.4 Periodic cubic spline

To ensure a more flexible shape of the function on the intervals $[0, \xi_1)$, $[\xi_1, \xi_2)$, …, $[\xi_L, L]$ and more smoothness at the knots, i.e. the existence of (at least some) derivatives at the knots, we can consider using cubic spline functions instead of linear ones.

Using an analogous notation as in Section 1.3, a cubic spline function with K knots $0 \leq \xi_1 < \xi_2 < ... < \xi_K < L$ can be defined on the interval $[0, L]$ as follows (see Friedman, Hastie and Tibshirani, 2001; Ramsay and Silverman, 2002; or Ramsay and Silverman, 2005):

$$c(m) = \beta_0 + \beta_1 m + \beta_2 m^2 + \beta_3 m^3 + \sum_{k=1}^{K} \delta_k (m - \xi_k)_+^3, \quad 0 \leq m \leq L, \tag{8}$$

where β_0, β_1, β_2, β_3 and δ_k, for $k = 1, ..., K$, are parameters. From the definition, it follows that the cubic spline function is continuous on the interval $[0, L]$. It is a piecewise cubic polynomial since it is a cubic polynomial on each of the intervals $[0, \xi_1)$, $[\xi_1, \xi_2)$, …, $[\xi_L, L]$ separately. The first and second derivatives exist at the knots, whereas the third derivative does not.

Further, we require that:

$$c(0) = c(L), \tag{9}$$

$$c(0)^{(1)} = c(L)^{(1)}, \tag{10}$$

$$c(0)^{(2)} = c(L)^{(2)}, \tag{11}$$

$$c(0)^{(3)} = c(L)^{(3)}, \tag{12}$$

where $c(0)^{(1)}$, $c(0)^{(2)}$, $c(0)^{(3)}$ denote the first, second and third right derivative at point 0, and $c(L)^{(1)}$, $c(L)^{(2)}$, $c(L)^{(3)}$ denote the first, second and third left derivate at point L. Analogously to Section 1.3, the constraints ensure that $c(m)$ returns to the same value after period L (the first constraint) and that "the connection of the two ends of the cycle" is smooth (the second, third and fourth constraint).[9] Using these constraints,

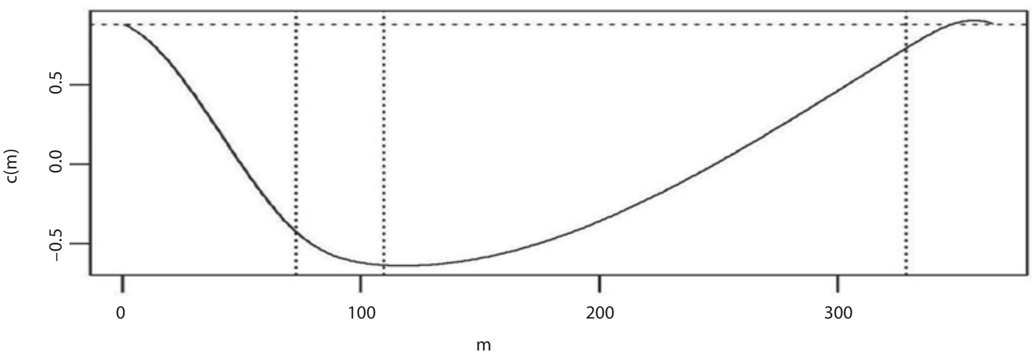
[9] If the first knot ξ_1 is placed at 0, then the third derivative of c(m) does not exist at 0.

the number of parameters is reduced by 4. The cubic spline function which fulfills these constraints will be called a periodic cubic spline, has $4 + K - 4 = K$ parameters and can be written as a linear combination of K basis functions. Figure 2 presents an example of a periodic cubic spline.

If the periodic cubic spline is sampled at discrete values of $m = 1, 2, \dots, L$, it can be used as a representation of the seasonal component $\{S_{1,m(t)}\}$ of Formula (1).

If local features (sudden changes and bumps) are a priori expected to be present in some regions of the seasonal cycle, more knots can be placed in that region to allow for a more flexible representation of the seasonal cycle in that region (Ramsay and Silverman, 2005).

Figure 2 An example of a periodic cubic spline (solid line) with L = 365 and three knots placed at 73, 110, 329 (dashed vertical lines)



Source: Own construction

1.5 Model for weekly seasonality

To model the weekly seasonal cycle $\{S_{2,n(t)}\}$, we consider only the most common approach based on dummy variables. Specifically, we write:[10]

$$S_{2,n} = \sum_{k=2}^{5} \alpha_k \lambda_k, \quad n = 1, \dots, 5, \tag{13}$$

where α_k, for $k = 2, \dots, 5$, are parameters and λ_k, for $k = 2, \dots, 5$, is a dummy variable which is equal to 1 if $k = n$, and to 0 otherwise.

1.6 Model selection

To find an optimal model for $\{X_t\}$, various representations of the yearly seasonal cycle (dummy variables, Fourier basis, periodic linear splines or periodic cubic splines) as well as various numbers of parameters for the yearly seasonal cycle (i.e. various numbers of sines/cosines or various numbers of knots) can be explored. The selection of the best model from the candidate models can be based on information criteria such as the Akaike information criterion (AIC) or the Bayesian information criterion (BIC) which are defined as:

$$AIC = 2p - 2 \cdot \hat{l}, \tag{14}$$

$$BIC = \log(N)\, p - 2 \cdot \hat{l}, \tag{15}$$

[10] For simplicity, we write $S_{2,n}$ instead of $S_{2,n(t)}$.

where p is the total number of estimated parameters in the model of Formula (1) and $\hat{l}$ is the natural logarithm of the maximized likelihood function. Models with the lowest values of AIC or BIC are considered as optimal models.

The terms $2p$ and $\log(N)\,p$ in Formulas (14) and (15) can be regarded as penalties. Consequently, BIC penalizes a model with a larger number of parameters more heavily than AIC provided that $\log(N) > 2$, which is the case of our data. AIC may lead to selection of too big a model. On the other hand, provided N is large enough, BIC may easily select too small a model. Consequently, we will work with both the criteria while selecting the optimal model, giving a little bit more preference to AIC.

A question arises as to what extent a model with the lowest value of AIC is better compared to another model M with a higher value of AIC. If the model with the lowest value of AIC is assigned "relevance" (weight) equal to 1, then the relevance of model M could be given as (Claeskens and Hjort, 2008):

$$\exp\left(-\frac{\Delta AIC_M}{2}\right), \tag{16}$$

where $\Delta AIC_M \geq 0$ is the difference between AIC of model M and AIC of the model with the lowest value of AIC. An analogous formula applies to comparing models by their BIC values. Namely, the "relevance" (weight) of model M is given as (Claeskens and Hjort, 2008):

$$\exp\left(-\frac{\Delta BIC_M}{2}\right), \tag{17}$$

where $\Delta BIC_M \geq 0$ is the difference between BIC of model M and BIC of the model with the lowest value of BIC.

2 EMPIRICAL ANALYSIS

In this section, we will use the above mentioned approaches to model yearly seasonality in the daily time series of realized volatility of several stock market indices:

- S&P 500, DJIA and NASDAQ 100 represent the American stock market.
- FTSE 100 represents the London Exchange. DAX, Euro STOXX 50 (both Germany), CAC 40 (France), AEX Index (Netherlands), Swiss Market Index (Switzerland), IBEX 35 (Spain) and FTSE MIB (Italy) represent the Continental European stock exchange.
- Nikkei 225 (Japan), Hang Seng (Hong Kong), KOSPI Composite Index (South Korea), FT Straits Time (Singapore) represent the Asian stock exchange.
- IPC Mexico and Bovespa Index represent the Mexican and Brazilian stock exchange.
- All Ordinaries represent the Australian stock exchange and the S&P/TSX Composite Index the Canadian one.

The data cover the period from January 3, 2000 till July 30, 2017[11] and have been obtained from Heber et al. (2009). Specifically, realized volatility (from the open to the close of the trading day) is available for each of the 19 indices being given as the square root of the sum of squared 5-minute log returns.[12] Details about the calculation of the realized volatility can be found in Liu, Patton and Sheppard (2012).

In Figure 3, we present the realized volatility time series for S&P 500, the logarithm of the time series and the annualized version of the realized volatility time series given in percentages obtained by multiplying the realized volatility time series by the square root of 252 and by 100.

[11] For some indices data are available only for shorter time spans.
[12] A trading day, which has, for example, 6.5 hours, has a total of 78 5-minute log returns.

We can see that periods of high realized volatility are present during the dot-com bubble (years 2000–2001), during the stock market downturn in 2002 and especially during the U.S. subprime mortgage crisis (2007–2009), which overlaps with the global financial crisis (2007–2008) and the Great Recession (2007–2009). Increased levels of volatility can also be observed during the recovery from the Great Recession (2010–2012), during the Chinese stock market crisis (2015–2016) and the Russian financial crisis (2014–2017).

The phenomena of volatility clustering and mean-reversion can be clearly discerned from the figure, clusters of high volatility being followed by clusters of low volatility. No obvious yearly seasonal pattern in realized volatility can be seen in the figure.

In Table 1, descriptive statistics (mean, standard deviation, coefficient of variation, minimum, maximum, skewness and kurtosis) are given for the realized volatility time series of S&P 500, FTSE 100, KOSPI Composite Index 100 and Bovespa Index. We can see that the average values are quite similar for the four indices, the coefficients of variation ranging from 0.49 to 0.69. The unconditional distributions of the realized volatility time series are highly skewed and leptokurtic.

Analogous descriptive statistics for the *natural logarithm* of the four realized volatility time series are presented in Table 2. We can see that the logarithm of the realized volatility time series is more Gaussian. This is also in agreement with Figure 3. Consequently, in further analysis, the natural logarithm of the realized volatility time series will be used. This is no drawback since the natural logarithm has a straightforward interpretation, its change being directly related to percentage change in the original volatility time series.

Table 1 Descriptive statistics for four realized volatility time series

	Mean	Standard deviation	Coefficient of variation	Min.	Max.	Skewness	Kurtosis
S&P 500	0.009	0.006	0.689	0.001	0.088	3.135	17.838
FTSE 100	0.008	0.005	0.623	0.002	0.068	2.910	16.168
KOSPI CI	0.010	0.006	0.616	0.002	0.077	2.461	11.753
Bovespa Index	0.013	0.006	0.486	0.003	0.082	3.492	21.714

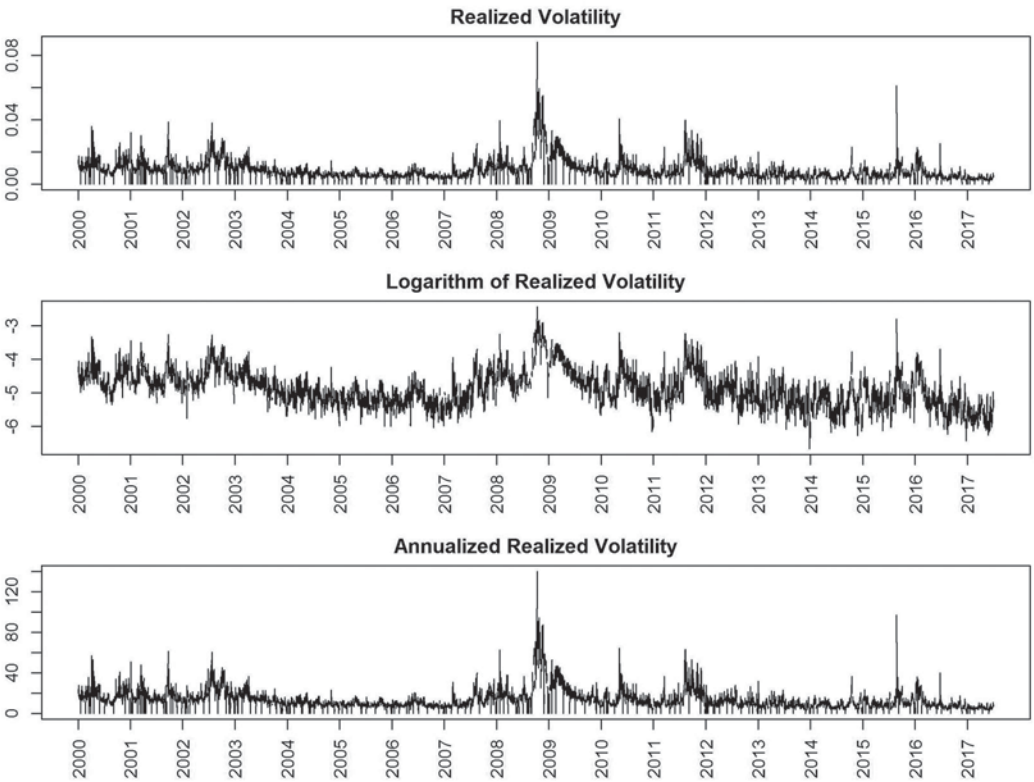
Source: Own construction

Table 2 Descriptive statistics for the natural logarithm of four realized volatility time series

	Mean	Standard deviation	Coefficient of variation	Min.	Max.	Skewness	Kurtosis
S&P 500	–4.886	0.553	0.113	–6.667	–2.430	0.410	0.313
FTSE 100	–4.985	0.506	0.101	–6.242	–2.688	0.514	0.194
KOSPI CI	–4.778	0.526	0.110	–6.040	–2.563	0.362	-0.194
Bovespa Index	–4.398	0.385	0.087	–5.756	–2.499	0.619	1.697

Source: Own construction

Figure 3 Time series of realized volatility of S&P 500 based on 5-minute log returns (top plot), the natural logarithm of the time series (middle plot) and the annualized version of the time series given in percentages (bottom plot).The ticks on the x-axis correspond to the start of calendar years



Source: Own construction

2.1 Seasonality models

We assume the model of Formula (1) and work with the natural logarithm of realized volatility based on 5-minute log returns. For each of the 19 time series, each time series being associated with one stock market index, six different approaches to yearly seasonal cycle modeling will be assumed:

1.) Intercept only, which corresponds to no yearly seasonality. This approach to yearly seasonality modeling will be denoted as N.
2.) Intercept + 11 dummy variables, which amounts to 12 parameters. Denoted as D.
3.) Fourier basis functions with $K = 2, 3, 5, 8$ and 11 (see Formula (4)), which corresponds to 5, 7, 11, 17 and 23 parameters. Denoted as F5, F7, F11, F17 and F23.
4.) Periodic linear splines with the following number of knots (parameters): $K = 5, 7, 11, 17$ and 23. The knots are placed equidistantly, see below. Denoted as L5, L7, L11, L17 and L23.
5.) Periodic cubic splines with the following number of knots (parameters): $K = 5, 7, 11, 17$ and 23. The knots are placed equidistantly, see below. Denoted as C5, C7, C11, C17 and C23.
6.) Periodic cubic splines with the following number of knots (parameters): $K = 5, 7, 11, 17$ and 23. *Expert* placement of knots is used, see below. Denoted as E5, E7, E11, E17 and E23.

In approaches L and C, the knots will be placed equidistantly throughout the year so that the distance between two neighboring knots is constant, the first knot being placed at the beginning of the calendar year.

Concerning approach E, the first knot is placed at the beginning of the calendar year and the second one at the start of December. Every further *odd* knot is placed within the interval from the start of the calendar year to the start of December (11-month interval) and every *even* knot is placed within the interval from the start of December to the start of the calendar year (1-month interval). In both these intervals the knots are placed equidistantly. Approach E thus places more knots to December than to any other month of the year. This is an example of an expert placement of the knots which can be supported by the fact that the end of the calendar year is a very special period overlapping with Christmas in many cultures and countries, and with an upcoming new year.[13]

Concerning the weekly seasonal cycle, two approaches will be assumed for its modeling:

1.) No weekly seasonal cycle. Denoted as N.

2.) 4 dummy variables. Denoted as D.

The six approaches to modeling the yearly seasonal cycle will be explored in combination with the two possible approaches to modeling the weekly seasonal cycle. Table 3 provides a summary of the approaches explored. The approaches used for yearly seasonal cycle modeling are given in columns, whereas the approaches used for weekly seasonal cycle modeling are given in rows. The number in each cell is the total number of models explored for the given combination of approaches, the models in the cell differing in the number of parameters used for the yearly seasonal cycle model (5, 7, 11, 17 or 23).

A given combination of a model for yearly seasonality and a model for weekly seasonality will by denoted, for example, as F11 + D, which means that Fourier basis functions with eleven parameters (including intercept) were used to model the yearly seasonal cycle and dummy variables were used to model the weekly cycle.

A total of 44 different models for the seasonal parts will be explored for each of the 19 time series of the natural logarithm of realized volatility. For each of the 44 models, the following ARMA model for $\{E_t\}$ (see Formulas (1) and (2)) will be considered:

$$E_t = \phi_1 E_{t-1} + \phi_2 E_{t-2} + \phi_6 E_{t-6} + \theta_1 \varepsilon_{t-1} + \epsilon_t, \quad t = 1, \ldots, N \; , \qquad (18)$$

since it provides a good approximation to the HAR model (Corsi, 2009) commonly used for realized volatility modeling.

R software (R Core Team, 2017) and the following contributed R packages have been used in the analysis: *forecast* (Hyndman, 2017; Hyndman and Khandakar, 2008) and *pbs* (Wang, 2013).

Employing the *Arima()* function from the *forecast* package (Hyndman, 2017) Gaussian maximum likelihood estimation will be used to estimate the parameters of Formula (1). Further, the best combination of the yearly and weekly seasonal cycle (i.e. the best model) will be selected according to the value of AIC or BIC. The residuals of the selected model are checked whether they are uncorrelated, homoskedastic, normal and whether no outliers are present (see Section 2.3).

Since it is impossible to simply interpret the individual parameters of the Fourier or spline basis expansion, we will neither interpret the individual parameter estimates nor will we assess the accuracy of the estimates in our analysis. On the other hand, we will interpret the estimated seasonal cycle (i.e. a linear combination of parameters) and will assess the accuracy of the estimate of the seasonal cycle, se further.

[13] Ramsay and Silverman (2005) state that a researcher may want to have more knots over regions where the function to be estimated exhibits the most complex variations. We presume that the knowledge of such regions is often a question of an expert understanding of the problem at hand. Consequently, different placements of the knots are expected for different applications and are subjective to some extent. We believe that December is a reasonable choice for the placement of more knots in our application.

Table 3 Number of models used for the various combinations of approaches to yearly and weekly seasonal cycle modeling. The models in each cell differ by the number of parameters used for yearly seasonal cycle modeling

		Yearly seasonal cycle					
		N	D	F	L	C	E
Weekly seasonal cycle	N	1	1	5	5	5	5
	D	1	1	5	5	5	5

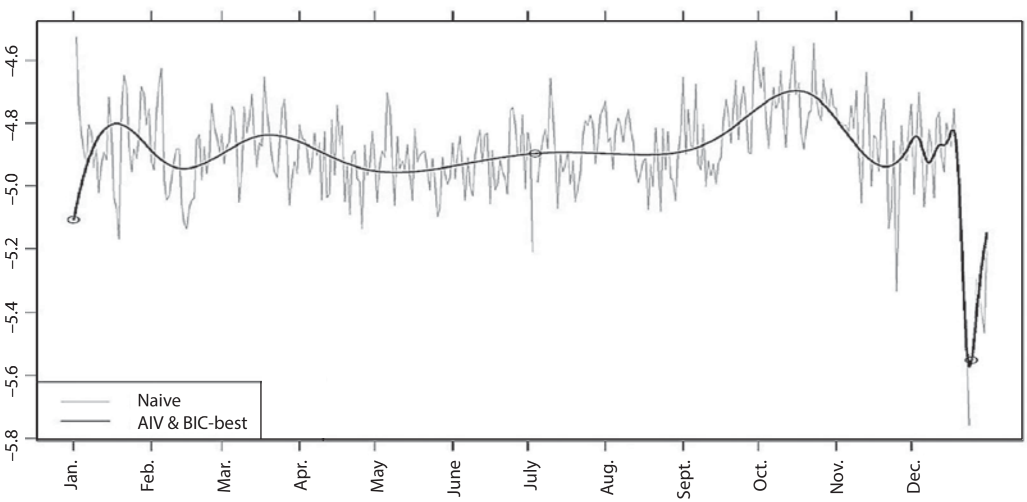
Source: Own construction

2.2 Results for S&P 500

At first, results for the natural logarithm of realized volatility of S&P 500 will be presented. The best model among the 44 models of Table 3 according to both AIC and BIC is model E17 + D, i.e. model E17 for yearly seasonality combined with model D for weekly seasonality.

Figure 4 presents a naïve estimate of the yearly seasonal cycle[14] and the estimated yearly seasonal cycle for the best model (E17 + D). The estimated cycle (for model E17 + D) has a minimum on December 24 and a maximum on October 16. The shift from the minimum to the maximum value corresponds to a 140 percent increase of volatility. The estimated seasonal cycle exhibits a deep trough from December 18 till the start of January. Another, higher and flatter, bottom in volatility occurs during May. Typical variations of the estimated seasonal cycle throughout the year correspond to an approximately 10 percent change in volatility.

Figure 4 Naïve estimate of the yearly seasonal cycle (gray) in the logarithm of realized volatility of S&P 500 together with the estimated cycle from the best model (E17 + D) according to AIC and BIC (black). The three points on the estimated cycle correspond to calendar days (Jan 1, Jul 4 and Dec 25) where in fact no trading ever happens due to public holidays

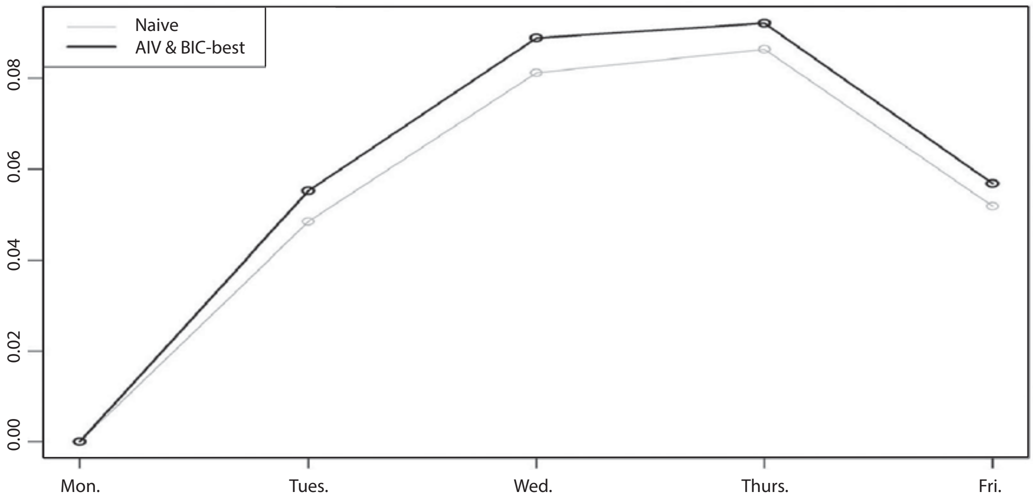


Source: Own construction

[14] A naïve estimate of the yearly seasonal cycle (including the intercept/level) is obtained by calculating the average of the time series separately for each calendar day of the year. In this way we get 365 average values, one average value being associated with one day of the calendar year.

Figure 5 presents a naïve estimate of the weekly seasonal cycle[15] and the estimated weekly cycle from the best model (E17 + D). The highest level occurs on Thursday and the lowest on Monday, the shift from the minimum to the maximum value corresponding to an approximately 10 percent change in volatility.

Figure 5 Naïve estimate of the weekly seasonal cycle (gray) in the logarithm of realized volatility of S&P 500 together with the estimated cycle from the best model (E17 + D) according to AIC and BIC (black)



Source: Own construction

Table 4 presents the "relevance" (weight) for the *best* model according to AIC (BIC) in the given *group of models* associated with the cell of the table calculated according to Formula (16) (Formula (17)) with respect to the *global best* model according to AIC (BIC). In other words, the *best* model in *the group of models* is compared to the *global best* model (E17 + D) using Formula (16) (Formula (17)). The relevance for the cell which contains the global best model (E17 + D) is 1, the values of relevance in other cells being rather small. This suggests that the combination of approach E used to model the yearly seasonal cycle and approach D to model the weekly seasonal cycle performs considerably better than any other explored combination of approaches.

Table 4 "Relevance" for the best model according to AIC or BIC in the given group of models associated with the cell of the table calculated according to Formula (16) (AIC, the first number) and Formula (17) (BIC, the second number) with respect to the global best model according to AIC or BIC

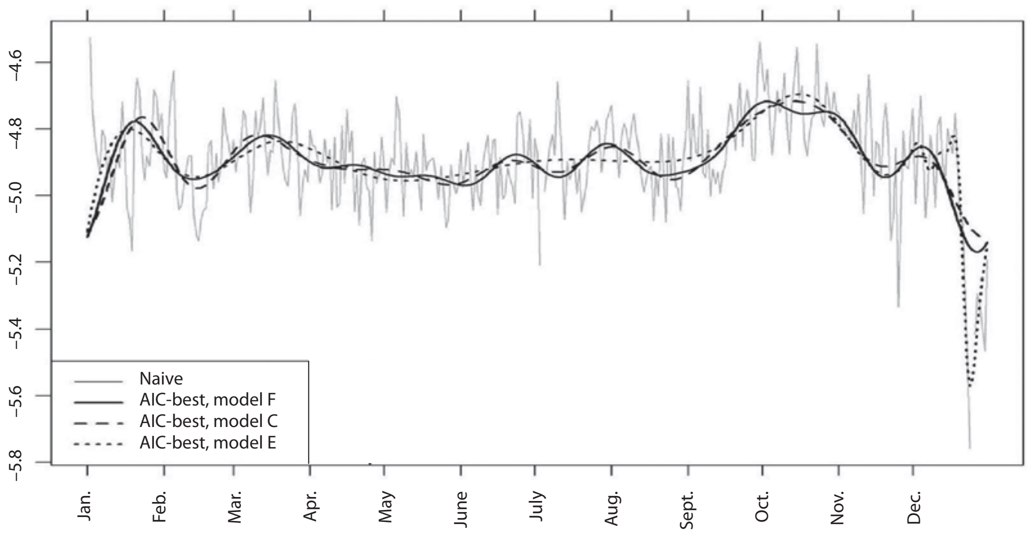
		Yearly seasonal cycle					
		N	D	F	L	C	E
Weekly seasonal cycle	N	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
	D	0, 0.03	0, 0	0, 0	0, 0	0, 0	1, 1

Source: Own construction

[15] A naïve estimate of the weekly seasonal cycle is obtained by calculating the average of the time series of the natural logarithm of realized volatility separately for different days of the week (Monday, Tuesday, Wednesday, Thursday and Friday). Further, the average for Monday is subtracted from the five averages (for Monday, Tuesday, Wednesday, Thursday and Friday).

To compare approaches to modeling the yearly seasonal cycle based on smooth functions, i.e. approaches F, C and E, in Figure 6, we present the estimated yearly seasonal cycle for the best F approach according to AIC, the best C approach according to AIC and the best E approach according to AIC. We can clearly observe that the advantage of the E approach is the expert placement of the knots which allowed for more flexibility of the seasonal cycle in December. On the other hand, in order to capture the trough in December with the use of Fourier basis functions, it would have been necessary to use a much larger value of *K*.

Figure 6 Comparison of the estimates of the yearly seasonal cycle in the natural logarithm of realized volatility of S&P 500 for the AIC-best approach among the F approaches (black), the AIC-best approach among the C approaches (dashed) and the AIC-best approach among the E approaches (dotted). The naïve estimate is presented in gray



Source: Own construction

2.3 Results for all the indices

Table 5 presents the results for the 19 indices and the corresponding time series of the natural logarithm of realized volatility based on 5-minute log returns. Specifically, the number of time series is given for which the corresponding combination of the yearly and weekly seasonal cycle model resulted in the *lowest* value of AIC or BIC.

Table 6 is constructed as follows. 19 tables analogous to Table 4 are constructed containing the values of "relevance" according to AIC and BIC. Each of the 19 tables corresponds to one specific time series. Further, for each cell of the table, we calculate (separately for AIC and BIC) the average value of the cell across the 19 tables, i.e. we obtain the average value of "relevance" based on AIC and the average value of "relevance" based on BIC. These average values are reported in Table 6.

The results from Tables 5 suggest that approach E is presumably the most appropriate approach for modeling the yearly seasonal cycle. It wins 17 times (out of 19) according to AIC and 6 times (out of 19) according to BIC. Even in terms of the "average relevance" (see Table 6) it performs very well. Model N for yearly seasonality also performs rather well as assessed by BIC. This suggests that if we use a very heavy penalty on the number of parameters, as is the case in BIC, no yearly seasonality is suggested as an appropriate model in some instances. The other approaches to modeling the yearly seasonality (D, F, L, C) do not perform very well, the reason presumably being that they are not capable of capturing the deep trough in the yearly seasonal cycle which occurs in the second half of December.

Table 5 Number of time series of the natural logarithm of realized volatility for which the corresponding combination of the yearly seasonal cycle model and the weekly seasonal cycle model resulted in the lowest value of AIC (the first number in the cell) or BIC (the second number)

		Yearly seasonal cycle					
		N	D	F	L	C	E
Weekly seasonal cycle	N	0, 3	0, 0	0, 0	0, 0	0, 0	0, 0
	D	1, 9	1, 1	0, 0	0, 0	0, 0	17, 6

Source: Own construction

Table 6 Average value of "relevance" according to AIC (first number) and BIC (second number); see text for details

		Yearly seasonal cycle					
		N	D	F	L	C	E
Weekly seasonal cycle	N	0, 0.16	0, 0	0, 0	0, 0	0, 0	0.02, 0
	D	0.05, 0.49	0.05, 0.06	0, 0	0, 0	0, 0	0.89, 0.32

Source: Own construction

For each of the 19 time series of the natural logarithm of realized volatility the following is reported in Table 7:

- **Best**: The best model according to *AIC*.
- **Day of year, min. and max.**: The estimated day of the year (in the range 1–365) where the minimum and maximum value of the *yearly* seasonal cycle occurs according to the best model selected with the use of AIC.
- **Day of week, min. and max.**: The estimated day of the week (1 = Mon, 2 = Tue, 3 = Wed, 4 = Thu, 5 = Fri) where the minimum and maximum value of the *weekly* seasonal cycle occurs according to the best model selected with the use of AIC.
- **sd:** The standard deviation of the estimated yearly cycle from the best model according to AIC.
- **R**: The "relevance" calculated according to Equation 16 of the AIC-best model where *no yearly* seasonality is present with respect to the global AIC-best model. Low values of "relevance" suggest that yearly seasonality is present in the time series.[16]
- **aver. std. err.**: Average of 365 standard errors of the estimated yearly seasonal cycle for different days of the year.
- **resid. diagn.:** The assumption of no autocorrelation, homoskedasticity and normality of error was assessed in the AIC-best model using the residuals of the model. Formal hypothesis tests were *not* used since they would have large power due to the length of the time series and would lead to the rejection of the null hypotheses even for minor and practically irrelevant deviations from the null hypothesis. Instead, qualitative assessment with focus given on practically important deviations from the null hypothesis was used, employing autocorrelation function of residuals, autocorrelation function of squared residuals, QQ plots and time series of residuals. An "OK" in the column stands for satisfied assumptions. "SNN" stands for slight non-normality, "NN" for non-normality, "SH" for slight heteroskedasticity and "OUT" for outliers.

[16] The X symbol in the R column implies that the estimation procedure did not converge while fitting the model with no yearly seasonality.

Table 7 See text for details

	Best	Day of year, min. and max.	Day of week, min. and max.	sd	R	aver. std. error	resid. diagn.
S&P 500	E17 + D	358, 289	1, 4	0.11	0	0.12	OK
DJIA	E17 + D	358, 289	1, 4	0.10	0	0.11	OK
NASDAQ 100	D + D	-	1, 3	0.09	X	0.13	SNN
FTSE 100	E17 + D	358, 289	1, 5	0.09	0	0.12	SNN
DAX	E17 + D	360, 289	1, 4	0.09	0	0.12	SNN
Euro STOXX 50	E17 + D	359, 290	1, 5	0.11	0	0.10	NN, OUT
CAC 40	E17 + D	358, 288	1, 5	0.10	0	0.11	SH, SNN
AEX	E23 + D	359, 283	1, 5	0.11	0	0.11	SNN
Swiss Index	E11 + D	360, 27	1, 4	0.06	0	0.09	SNN
IBEX 35	E17 + D	358, 337	1, 4	0.08	X	0.12	SNN
FTSE MIB	E17 + D	359, 337	1, 5	0.08	0	0.11	SH, SNN
Nikkei 225	E17 + D	359, 18	1, 5	0.07	0	0.09	SH, NN
Hang Seng	E17 + D	358, 302	5, 4	0.07	0	0.10	SNN
KOSPI	N + D	-	1, 4	-	1	0.16	OUT
FT Straits Time	E23 + D	360, 286	1, 3	0.06	0	0.09	SNN
IPC Mexico	E17 + D	359, 19	1, 3	0.08	0	0.10	SNN
Bovespa	E23 + D	360, 293	1, 4	0.10	0	0.07	SNN
All Ordinaries	E11 + D	361, 277	2, 3	0.05	0.03	0.10	SNN
S&P/TSX	E17 + D	358, 292	1, 3	0.09	0	0.13	OK

Source: Own construction

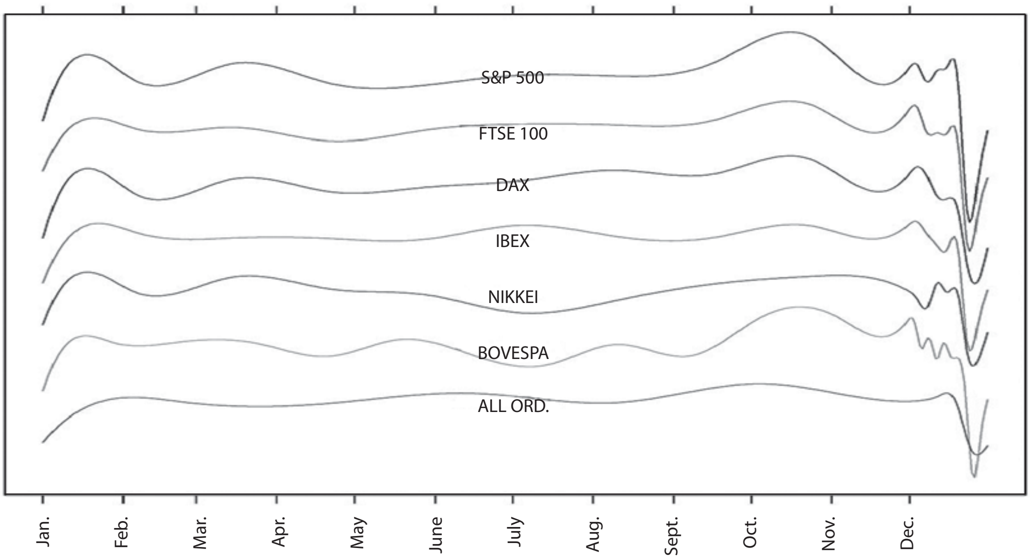
We can clearly see that according to AIC the model E17 + D performs mostly the best. The minimum of the yearly seasonal cycle occurs at the end of the calendar year in all the time series, the maximum occurring mostly in October or at the beginning of December or during January. Figure 7 presents the estimated seasonal cycles (disregarding the level of the cycle) in the best model according to AIC for 7 different indices.

Concerning the weekly seasonal cycle, the lowest volatility occurs mostly on Mondays, while the highest occurs during the second part of the week (Wednesday, Thursday or Friday).

The assumptions of no autocorrelation, heteroskedasticity and normality of errors and the presence of no outliers were reasonably satisfied[17] in most of the AIC-best models, except for Euro Stoxx 50 (where more severe non-normality and outliers are present), Nikkei (where more severe non-normality is present) and KOSPI (which exhibits outliers).

[17] In the sense that no deviations or only slight deviations from the assumptions were detected.

Figure 7 Comparison of the estimated yearly seasonal cycles for the best models according to AIC for 7 indices (S&P 500, FTSE 100, DAX, IBEX, NIKKEI, BOVESPA, ALL ORDINARIES). The levels of the cycles should not be compared since the estimated cycles are intentionally plotted at different levels to allow for a better comparison of their shapes



Source: Own construction

CONCLUSION

We have demonstrated the usefulness of various basis expansion methods in representing the yearly seasonal cycle in the natural logarithm of realized volatility based on 5-minute log returns of 19 stock market indices. Cubic splines with an expert placement of knots seem to be an appealing approach which suggests that yearly seasonality is present in most of the time series. These findings are important for future research since they play a crucial role in decisions regarding asset allocation and risk management, and should also be taken into the account while pricing options.

The presented approaches can potentially be used for modeling of other time series containing long seasonal periods such as daily time series of number of products sold, car accidents, electricity consumption and traffic volume.

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Economic Assessment of the Development Level of the Central Federal District Regions of the Russian Federation: Econometric Approach

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Abstract

The article is devoted to assessment of the development level of the Central Federal district, based on the study of gross domestic product by region by Russian Classification of Economic Activities grouped by institutional sector of the economy. We apply the author's methodological toolkit that includes the formation of individual and integrated indicators of the functioning of complex systems, considered as socio-ecological and economic systems. The algorithm of constructing indicators is based on aggregate data and econometric modeling and takes into account both individual assessment indicators and specific conditions for these systems functioning (factors of condition and impact). We present the methodology of forming the harmonic coefficient characterizing the equilibrium of socio-ecological and economic systems economic development by activities, the latter is one of the criteria of complex systems stability. This approach was tested on the example of Central Federal district regions using Russian Federation Federal State Statistics Service data for 2007–2015.

Keywords

Regional economy, socio-ecological and economic systems, econometric modeling, integral indicators, balance

JEL code

C10, P25, R11, R15

INTRODUCTION

Assessment of the state of the regions, the features of their functioning, development and resistance to the impact of external negative processes requires further development of existing approaches, the construction of refined or the creation of new models based on traditional and time-tested solutions

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in the study of complex systems existing in Russian and worldwide practice. The region as an administrative--territorial unit is a socio-ecological and economic system (SEES), and its study is connected with a set of methodological, conceptual, descriptive and interpretational problems inherent in most objects in the modern world. The identification of SEES, their analysis and assessment depend on the viewpoint, used approaches, methods of data processing and their presentation, that's why we can observe their wide variety. Modern economists, sociologists and geographers chose the indicators for assessing the state of the regions according to their applied tasks, research objectives as well as the generality of the category under study and expert conclusions (Aivazian, 2012). It determines a wide variety of approaches, techniques, models, as well as the use of private and integral indicators. The study of the economic component of the SEES in conjunction with various factors (conditions) of its functioning enables to take an objective approach to the formation of integrated strategies for the socio-economic and ecological development of the region, thereby ensuring reliable positions of the state in the world political arena.

The aim of this paper is to assess the level of economic development of the Central Federal district (CFD) regions taking into account the specific factors of their operation and also to evaluate the balance of economy by the study of certain types of economic activities.

The key method is supposed to use the author's approach presented in the part Data and Methods. It is based on the methods of system analysis, econometric modeling, correlation and regression analysis and the convolution algorithm of data.

1 SURVEY AND LITERATURE REVIEW

Economic evaluation of the region includes a number of indicators, the gross domestic product by region (GDP by region), usually calculated per capita at purchasing power parity, belongs to the most generalized ones.. The System of National Accounts (SNA) (Zander, 2012) is a commonly used methodology for calculating GDP by region. Econometric approach to clustering and regional estimation of GDP by region is considered in the work by (Aivazyan et al., 2016). Other macroeconomic indicators (indicators of economic potential use) estimating the economy of a subject of the Russian Federation include unemployment rate, consumption of fixed capital, capital productivity, material and capital intensity (Arzhenovskiy, 2014). A more detailed assessment can be made by studying the particular spheres of goods and services production, the first of them includes industry, agriculture and forestry, construction (Morozova, 2012). Service industries are divided into transport and communication services, trade and procurement, public catering and other services (market, non-market, including science, health, education, defense, management). Kolesnikov et Tolstoguzov (2016) put forward the plan of studying the regional economy state by the index of added value (the ratio of gross value added per capita to the average for Russia) for certain economy sectors and the sections included herein (types of activities according to Federal State Statistics Service of the Russian Federation – ROSSTAT). Within the framework of the integrated model of the regional economy functioning, where attention is paid to the mechanism of relations between economic entities, Bozo (2012) adduces 6 blocks that reflect the production of goods, market and non-market services, demography, government revenues and expenditures, basic balance relations, as well as the indicators of socio-economic development. A number of scientists use a cluster approach to the economic development assessment (Enright, 1996).

In a number of recent studies, the assessment of the regional economy condition is replaced by a study of its potential,i.e. hidden opportunities to ensure the production of goods and services within the economic activities of economic entities in order to meet the needs of the population. So the Ministry of Economic Development of the Russian Federation (formerly the Ministry of Economic Development and Trade of the Russian Federation) offered the methodology to assess the economic potential of the region that has 12 indicators including GDP by region per capita at purchasing power parity (Chernitsky, 2014).

The methodology of the Expert RA (Russian rating agency) contains 9 indicators estimating the investment potential and appeal of the regions.[2] However, the level of economic potential can only give an idea of the possible state of the region, provided rational use of its resources, but does not allow to assess the real state of the economy. At the same time, GDP by region and its derivatives characterize the functioning of the region and its results but not the potential, therefore, they should not be regarded as evaluation indicators. So, the study of the region's economic potential should be aimed at solving problems related to the analysis of conditions or factors of economic growth that form its investment appeal.

Another approach to the analysis of the state of economy is connected with the study of economic growth (Akerlof et al., 2001; Rothschild et Stiglitz, 1976; Schumpeter, 1935; Solow, 1956). The economic growth or sustained increase in the country's (or region's) production capacity, meaning the production of goods and services, is determined by many significant factors that make up the socio-economic and natural resource potential (Lipsits, 2006).

Lipsits (2006) distinguishes five indicators: capital-labor ratio (the average cost of physical production capital per person employed in economic activities); used technologies; the educational level of employees; methods for the allocation of limited resources; scale of production and its effect.

According to the proposed methodology of the United Nations Development Program (UNDP), indicators of sustainable development are divided into 3 categories on the basis of their target: driving force, state and response.[3] A similar analytical scheme of "impact – changes – consequences" was offered by Muhina et al. (1978) for the study of complex systems. The Organization for Economic Cooperation and Development (OECD) scheme included a theme, subtopic and indicator (Tarasova et Kruchina, 2006).

The development of the methodologies studying the sustained growth at the mesolevel leads to a change in the indicators of the state of complex systems. Gurban et al. (2016) use the growth rate measures of the corresponding indicators (GDP by region, price index, imports and exports, investments), when they treat the region as a socio-economic system according to the scheme "ecology – population – economy". Shabashev et al. (2016) identified 5 macroeconomic factors closely connected to social and demographic indicators that affect the development of the Russian Federation (RF) subjects. They are considered within the framework of a structural model formed with the help of multidimensional statistical analysis method SEPATH. Meanwhile the problem of choice of the productive and factor attributes remained open (the cause-effect relations were studied). Chichkanov et Belyaevskaya-Plotnik (2016) define indicators in terms of their threat to economic security (obsolete structure, technologies, renewal of fixed assets, profitability and energy costs).

The analysis of modern research has shown that there is no common approach to the choice of indicators of the regional economic condition. The wide variety of the latter and sometimes the confusion of the used indicators of regional economic assessment make it difficult to evaluate the state and the prospects of SEES development.

The first of such inaccuracies is to unite in a group the indicators that characterize different elements of one of the schemes divided by the target, for example, "driving force, state and response," or using the same indicator to describe the various blocks of the triad.

The second is the incorrect use of static and dynamic indicators while studying either the state (for example, the growth rate of GDP by region per capita) or the development of the complex system under study (for example, the unemployment rate).

The third problem is the simultaneous use of indicators, some of which are conditions (factors), and others are the result of the functioning of the system under consideration, and this situation leads to a violation of the cause-effect relationships characteristic of the SEES. Meanwhile, the performance

[2] The Rating Agency «Rating of RA» [online]. <http://raexpert.ru/ratings/regions/2015/regions_2015.pdf>. [cit. 18.9.2017].
[3] The United Nations development programme (UNDP) [online]. <http://www.undp.ru>. [cit. 18.9.2017].

indicators of socio-ecological-economic systems and the factors (conditions) for their achievement often belong to one and the same group, that is, the latter also act as an assessment, not being it in fact. Some of them are the derivatives from other indicators (for example, GDP by region, gross output and intermediate consumption are assessed simultaneously, and the aim of the study is not the formation and composition of the gross domestic product by region), so we can make a conclusion that the selected characteristics of the assessment are redundant.

The factorial and productive features of the SEES should be strictly differentiated according to the directions of cause-effect relationships (direct or inverse). So the growth of GDP by region depends on the investment amount. In its turn, the high GDP by region influences the investment appeal of the region. Therefore, their interrelation should be described by a ratio that takes into account the degree and direction of this dependence.

The fourth aspect is related to the use of absolute and relative characteristics in one group, because it makes it difficult to analyze and make managerial decisions.

The fifth problem deals with the substitution of the concept of the economic state of the system by economic or investment potential, as well as the level of economic development or economic growth. Herewith, both economic security and stability of the SEES can be studied.

The sixth group combines the ways of forming integrated indicators, where there is no common methodology for their design. In most of the works dealing with the integral assessment the arithmetic or geometric mean approaches are used (for example, while studying the quality of life) (Mishra et Nathan, 2013). A number of researchers use the weights of each of the individual indicators, but their weights are determined by experts. The FEA (Functioning Environment Analysis) method, which is a Russian analogue of the DEA (DEA – Data Envelopment Analysis) method (Charnes et al., 1978) makes it possible to construct an integral index by solving the non-linear programming problem (determine the weight of subindexes) (Krivonozhko et Lychev, 2010). However, their usage violates the conditions of dimensionlessness and normalization.

2 DATA AND METHODS

To eliminate the identified inconsistencies and misunderstandings of the goals and results of ongoing research, and to select indicators for assessing the state and functioning of complex systems it is important to adhere to the following recommendations.

Determine the object of research, the level of detailed elaboration of its structural features and the existing cause-effect relationships according to the model and the hypothesis.

Choose (form) a scheme for studying the system and determine the direction of the research (state, dynamics, result of functioning, potential, safety, stability, integrated assessment, etc.).

Describe and justify the choice of factor and performance indicators. Divide them into groups (social, ecological, economic), corresponding to the chosen scheme and taking into account the available information base. The research information base must meet the criteria of reliability, veracity, completeness (sufficiency) and relative availability.

Justify the choice of the method and construct the integral index. The integral indicator and the process of its construction must have the following properties:

Universality. The possibility of using the approach in any field of activity.

Dimensionlessness. It allows you to compare characteristics that are of a different nature and refer to different processes (for example, ecological, economic and social). Dimensionlessness can be achieved through the procedure of standardization.

Normalization. Bringing the indicators to a scale from 0 to 1 provides a visual representation of the data.

Normability. It provides the ability to compare the actual data with the norm, that is calculated for a particular SEES functioning under its specific conditions.

Take into account the nexus between private performance indicators. The necessity is justified by considering the object of research as a complex system.

Consider the specific conditions of functioning. It assumes the account of the SEES state in the quantitative assessment of private and integral indicators.

The approach can prevent the mistakes that are common while studying the SEES.

The individual performance indicator is determined as the ratio of the actual and normative indicator values for the selected region (Zhuravlev et al., 2013):

$$\xi_{k,i}(t) = \frac{y_{k,i}^{0}(t)}{\hat{y}_{k,i}^{0}(t)}, \tag{1}$$

where $y_{k,i}^{0}(t)$, $\hat{y}_{k,i}^{0}(t)$ are actual and normative values of standardized individual performance indicators which are specific for explored region, k is the region number, t is the time parameter ($t = 1..T$), $i = 1..m$, m is the amount of individual performance indicators, the index "0" indicates that the normalization procedure has been carried out (bringing to the scale from 0 to 1), and after standardization:

$$y_{k,i}^{0}(t) = \frac{y_{k,i}^{*} - \min\{y_{k,i}^{*}, \hat{y}_{k,i}^{*}\}}{\max\{y_{k,i}^{*}, \hat{y}_{k,i}^{*}\} - \min\{y_{k,i}^{*}, \hat{y}_{k,i}^{*}\}}, \tag{2}$$

$$\hat{y}_{k,i}^{0}(t) = \frac{\hat{y}_{k,i}^{*} - \min\{y_{k,i}^{*}, \hat{y}_{k,i}^{*}\}}{\max\{y_{k,i}^{*}, \hat{y}_{k,i}^{*}\} - \min\{y_{k,i}^{*}, \hat{y}_{k,i}^{*}\}}. \tag{3}$$

Here $y_{k,i}^{*}, \hat{y}_{k,i}^{*}$ are standardized individual performance indicators, defined by the formulas:

$$y_{k,i}^{*}(t) = \frac{y_{k,i} - M(y_i(t))}{\sigma(y_i(t))}, \tag{4}$$

$$\hat{y}_{k,i}^{*}(t) = \frac{\hat{y}_{k,i} - M(\hat{y}_i(t))}{\sigma(\hat{y}_i(t))}, \tag{5}$$

where $M(y_i(t))$, $M(\hat{y}_i(t))$, $M(y_i(t))$, $\sigma(\hat{y}_i(t))$ are expected value and standard deviation, respectively.

A generalized performance indicator is calculated as the ratio of individual performance indicators (actual and normative) (Zhukov, 2014):

$$\xi_k(t) = \frac{\sqrt{\sum_{p=1}^{m}\sum_{q=1}^{m} r_{pq} \cdot y_{k,p}^{0}(t) \cdot y_{k,q}^{0}(t)}}{\sqrt{\sum_{p=1}^{m}\sum_{q=1}^{m} \hat{r}_{pq} \cdot \hat{y}_{k,p}^{0}(t) \cdot \hat{y}_{k,q}^{0}(t)}}, \tag{6}$$

where r_{pq} and $\hat{r}_{pq}$ are the corresponding paired correlation coefficients.

If $\xi_k(t) \geq 1$, then we can assume them satisfactory otherwise we are to take measures aimed at the achievement of the norm that is calculated for each k of the object.

The application the proposed approach makes it possible to meet all the requirements for integrated assessment indicators.

The harmonic coefficient characterizing the balance of the system's functioning results can be determined by the formula (Zhukov, 2016, 2017):

$$K_k = 1 - \frac{\sigma(\xi_{k,i})}{M(\xi_{k,i})}, \tag{7}$$

where $M(\xi_{k,i})$ is expected value, $\sigma(\xi_{k,i})$ is standard deviation. The closer to the one K_k, the more harmonic the functioning of the object under research is. This indicator does not characterize its specialization, but shows the degree of compliance of the indicators under consideration with the norms, taking into account specific conditions.

The standardized models of the additive form were chosen as the models for forming the norms:

$$\hat{y}_i^* = \sum_{j=1}^{n} C_{i,j} \cdot x_j^* + \sum_{s=1}^{s} D_{i,s} \cdot z_s^*, \tag{8}$$

where n is the number of state factors, s is the number of impact factors, $C_{i,j}$, $D_{i,s}$, are corresponding weight coefficients between i productive (result of functioning of system) and j and s standardized factors of x_j^* state and z_s^* impact. When substituting actual values x_j^* and z_s^* in (8) for k region you can get a individual norm. Herewith:

$$x_j^* = \frac{x_j - M(x_j)}{\sigma(x_j)}, \tag{9}$$

$$z_s^* = \frac{z_s - M(z_s)}{\sigma(z_s)}, \tag{10}$$

where x_j, z_s are the actual values of factors of state and impact in absolute units of measurement.

As the alternatives to (8) their logarithmic analogs were considered:

$$\ln(\hat{y}_i^*) = \sum_{j=1}^{n} C_{i,j} \cdot \ln(x_j^*) + \sum_{s=1}^{s} D_{i,s} \cdot \ln(z_s^*), \tag{11}$$

that is the equivalent to the representation of non-linear models, and in case of using labor and capital – to the explanatory variables in the form of Cobb-Douglas (Cobb et Douglas, 1928):

$$\hat{y}_i^* = \prod_{j=1}^{n} x_j^{*C_{i,j}} \cdot \prod_{s=1}^{n} z_s^{*D_{i,j}}. \tag{12}$$

The coefficients of the model are determined with the help of a step-by-step least squares method (determination of important factors).

So, the actual indicator means real index value which characterizes the level of development of the region. The normative indicator is value which is calculated using the model for the region (Formula (8), (11) or (12)). When substituting actual and normative values in Formula (1) or (6) you can get individual performance indicator or generalized performance indicator correspondingly.

The information base of the research is represented by the ROSSTAT data for the regions of the Central Federal district in 2007–2015,[4] that also embrace:

- performance indicators (private assessment indicators) that include GDP by regions by economic activities (Russian Classification of Economic Activities (NACE (OKVED)) was used in the Russian Federation till 2015)[5] grouped by institutional sector of the economy;
- factors of state and impact from the qualitative point of view, they have a social, economic and socio-economic meaning.

4 Federal State Statistics Service of the Russian Federation (ROSSTAT) [online]. <http://www.gks.ru>. [cit. 20.6.2017].

5 Russian Classification of Economic Activities (NACE (OKVED)) [online]. <http://www.gks.ru/bgd/free/b02_60/Main.htm>. [cit. 18.6.2017].

"Section B. Fisheries", was not included in the analysis due to its small share (less than 0.1%) in the GDP by region structure for the regions of the Central Federal district. The variables that we use are grouped according to the division offered by Kolesnikov et Tolstoguzov (2016) and their descriptive statistics is represented in Table 1 to Table 3.

Table 1 Descriptive statistics (private assessment indicators)

№	Variables	Description	№	Variables	Description
1	y_1, y_1^*, $\hat{y}_1$, $\hat{y}_1^*$	Commodity institutional sector	4.3	$y_{4.3}$, $y_{4.3}^*$, $\hat{y}_{4.3}$, $\hat{y}_{4.3}^*$	Section I. Transport, storage and communications (H, J)
1.1	$y_{1.1}$, $y_{1.1}^*$, $\hat{y}_{1.1}$, $\hat{y}_{1.1}^*$	Section A. Agriculture, hunting and forestry (A)	4.4	$y_{4.4}$, $y_{4.4}^*$, $\hat{y}_{4.4}$, $\hat{y}_{4.4}^*$	Section J. Financial intermediation (K)
1.2	$y_{1.2}$, $y_{1.2}^*$, $\hat{y}_{1.2}$, $\hat{y}_{1.2}^*$	Section C. Mining and quarrying (B)	4.5	$y_{4.5}$, $y_{4.5}^*$, $\hat{y}_{4.5}$, $\hat{y}_{4.5}^*$	Section K. Real estate, renting and business activities (L, M, N)
2	y_2, y_2^*, $\hat{y}_2$, $\hat{y}_2^*$	Manufacturing institutional sector	5	y_5, y_5^*, $\hat{y}_5$, $\hat{y}_5^*$	Institutional sector of non-market services
2	y_2, y_2^*, $\hat{y}_2$, $\hat{y}_2^*$	Section D. Manufacturing (C)	5.1	$y_{5.1}$, $y_{5.1}^*$, $\hat{y}_{5.1}$, $\hat{y}_{5.1}^*$	Section E. Electricity, gas and water supply (D, E)
3	y_3, y_3^*, $\hat{y}_3$, $\hat{y}_3^*$	Construction institutional sector	5.2	$y_{5.2}$, $y_{5.2}^*$, $\hat{y}_{5.2}$, $\hat{y}_{5.2}^*$	Section L. Public administration and defence; compulsory social security (O)
3	y_3, y_3^*, $\hat{y}_3$, $\hat{y}_3^*$	Section F. Construction (F)	5.3	$y_{5.3}$, $y_{5.3}^*$, $\hat{y}_{5.3}$, $\hat{y}_{5.3}^*$	Section M. Education (P)
4	y_4, y_4^*, $\hat{y}_4$, $\hat{y}_4^*$	Institutional sector of market services	5.4	$y_{5.4}$, $y_{5.4}^*$, $\hat{y}_{5.4}$, $\hat{y}_{5.4}^*$	Section N. Health and social work (Q)
4.1	$y_{4.1}$, $y_{4.1}^*$, $\hat{y}_{4.1}$, $\hat{y}_{4.1}^*$	Section G. Wholesale and retail trade: repair of motor vehicles, motorcycles and personal and household goods (G)	5.5	$y_{5.5}$, $y_{5.5}^*$, $\hat{y}_{5.5}$, $\hat{y}_{5.5}^*$	Section O. Other community, social and personal services activities (R, S)
4.2	$y_{4.2}$, $y_{4.2}^*$, $\hat{y}_{4.2}$, $\hat{y}_{4.2}^*$	Section H. Hotel and restaurants (I)			

Note: Variables without any extra characters are variables in absolute units, * standardized variables, ^ model (calculated) variables, () NACE Rev. 2. sections.
Source: ROSSTAT, own construction

The non-market sector includes kinds of activities which are implemented in markets with regulated pricing mainly.

Table 2 Descriptive statistics (state factors)

№	Variables	Description	№	Variables	Description
6	The cost of fixed production assets at full accounting value at the end of the year by types of economic activity		7.4	$x_{3,2}$, $x^*_{3,2}$	Section F
6.1	$x_{1.1,1}$, $x^*_{1.1,1}$	Section A (A)	7.5	$x_{4.1,2}$, $x^*_{4.1,2}$	Section G (G)
6.2	$x_{1.2,1}$, $x^*_{1.2,1}$	Section C (B)	7.6	$x_{4.2,2}$, $x^*_{4.2,2}$	Section H (I)
6.3	$x_{2,1}$, $x^*_{2,1}$	Section D (C)	7.7	$x_{4.3,2}$, $x^*_{4.3,2}$	Section I (H, J)
6.4	$x_{3,1}$, $x^*_{3,1}$	Section F (F)	7.8	$x_{4.5,2}$, $x^*_{4.5,2}$	Section K (L, M, N)
6.5	$x_{4.1,1}$, $x^*_{4.1,1}$	Section G (G)	7.9	$x_{5.1,2}$, $x^*_{5.1,2}$	Section E (D, E)
6.6	$x_{4.3,1}$, $x^*_{4.3,1}$	Section I (H, J)	7.10	$x_{5.3,2}$, $x^*_{5.3,2}$	Section M (P)
6.7	$x_{5.1,1}$, $x^*_{5.1,1}$	Section E (D, E)	8	x_3, x^*_3	Average annual population
7	Average annual number of persons employed by types of economic activities		9	Transport	
7.1	$x_{1.1,2}$, $x^*_{1.1,2}$	Section A (A)	9.1	$x_{4.1}$, $x^*_{4.1}$	Passenger turnover of public buses
7.2	$x_{1.2,2}$, $x^*_{1.2,2}$	Section C (B)	9.2	$x_{4.2}$, $x^*_{4.2}$	Departure of passengers by public railway transport
7.3	$x_{2,2}$, $x^*_{2,2}$	Section D (C)	10	x_5, x^*_5	Morbidity per 1 000 of population, registered diseases diagnosed in patients for the first time in life

Note: Variables without any extra characters are variables in absolute units, * standardized variables, ^ model (calculated) variables, () NACE Rev. 2. sections.
Source: ROSSTAT, own construction

Table 3 Descriptive statistics (impact factors)

№	Variables	Description	№	Variables	Description
11	Investments in fixed capital by kinds of economic activities		12	z_2, z_2^*	Organic fertilizers per 1 ha of agricultural crops (in terms of 100% nutrients)
11.1	$z_{2,1}$, $z_{2,1}^*$	Section D (D)	13	Consolidated budget expenditures (by object)	
11.2	$z_{4.1,1}$, $z_{4.1,1}^*$	Section G (G)	13.1	$z_{5.2,3}$, $z_{5.2,3}^*$	Social policy
11.3	$z_{4.2,1}$, $z_{4.2,1}^*$	Section H (I)	13.2	$z_{5.3,3}$, $z_{5.3,3}^*$	Education
11.4	$z_{4.5,1}$, $z_{4.5,1}^*$	Section K (L, M, N)			

Note: Variables without any extra characters are variables in absolute units, * standardized variables, ^ model (calculated) variables, () NACE Rev. 2. sections.
Source: ROSSTAT, own construction

The model includes only substantial factors, significant at the level of no more than 5%. At the first stage, all the factors of state and impact were analyzed according to the sections of the annual statistical compendium ROSSTAT (more than 20 variables). The number of observations for each variable made 153 (input data as of 2007–2015 for 17 CFD regions).

We used least square method (backward selection) to select them.

Every absolute indicator represented in terms of value was adjusted according to purchasing power parity (PPP) in US dollars so that we could make a comparison with the international level.

3 RESULTS AND DISCUSSION

The conducted research resulted in the corresponding models in Formulas (8) and (11), whose specification is represented in Table 1 to Table 3 using author's expert system (beta version) (Zhukov, 2015):

$$\hat{y}^*_{1.1} = \underset{(0.001)}{0.668} \cdot x^*_{1.1,1} + \underset{(0.001)}{0.259} \cdot x^*_{1.1,2} + \underset{(0.035)}{0.611} \cdot z^*_2 \,, (R^2 = 0.874), \tag{13}$$

$$\ln(\hat{y}^*_{1.1}) = \underset{(0.073)}{0.706} \cdot \ln(x^*_{1.1,1}) + \underset{(0.087)}{0.241} \cdot \ln(x^*_{1.1,2}) + \underset{(0.030)}{0.006} \cdot \ln(z^*_2) \,, (R^2 = 0.824), \tag{14}$$

$$\hat{y}^*_{1.2} = \underset{(0.001)}{0.604} \cdot x^*_{1.2,1} + \underset{(0.025)}{0.346} \cdot x^*_{1.2,2} \,, (R^2 = 0.889), \tag{15}$$

$$\ln(\hat{y}^*_{1.2}) = \underset{(0.090)}{0.707} \cdot \ln(x^*_{1.2,1}) + \underset{(0.111)}{0.243}{*} \cdot \ln(x^*_{1.2,2}) \,, (R^2 = 0.879), \tag{16}$$

$$\hat{y}^*_2 = \underset{(0.001)}{0.396} \cdot x^*_{2,1} + \underset{(0.002)}{0.395} \cdot x^*_{2,2} + \underset{(0.001)}{0.226} \cdot z^*_{2,1} \,, (R^2 = 0.964), \tag{17}$$

$$\ln(\hat{y}^*_2) = \underset{(0.047)}{0.492} \cdot \ln(x^*_{2,1}) + \underset{(0.052)}{0.271} \cdot \ln(x^*_{2,2}) + \underset{(0.031)}{0.324} \cdot \ln(z^*_{2,1}) \,, (R^2 = 0.928), \tag{18}$$

$$\hat{y}^*_3 = \underset{(0.001)}{0.170} \cdot x^*_{3,1} + \underset{(0.003)}{0.786} \cdot x^*_{3,2} \,, (R^2 = 0.888), \tag{19}$$

$$\ln(\hat{y}_3^*) = \underset{(0.076)}{0.391} \cdot \ln(x_{3,1}^*) + \underset{(0.121)}{0.464} \cdot \ln(x_{3,2}^*)\,,\,(R^2 = 0.664), \tag{20}$$

$$\hat{y}_{4.1}^* = \underset{(0.001)}{0.715} \cdot x_{4.1,1}^* + \underset{(0.003)}{0.168} \cdot x_{4.1,2}^* + \underset{(0.001)}{0.121} \cdot z_{4.1,1}^*\,,\,(R^2 = 0.966), \tag{21}$$

$$\ln(\hat{y}_{4.1}^*) = \underset{(0.058)}{0.662} \cdot \ln(x_{4.1,1}^*) + \underset{(0.093)}{0.180} \cdot \ln(x_{4.1,2}^*) + \underset{(0.039)}{0.126*} \cdot \ln(z_{4.1,1}^*)\,,\,(R^2 = 0.844), \tag{22}$$

$$\hat{y}_{4.2}^* = \underset{(0.001)}{0.758} \cdot x_{4.2,2}^* + \underset{(0.001)}{0.229} \cdot z_{4.2,1}^*\,,\,(R^2 = 0.942), \tag{23}$$

$$\ln(\hat{y}_{4.2}^*) = \underset{(0.001)}{0.711} \cdot \ln(x_{4.2,2}^*) + \underset{(0.001)}{0.244} \cdot \ln(z_{4.2,1}^*)\,,\,(R^2 = 0.781), \tag{24}$$

$$\hat{y}_{4.3}^* = \underset{(0.001)}{0.171} \cdot x_{4.3,1}^* + \underset{(0.004)}{0.553} \cdot x_{4.3,2}^* + \underset{(0.001)}{0.138*} \cdot x_{4,1}^* + \underset{(0.001)}{0.141*} \cdot x_{4.2}^*\,,\,(R^2 = 0.968), \tag{25}$$

$$\ln(\hat{y}_{4.3}^*) = \underset{(0.047)}{0.182} \cdot \ln(x_{4.3,1}^*) + \underset{(0.109)}{0.391} \cdot \ln(x_{4.3,2}^*) + \underset{(0.040)}{0.205} \cdot \ln(x_{4.1}^*) + \underset{(0.037)}{0.260} \cdot \ln(x_{4.2}^*)\,,\,(R^2 = 0.895), \tag{26}$$

$$\hat{y}_{4.4}^* = \underset{(0.001)}{0.969} \cdot x_3^*\,,\,(R^2 = 0.940), \tag{27}$$

$$\ln(\hat{y}_{4.4}^*) = \underset{(0.054)}{0.976} \cdot \ln(x_3^*)\,,\,(R^2 = 0.853), \tag{28}$$

$$\hat{y}_{4.5}^* = \underset{(0.004)}{0.576} \cdot x_{4.5,2}^* + \underset{(0.001)}{0.408} \cdot z_{4.5,1}^*\,,\,(R^2 = 0.959), \tag{29}$$

$$\ln(\hat{y}_{4.5}^*) = \underset{(0.066)}{0.576} \cdot \ln(x_{4.5,2}^*) + \underset{(0.045)}{0.167} \cdot \ln(z_{4.5,1}^*)\,,\,(R^2 = 0.898), \tag{30}$$

$$\hat{y}_{5.1}^* = \underset{(0.001)}{0.567} \cdot x_{5.1,1}^* + \underset{(0.004)}{0.418} \cdot x_{5.1,2}^*\,,\,(R^2 = 0.929), \tag{31}$$

$$\ln(\hat{y}_{5.1}^*) = \underset{(0.055)}{0.507} \cdot \ln(x_{5.1,1}^*) + \underset{(0.086)}{0.465} \cdot \ln(x_{5.1,2}^*)\,,\,(R^2 = 0.860), \tag{32}$$

$$\hat{y}_{5.2}^* = \underset{(0.001)}{0.250} \cdot x_3^* + \underset{(0.001)}{0.749} \cdot z_{5.2,3}^*\,,\,(R^2 = 0.979), \tag{33}$$

$$\ln(\hat{y}_{5.2}^*) = \underset{(0.042)}{0.475} \cdot \ln(x_3^*) + \underset{(0.032)}{0.534} \cdot \ln(z_{5.2,3}^*)\,,\,(R^2 = 0.942), \tag{34}$$

$$\hat{y}_{5.3}^* = \underset{(0.001)}{0.225} \cdot x_{5.3,2}^* + \underset{(0.001)}{0.781} \cdot z_{5.3,3}^*\,,\,(R^2 = 0.982), \tag{35}$$

$$\ln(\hat{y}_{5.3}^*) = \underset{(0.039)}{0.370} \cdot \ln(x_{5.3,2}^*) + \underset{(0.024)}{0.597} \cdot \ln(z_{5.3,3}^*)\,,\,(R^2 = 0.950), \tag{36}$$

$$\hat{y}^*_{5.4} = \underset{(0.001)}{0.993} \cdot x^*_3 + \underset{(0.001)}{0.054} \cdot x^*_5, \; (R^2 = 0.962), \tag{37}$$

$$\ln(\hat{y}^*_{5.4}) = \underset{(0.037)}{0.963} \cdot \ln(x^*_3) + \underset{(0.117)}{0.051} \cdot \ln(x^*_5), \; (R^2 = 0.887), \tag{38}$$

$$\hat{y}^*_{5.5} = \underset{(0.001)}{0.466} \cdot x^*_3 + \underset{(0.001)}{0.518} \cdot z^*_{5.5,3}, \; (R^2 = 0.958), \tag{39}$$

$$\ln(\hat{y}^*_{5.5}) = \underset{(0.078)}{0.416} \cdot \ln(x^*_3) + \underset{(0.058)}{0.563} \cdot \ln(z^*_{5.5,3}), \; (R^2 = 0.883). \tag{40}$$

Here is denoted: () standard errors, * the statistical significance at 5% level, other coefficients show statistical significance at 1% level, ln(..) logarithmic model, R^2 determination coefficient.

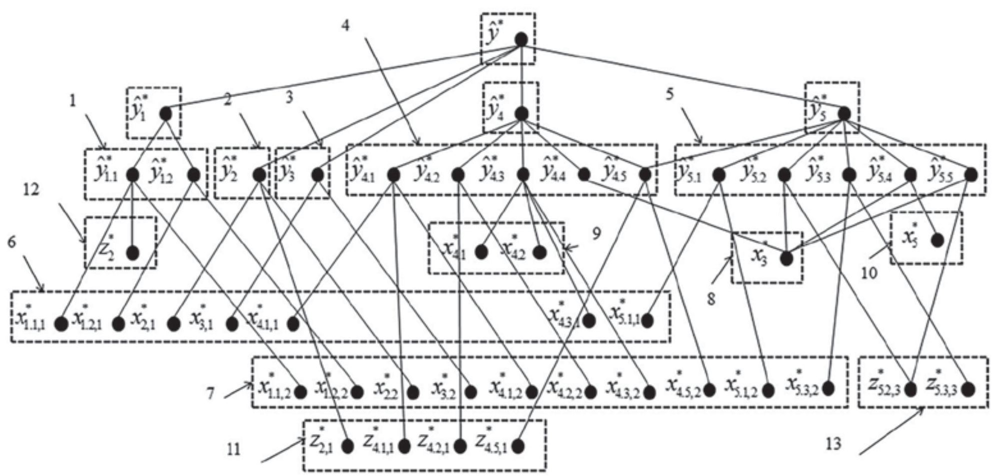
For these models the coefficient of determination are statistically significant at 1% level. For assessment the F-test was used. For minimal $R^2 = 0.664$ from these model (Formula (20)): F = 147.920 >> critical F = 4.749 ($p_1 = 2$, $p_2 = 150$, $\alpha = 0.01$). Also coefficients of models are statistical significant at 5% level (Formulas (16) and (25)). Coefficients of other models are statistical significant at 1% level. For assessment t-test was used.

The visualization of the detected significant links is shown in Figure 1.

Most of the performance indicators correlate with the size of fixed assets and the employment of the population, which is confirmed by earlier studies (Aivazyan et al., 2016). The market services sector depends on the economic (investments) and socio-economic (employment, population) factors.

While studying the influence of the state and impact factors on the non-market services sector, social (population, morbidity) and socioeconomic factors (passenger turnover and departure of passengers by rail, consolidated budget expenditures (social policy, education)) turn out to be predominant, and from a qualitative point of view, it can be easily explained.

Figure 1 The scheme that reveals the connections of factorial and productive features (performance indicators)



Note: 1 commodity sector, 2 manufacturing industries sector, 3 construction sector, 4 market services sector, 5 non-market services sector, 6 fixed assets value (FA) (by types of activities), 7 average annual number of persons employed (by types of activities), 8 average annual population, 9 passenger turnover and departure of passengers by railway transport, 10 morbidity per 1 000 people, 11 investments in fixed assets (by types of economic activities), 12 introduction of organic fertilizers, 13 expenses of the consolidated budget (under sections).
Source: Own construction

The integrated (generalized) performance indicators for the CFD regions were calculated with the help of Formulas (6) and (8)–(11) (see Table 4 and Table 5).

Table 4 The values of the integral performance indicators ξ_k *(t)* for the regions of the CFD in 2010–2015 years

№	Region/Year	2010	2011	2012	2013	2014	2015	Ranking
1	Belgorod	0.913 / 1.042	1.101 / 1.073	1.026 / 1.079	1.008 / 1.082	1.013 / 1.097	1.016 / 1.073	10
2	Bryansk	0.871 / 0.869	0.895 / 0.923	0.999 / 0.991	1.003 / 0.968	1.049 / 1.002	0.932 / 0.921	14
3	Vladimir	0.740 / 0.903	0.816 / 0.942	0.847 / 0.976	0.817 / 0.939	0.826 / 0.947	0.806 / 0.937	17
4	Voronezh	0.752 / 0.899	0.929 / 0.955	1.034 / 1.015	0.977 / 0.989	1.038 / 0.993	1.06 / 1.003	8
5	Ivanovo	0.827 / 0.811	0.739 / 0.768	0.835 / 0.814	0.883 / 0.819	0.859 / 0.805	0.825 / 0.740	16
6	Kaluga	0.946 / 1.008	1.137 / 1.061	1.102 / 1.062	1.039 / 1.028	1.070 / 1.052	0.962 / 0.967	13
7	Kostroma	1.821 / 1.114	2.606 / 1.416	2.776 / 1.537	3.027 / 1.473	3.047 / 1.552	3.427 / 1.662	1
8	Kursk	1.104 / 0.991	1.116 / 1.001	1.102 / 1.020	1.087 / 1.030	1.141 / 1.054	1.142 / 1.042	5
9	Lipetsk	0.881 / 0.953	0.962 / 1.003	0.955 / 1.019	0.939 / 0.974	1.055 / 1.032	1.061 / 1.019	7
10	Moscow	1.006 / 0.968	1.008 / 0.97	1.037 / 0.982	0.983 / 0.964	0.983 / 0.971	0.996 / 0.965	11
11	Orel	1.358 / 1.018	1.163 / 1.084	1.305 / 1.145	1.419 / 1.247	1.448 / 1.196	1.435 / 1.121	3
12	Ryazan	0.978 / 0.936	1.086 / 0.997	1.210 / 1.080	1.206 / 1.095	1.226 / 1.088	1.146 / 1.023	4
13	Smolensk	0.917 / 0.939	0.977 / 0.991	1.022 / 1.024	1.039 / 1.030	1.018 / 0.999	0.983 / 0.960	12
14	Tambov	0.942 / 0.914	1.104 / 1.004	1.175 / 1.047	1.168 / 1.076	1.343 / 1.135	1.461 / 1.104	2
15	Tver	1.002 / 1.021	1.049 / 1.058	0.959 / 1.023	0.990 / 1.010	1.003 / 1.029	1.060 / 1.015	9
16	Tula	0.749 / 0.852	0.847 / 0.900	0.863 / 0.933	0.848 / 0.922	0.891 / 0.931	0.882 / 0.918	15
17	Yaroslavl	0.891 / 0.944	0.921 / 0.999	0.959 / 1.016	1.007 / 1.015	1.046 / 1.048	1.081 / 1.043	6

Note: / – linear / logarithmic model calculations.
Source: Own calculations

Table 4 shows that the largest value of the indicator belongs to the Kostroma region, and the smallest to the Vladimir region. The value above 1 means that within the current state and impact factors subject's performance satisfies the norm. The differences in the indicator values for the linear and logarithmic models, while preserving a general trend, can be explained by the shift in the norm boundary because of its functional form's choice. Until 2014 inclusive the indicator dynamics is positive for most regions except for the Moscow and Ryazan regions, where there are small fluctuations in the overall trend. In 2015, the integral indicator value that determined the level of regional development decreased due to the stagnation of macroeconomic processes, as well as the unjustified change in purchasing power parity (12.7% in 2015 against 3.8% in 2014). The similar changes characterize the generalized indicators by the economic sector.

Anomalous value for the Kostroma region in the commodity sector (ξ_1 = 23.102), is explained by the lack of fixed assets (234 million rubles) and the employment (0.4 thousand people) in the sector, that gives the minimal value for the norm having 0,1% GDP by region in its structure. The corresponding values are in the Belgorod region (the part of GDP by region is 12.4%, fixed assets – 65 505 million rubles, employment – 22.9 thousand people). Logarithmic model allows to smoothen its inheterogeneity (ξ_1 = 1.851). A similar situation is observed for the Tambov region in the construction sector (ξ_3 = 44.005 / 3.563), which is 13.5% of GDP by region at a relatively low cost of fixed assets (5 664 million rubles) and minimum employment among the regions of the Central Federal district (17.6 thousand people).

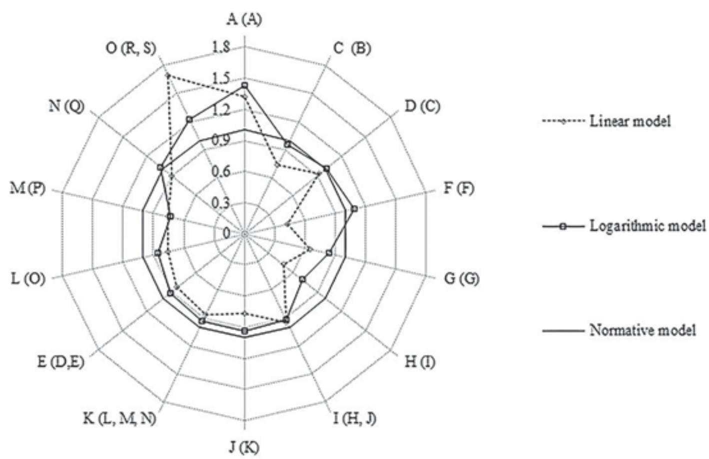
Table 5 The value of the integral performance indicators ξ_k for the CFD by the types of economic activities for 2015

№	Region/Indicator	ξ_1	ξ_2	ξ_3	ξ_4	ξ_5
1	Belgorod	0.953 / 1.077	1.363 / 1.156	1.339 / 1.148	1.232 / 1.159	1.126 / 1.005
2	Bryansk	1.033 / 1.122	1.665 / 1.149	0.978 / 0.813	1.144 / 1.072	0.798 / 0.751
3	Vladimir	1.538 / 1.169	0.924 / 1.013	0.427 / 1.08	0.695 / 0.839	1.01 / 0.937
4	Voronezh	1.449 / 1.137	1.073 / 1.038	1.146 / 1.088	0.968 / 1.013	1.011 / 0.945
5	Ivanovo	1.236 / 0.842	0.372 / 0.388	0.364 / 0.000	0.905 / 0.797	1.034 / 0.873
6	Kaluga	1.638 / 1.208	0.607 / 0.783	0.826 / 0.941	0.885 / 0.849	1.485 / 1.101
7	Kostroma	23.102 / 1.851	1.570 / 0.749	1.383 / 3.909	4.583 / 1.773	3.445 / 1.537
8	Kursk	1.061 / 1.125	3.546 / 1.731	1.849 / 1.455	0.891 / 0.857	1.254 / 1.020
9	Lipetsk	1.182 / 1.105	1.389 / 1.177	1.664 / 1.443	0.774 / 0.931	1.055 / 0.932
10	Moscow	0.810 / 1.002	0.972 / 1.016	1.368 / 1.051	0.989 / 0.966	1.033 / 0.953
11	Orel	1.264 / 1.275	4.268 / 1.016	3.835 / 2.070	1.305 / 0.859	1.785 / 1.092
12	Ryazan	1.615 / 1.162	1.102 / 1.010	0.625 / 0.738	1.237 / 1.127	1.239 / 0.994
13	Smolensk	0.938 / 0.941	0.984 / 0.866	0.643 / 0.809	1.201 / 1.120	1.130 / 0.932
14	Tambov	1.493 / 1.252	1.190 / 0.960	44.005 / 3.563	1.063 / 1.002	1.162 / 0.854
15	Tver	1.048 / 0.866	1.147 / 1.076	0.920 / 0.944	1.300 / 1.162	1.020 / 0.980
16	Tula	1.023 / 1.003	1.164 / 1.077	0.672 / 0.859	0.888 / 0.941	0.860 / 0.858
17	Yaroslavl	1.442 / 1.097	0.869 / 0.947	1.154 / 1.045	1.192 / 1.074	1.199 / 1.048

Note: ξ_1 commodity institutional sector, ξ_2 manufacturing activities institutional sector, ξ_3 construction institutional sector, ξ_4 institutional sector of market services, ξ_5 institutional sector of non-market services.
Source: Own calculations

In 2015 the lowest value of integral performance indicator (ξ = 0.806 / 0.937), it is related to the functioning of individual industries characterized by private indicators by types of economic activity (see Figure 2).

Figure 2 The diagram of individual performance indicators by NACE (OKVED) sections for the Vladimir region



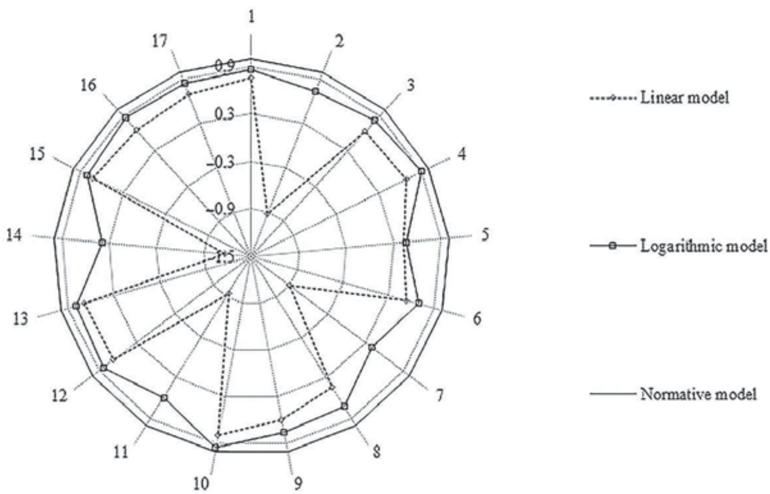
Note: A-O are NACE rev 1.1. sections, () NACE Rev. 2. sections.
Source: Own construction

Figure 2 shows that the norm is not achieved according to all the indicators except for sections A (A), N (Q) and O (R, S) which make the asymmetry of economic development. The worst values for linear and logarithmic models are observed in sections G (G) and H (I) that can be interpreted as the irrational investment, the engagement of fixed assets and wholesale and retail trade workers as well as the hotel and restaurant business. There are some differences of assessment in sector F and they are explained by the disbalance between the usage of fixed assets (shortage, $C_{3,1} = 0.170$) and the construction sector workers (overemployment, $C_{3,2} = 0.786$), they can not be detected while studying the indicator at the logarithmic model $C_{3,1} = 0.391$, $C_{3,2} = 0.464$).

The performance uniformity analysis of SEES functioning showed the differentiation of the harmonic coefficient (see Formula 7) for the CFD regions in 2015 (see Figure 3).

As we can see from figure 3 the greatest disbalance in the economic assessment indicators is observed for Bryansk, Kostroma, Orel and Tambov regions, moreover, the form of the linear representation is identical to the logarithmic mapping. The differences are in numerical values and scaling, that is explained by the oscillations smoothing effect while using the logarithmic procedure.

Figure 3 Harmonic coefficient for the CFD regions in 2015



Note: 1 – Belgorod, 2 – Bryansk, 3 – Vladimir, 4 – Voronezh, 5 – Ivanovo, 6 – Kaluga, 7 – Kostroma, 8 – Kursk, 9 – Lipetsk, 10 – Moscow, 11 – Orel, 12 – Ryazan, 13 – Smolensk, 14 – Tambov, 15 – Tver, 16 – Tula, 17 – Yaroslavl.
Source: Own construction

So we have the grounds to conclude that the use of non-linear models particularly in Cobb-Douglas form is preferable in the study of regions with a strong differentiation of the GRP structure.

CONCLUSION

The main result of the study is the solution how to assess the economic development of the Central Federal District regions using the methodology for the formation of individual and integral (generalized) performance indicators of complex systems, considered as socio-ecological and economic systems. In this regard, further analysis of each object under consideration does not require the comparative evaluation of other systems because each region has its own norms that take into account the specific conditions of its functioning. The methodology includes the data convolution and it considers the mutual influence of performance indicators characteristic for any SEES. It is an advantage over other approaches. At the same time, all the requirements for integral indicators of the object's assessment are met. Within the framework of the study, a generalized economic indicator and performance indicators by economic sectors were calculated for each CFD region, and it enabled to find out both the specific features of SEES functioning and current problems related to state and impact factors.

Also, the generated harmonic coefficient that characterizes the functioning uniformity of the SEES can help management bodies to eliminate the asymmetry of ongoing processes by redistributing the resources at their disposal and changing the factors of state and impact.

The demonstrated approach is applicable to various territorial (economic entity, municipal entity, region, district, state) and sectoral levels. Results of the investigation can be interesting to a wide range of researchers, including the international studies.

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Impact of Different Questionnaire Design Characteristics on Survey Response Rates: Evidence from Croatian Business Web Survey

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Abstract

This paper aims to determinte whether or not certain questionnaire designs could lead to higher response rates compared to other questionnaire designs. To this end, the data from conducted business web survey on a sample of Croatian enterprises was used. In the survey nine different questionnaire designs were randomly distributed across the sample of enterprises. The difference between the questionnaire designs can be found in the number of questions that have been shown to respondents per questionnaire screen and in the kind of provided pictures. The conducted analyses have shown that, when all enterprises together are observed, questionnaire designs in which all questions were shown immediately to the respondents with "negative" pictures included and where questionnaire is divided into logical groups of questions with "positive" pictures, the highest response rates in the future business web surveys could be achieved. Thereby, the recommendation of questionnaire designs for enterprises stratified according to their size, legal form, main activity and location of their headquarters are also provided.

Keywords

Business web survey, Croatian enterprises, log-linear model, Pearson's chi-square test, response rate, questionnaire design

JEL code

C12, C83, M20

INTRODUCTION

In most cases, under a web survey it is understood a survey where respondents interact with the survey through their Internet browser either on their personal computer, tablet, smartphone or other similar device

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with an access to Internet (Dillman, Smyth, Christian, 2014). If a list-based sampling frame is available, sampling process can be done similarly as in the traditional survey by using sampling frame. However, sometimes the list-based sampling frames are not available and in such a case different approaches can be used as intercept surveys, pre-recruited panel surveys, entertainment polls, surveys using "harvested" e-mail lists, unrestricted self-selected surveys and volunteer (opt-in) panels (see Fricker, 2008).

The response rates of all surveys modes, including web surveys, are reporting to a decline (Carley-Baxter et al., 2009). Low response rates could lead to different problems of which the problem of bias is the most important one (Rindfuss et al., 2015). However, it has to be kept in mind that in survey research the response representativeness is more important than the response rate. But, if response rate carries representativeness, which is the case of most of web surveys, the response rate is important (Cook, Heath, Thompson, 2000). Additionally, response rate is especially important in web surveys because it has been shown by Lozar Manfreda et al. (2008) that, on average, web surveys show an 11% lower response rate compared to other survey modes. The same has been emphasized by Nulty (2008) inby comparison paper based and web surveys. Consequently, web surveys with extremely low response rates, surveys with response rates below 3%, are not rare now (Petchenik, Watermolen, 2011; Dumičić, Bregar, Žmuk, 2014).

Web surveys invitations to participate in a web survey are usually sent by e-mails. In order to increase response rates, in some cases, advance letters can be sent. Furthermore, the design of web surveys invitations may have an impact on response rate. Kaplowitz et al. (2011) have shown that different design elements of web survey invitations, like invitation mode, subject line, location of hyperlink, length of the invitation text, and survey time/effort estimate, show a significant impact on response rates. Furthermore, reminder intervals may also significantly affect the response rates (Lemon, 2007).

In order to increase response rates, respondents may be offered certain incentives which could be prepaid and promised, and they can be given in monetary and in non-monetary forms like gifts, products and similar (Bosnjak, Tuten, 2003). It has been shown that incentives, in general, have a significant positive impact on response rates but data quality problem with ethical consideration in line with the question of cost-effectiveness of incentives increases (Cobanoglu, Cobanoglu, 2003; Göritz, 2006, Singer, Ye, 2013; Cole, Sarraf, Wang, 2015).

The response rates could be impacted by different factors, among which it is worth to mention the lack of potential respondents' interest, excessive survey length and poor survey design, and the lack of interest (Dillman, Smyth, Christian, 2014). There are many theories that can be used to motivate potential respondents: cognitive dissonance theory (Festinger, 1957), reasoned action theory (Ajzen, Fishbein, 1980), adult-to-adult communication style (Comley, 2006), influence theory (Cialdini, 1984), leverage-saliency theory (Groves, Singer, Corning, 2000), cost-benefit theory (Singer, 2011), gamification theory (Lai, Bristol, Link, 2012). Some of these theories have also a direct impact on the questionnaire design. In order to avoid problems with survey length, poor survey design and technical difficulties, Monroe and Adams (2012) recommended that a pilot web survey should be conducted.

In order to improve further response rates, additional attention should be given to questionnaire design. When web surveys are observed, a decision whether to use scrolling or paging design should be made (Couper, 2008). Each of these two questionnaire designs have some advantages and disadvantages. Vehovar, Lozar Manfreda and Batagelj (2000), Forsman and Varedian (2002), Peytchev et al. (2006), and Mavletova and Couper (2014) have found that there is no statistically significant difference in response rates between those two questionnaire designs. However, Topoel, Das and Van Soest (2009) have shown that response rates decrease with an increase of the number of items appearing on a single screen. Still, it is not precisely defined how many questions per questionnaire screen should be given in order to achieve higher response rates (Schonlau, Fricker, Elliott, 2002; FluidSurveys, 2014).

Following the increase of Internet connection speed (Belson, 2017), the question of whether to includeg visual and interactive enhancements to questionnaires appears. Images or pictures can provide additional information and they can be used as questions, supplements or as incidentals (Couper, 2008). By smart use of pictures, a respondent can understand the questions easily and correctly (Couper, Tourangeau, Kenyon, 2004). However, pictures could be a source of bias if they are not carefully selected (Couper, Conrad, Tourangeau, 2007). Couper, Tourangeau and Kenyon (2004) did not found any evidence that pictures increase respondents' motivation to complete surveys. In other words, according to Couper, Tourangeau and Kenyon (2004) there is no significant impact of pictures on response rates in web surveys. Ganassali (2008) used simple "illustrations" in her survey and came up with the conclusion that the illustrations did not have an impact on the quality of the collected data. On the other hand, Deutskens et al. (2004) have shown that the visual presentation of the questionnaire had a significantly lower response rate than the textual presentation of the questionnaire. Kivu (2010) concluded that pictures have negative impact on dropouts in short questionnaires but positive in long ones.

The impact of using scrolling or paging design, or the impact of the number of questions per questionnaire screen, and the impact of pictures on responses rates is being inspected in general web surveys. However, this research is neglected when business web surveys are observed. Enterprises play a different role in society and have different characteristics as respondents than other individual respondents. Therefore, the impact of the number of questions per questionnaire screen and that of pictures on response rates could be different in business web surveys comparing to the impacts in general web surveys. The main effects of the number of questions per questionnaire screen and of pictures on response rates in business web surveys have already been investigated (Žmuk, 2017, 2018) but the common or interaction effect of these two questionnaire design elements has not been inspected yet. To this end, the main research aim of the paper is to found out which combination of these two questionnaire design elements, the number of questions per questionnaire screen shown to respondents and the kind of presented pictures, should result in the highest response rate in a business web survey. The research hypothesis claims that the simplest questionnaire design should lead to the highest response rate. In the observed case it is reffered to such questionnaire design where all questions, without pictures, are presented at once to respondents.

The paper is organized as follows. After introduction including brief literature review and the explanation of the main aim of the paper, in the first section the methodology used in the paper is desribed. In the first section the main emphasize is given to presenting survey approach to obtain necessary data for the analysis. In the second section results of conducted analyses are shown. In its first part the relationship between different questionnaire designs regards to the number of responses is observed, when different number of questions shown per questionnaire screen and different kind of pictures are included in the questionnaires. In the second part of the analysis section results from conducted log-linear analysis are shown. In the final section of the paper, the results of conducted analysis are discussed and included into the context of business web surveys. The final section provides conclusion.

1 METHODOLOGY APPROACH AND SURVEY RESPONSE RESULTS

In the paper, data from the business web survey conducted on a sample of Croatian enterprises are used. The topic of the web survey was the position of statistical methods in Croatian enterprises and their attitude towards statistical methods usefulness in their business. The web survey started on October 4, 2016 when the invitation e-mails for survey participation have been sent to active Croatian enterprises that, according to their legal form, are joint stock enterprises, limited liability enterprises or simple limited liability enterprises. The enterprise is assumed to be active if it had sent obligatory financial statements, which are related to business transactions conducted in 2015, to government institutions. Due to this, only corporations were observed in the survey. Namely, enterprises of other legal forms do not have so strict rules about sending and publishing financial statements. Consequently, for those enterprises it is difficult to detect if they are active and do some business or they exist only on the paper. Furthermore, if an enterprise is active, it is assumed that it will receive the invitation, read it and make a decision about conducting some action upon the received invitation. This process is very important to observe as a complete one because otherwise the response rates cannot be measured precisely enough. In order to determine response rate, only the enterprises that completed questionnaire will be taken into account.

The BizNet database was used as sampling frame (Croatian Chamber of Economy, 2017). The BizNet database includes directory of Croatian enterprises and is created and maintained by the Croatian Chamber of Economy. Identically, the BizNet database can be observed as an administrative source of data. Consequently, the data from the BizNet database are considered to be relevant, up-to-date and precise. According to the BizNet database, on the survey starting date there were 102 474 active corporations in Croatia. However, it turned out that not all corporations did provide their e-mail address. It seems that those corporations just do not use e-mail in their business because of different reasons (Just Add Content, 2017). Thus, 64 619 corporations or 63.06% of active corporations have been excluded from the survey because they could not receive the survey participation invitation letter which was sent by e-mail.

The survey participation invitation letter was sent to 37 855 corporations by e-mail. In the invitation letters unique hyperlinks for each corporation were provided. In that way, it has been followed by the information which of nine developed questionnaire versions have been fully completed. In all nine questionnaire versions the wording of questions was exactly the same. The first question in the questionnaire was a filter question to make distinction between corporations that use and that do not use statistical methods. To make sure that respondents provide the correct answer, a brief explanation of what it is understood under the use of statistical methods is given. After the filter question, corporations using statistical methods received 14 specific questions, whereas corporations not using statistical methods obtained four specific questions. At the end of questionnaire, five demographic questions and one optional question about respondents' suggestions were provided to both kinds of corporations. All questions in the questionnaire, except for the last one, were set to be obligatory. The different number of questions resulted in different survey time-limits needed to complete the questionnaire. In order to reduce the impact of different questionnaire lengths on response rate almost all questions were closed ones. Furthermore, in both cases questionnaire length could be considered as short one. Because of that the effect of different questionnaire lengths can be considered as negligible and it will not be analysed in more detail in the paper.

Whereas the question texts were the same, the questionnaire design was different in the nine developed questionnaire versions. The difference between the questionnaire versions can be found in a different number of questions that are shown to the respondent per questionnaire screen and in a different kind of pictures that are displayed to the respondent. Three levels of number of questions per questionnaire screen were introduced. At the first level only one question per questionnaire screen was presented to the respondent. In that case the respondent should answer the question presented in order to make progress in the survey and advance to next question. The second level is defined as a logical group of questions. In this case questions were grouped according to the topic. Enterprises using statistical methods had five groups of questions altogether whereas enterprises not using statistical methods had three groups of questions. The number of questions in the defined groups ranged from one (in case of filter question) to six questions. At the third level all questions were shown to the respondents immediately. It has to be emphasized that in the questionnaire design skip function was added. In other words, enterprises that use statistical methods were not able to see questions which were mentioned to be answered by enterprises that do not use statistical methods and vice versa.

Three different situations considering pictures in the questionnaire also were also created. In the first case, questionnaire did not include any picture. However, in other cases some pictures have been added to the questionnaire. In the second case, "positive" pictures and in the third case "negative" pictures were placed in the questionnaire. The difference between these two kinds of pictures subsists in what they suggest. For example, the question where the respondents were asked about the impact of statistical methods used in their business on average change of business results a bar chart with added arrow of change was placed. On the "positive" picture the bars are at each following period higher whereas added arrow emphasized the upward trend. On the other hand, on the "negative" picture the bars are at each following period lower whereas added arrow emphasized the downward trend. The questionnaire is short and because of that only few pictures were added to it. To the enterprises that use statistical methods five figures had been shown whereas enterprises that do not use statistical methods only two figures had been presented. It seemed that the number of pictures added to the questionnaire was very low. However, it should be taken into account that the number of questions in the questionnaire is 21, for enterprises that use statistical methods, and 11, for enterprises that do not use statistical methods. Furthermore, the pictures have been carefully chosen and placed next to the questions making a harmonious whole. The option to put a picture next to each question was excluded because the respondents could open the questionnaire in their smartphones and many pictures could cause some technical troubles to the respondents (especially at the questionnaire design where all questions are shown at one time).

Three levels of number of questions per questionnaire screen and three picture cases led to overall nine different questionnaire versions. The intention was that enterprises should be randomly associated to the different questionnaire versions. However, in order to have equally distributed enterprises across all nine questionnaire versions, systematic sampling was applied. In the process of systematic sampling, questionnaire versions have been randomly ordered and associated to enterprises that were alphabetically ordered. The process of systematic sampling was conducted by taking into account some characteristics of the enterprises like legal form (joint stock, limited liability, simple limited liability), size (small, medium, large) and main activity (industrial, trade, service, other). On that way, the number of contacted enterprises should be nearly the same at the overall level but also at each stratum level.

As mentioned earlier, the paper is focused on corporations, which Croatian law recognizes in a legal form of joint stock, limited liability and simple limited liability enterprises (Narodne novine, 2011). The stratification of enterprises by their size on small, medium and large enterprises was made according to the Accounting Act (Narodne novine, 2015). The National Classification of Economic Activities (Narodne novine, 2007) recognizes 21 different areas of enterprises activities. However, that seems to be a very ample and too detailed classification. Consequently, the number of main activities was reduced to only four by merging the original areas of enterprises' activities.

In the paper additional stratification of enterprises will be introduced. In this stratification enterprises are observed according to the location of their headquarters in Croatia. The stratification will be made according to the Nomenclature of territorial units for statistics (Eurostat, 2015). According to the Nomenclature of territorial units for statistics, at the second level of classification Croatia has been divided into Adriatic Croatia and Continental Croatia. It has to be emphasized that enterprises have not been associated to different questionnaire versions according to this stratification. Consequently, the difference in the number of contacted enterprises between the nine questionnaire versions will be more obvious here than at other characteristics of enterprises. In other words, approximately the same number of small enterprises will get each of the nine questionnaire versions. The same applies to medium, large, joint stock, limited liability, simple limited liability, industrial, trade, service and other enterprises. However, when enterprises from Adriatic Croatia or Continental Croatia are to be observed, all nine questionnaire versions will not be so equally distributed across the enterprises as it will be at previously mentioned categories. This problem will be solved later in analysis by using the weighting approach. In the weighting approach the number of contacted enterprises across all questionnaire versions will be equalized to the highest number of contacted enterprises at the observed strata level. The important thing is that this correction will impact on the number of enterprises that completed the questionnaire as well.

During the survey period two reminders were sent. The first survey reminder was sent on November 8, 2016 whereas the second reminder was sent on December 6, 2016. The survey period of collecting data officially ended on January 1, 2017. The analysis has shown that, from 37 855 contacted enterprises, 780 completed the questionnaires. The resulting Response Rate 1 or ratio of completed questionnaires and contacted enterprises (American Association for Public Opinion Research, 2016) is 2.06%. In Table 1 the number of contacted enterprises, the number of completed questionnaires and resulting Response Rates 1 for all enterprises according to the questionnaire versions are shown.

Table 1 Survey response results according to number of questions shown per questionnaire screen and according to kind of pictures included in the questionnaire, all enterprises

Number of questions shown	Category	Pictures			Total
		Without	Negative	Positive	
One question per screen	Contacted enterprises	4 207	4 209	4 208	12 624
	Response Rate 1 (in %)	2.38	2.09	2.38	2.28
Group of questions	Contacted enterprises	4 206	4 204	4 206	12 616
	Response Rate 1 (in %)	1.71	1.50	2.07	1.76
All questions	Contacted enterprises	4 206	4 204	4 205	12 615
	Response Rate 1 (in %)	2.28	2.43	1.71	2.14
Total	Contacted enterprises	12 619	12 617	12 619	37 855
	Response Rate 1 (in %)	2.12	2.01	2.05	2.06

Source: Own survey

According to Table 1 the highest Response Rate 1, of 2.43%, was achieved by enterprises that received the questionnaire version where all questions were immediately shown and where negative pictures were presented to the respondents. The Response Rate 1 is observed because this measure takes into account only completed questionnaires. On that way, the effect of different questionnaire designs on the response count is the most appropriately observed.

Table 2 Survey response results according to number of questions shown per questionnaire screen and according to kind of pictures included in the questionnaire, enterprises observed according to their size

Number of questions shown	Category	Pictures			Total
		Without	Negative	Positive	
Small enterprises					
One question per screen	Contacted enterprises	4 044	4 046	4 044	12 134
	Response Rate 1 (in %)	2.37	2.17	2.40	2.32
Group of questions	Contacted enterprises	4 042	4 041	4 043	12 126
	Response Rate 1 (in %)	1.76	1.48	2.05	1.76
All questions	Contacted enterprises	4 042	4 041	4 043	12 126
	Response Rate 1 (in %)	2.20	2.38	1.68	2.09
Total	Contacted enterprises	12 128	12 128	12 130	36 386
	Response Rate 1 (in %)	2.11	2.01	2.04	2.06

Source: Own survey

Table 2 Survey response results according to number of questions shown per questionnaire screen and according to kind of pictures included in the questionnaire, enterprises observed according to their size (continuation)

Number of questions shown	Category	Pictures			Total
		Without	Negative	Positive	
Medium enterprises					
One question per screen	Contacted enterprises	126	126	126	378
	Response Rate 1 (in %)	3.17	0.00	0.79	1.32
Group of questions	Contacted enterprises	127	125	126	378
	Response Rate 1 (in %)	0.79	2.40	3.17	2.12
All questions	Contacted enterprises	126	126	126	378
	Response Rate 1 (in %)	2.38	3.97	2.38	2.91
Total	Contacted enterprises	379	377	378	1134
	Response Rate 1 (in %)	2.11	2.12	2.12	2.12
Large enterprises					
One question per screen	Contacted enterprises	37	37	38	112
	Response Rate 1 (in %)	0.00	0.00	5.26	1.79
Group of questions	Contacted enterprises	37	38	37	112
	Response Rate 1 (in %)	0.00	0.00	0.00	0.00
All questions	Contacted enterprises	38	37	36	111
	Response Rate 1 (in %)	10.53	2.70	2.78	5.41
Total	Contacted enterprises	112	112	111	335
	Response Rate 1 (in %)	3.57	0.89	2.70	2.39

Source: Own survey

In Table 2 the numbers of contacted enterprises, the numbers of completed questionnaires and the values of Response Rates 1 of enterprises stratified by their size and for different questionnaire versions are shown. Since far more small enterprises than medium or large ones can be identified in the population, it is natural that the number of completed questionnaires in small enterprises is far above the numbers of completed questionnaires achieved at medium and at large enterprises. Despite very low number of completed questionnaires, large enterprises showed the highest overall Response Rate 1 (2.39%) whereas small enterprises the lowest one (2.06%). It has to be emphasized that response of large enterprises would be higher if they would not have some restrictive attitude to the participation in such research. Namely, large enterprises often have business decision not to participate in surveys because they do not want to share their business secrets. That could be applied to small enterprises but it is not as obvious as in larger enterprises where business secretes are of greater value.

In small enterprises the highest response rate was achieved at enterprises that received questionnaire where one question per questionnaire screen and positive pictures were shown. In medium-size enterprises the highest response rate was achieved in enterprises that received questionnaire where all questions were shown at once and negative pictures were included. The questionnaire version with all questions shown at once and no pictures achieved the highest response rate in large enterprises. Thus it is obvious that the questionnaire version which would results in the highest response rate is different between enterprises of different size.

Table 3 Survey response results according to number of questions shown per questionnaire screen and according to kind of pictures included in the questionnaire, enterprises observed according to their legal form

Number of questions shown	Category	Pictures			Total
		Without	Negative	Positive	
Joint stock enterprises					
One question per screen	Contacted enterprises	80	81	80	241
	Response Rate 1 (in %)	5.00	1.23	3.75	3.32
Group of questions	Contacted enterprises	81	80	81	242
	Response Rate 1 (in %)	0.00	2.50	3.70	2.07
All questions	Contacted enterprises	80	79	79	238
	Response Rate 1 (in %)	1.25	1.27	1.27	1.26
Total	Contacted enterprises	241	240	240	721
	Response Rate 1 (in %)	2.07	1.67	2.92	2.22
Limited liability enterprises					
One question per screen	Contacted enterprises	3 961	3 961	3 962	11 884
	Response Rate 1 (in %)	2.27	2.17	2.32	2.26
Group of questions	Contacted enterprises	3 959	3 960	3 959	11 878
	Response Rate 1 (in %)	1.74	1.41	1.94	1.70
All questions	Contacted enterprises	3 959	3 959	3 960	11 878
	Response Rate 1 (in %)	2.20	2.40	1.77	2.12
Total	Contacted enterprises	11 879	11 880	11 881	35 640
	Response Rate 1 (in %)	2.07	1.99	2.01	2.03
Simple limited liability enterprises					
One question per screen	Contacted enterprises	166	167	166	499
	Response Rate 1 (in %)	3.61	0.60	3.01	2.40
Group of questions	Contacted enterprises	166	164	166	496
	Response Rate 1 (in %)	1.81	3.05	4.22	3.02
All questions	Contacted enterprises	167	166	166	499
	Response Rate 1 (in %)	4.79	3.61	0.60	3.01
Total	Contacted enterprises	499	497	498	1 494
	Response Rate 1 (in %)	3.41	2.41	2.61	2.81

Source: Own survey

According to the results from Table 3, the highest overall response rate was achieved in simple limited liability enterprises (2.81%) and the lowest one in limited liability enterprises (2.03%). The joint stock enterprises reached response rate of 2.22%. In joint stock enterprises the highest response rate was achieved in enterprises that received questionnaire without pictures and with one question per questionnaire screen shown whereas in limited liability enterprises the highest response rate was achieved in enterprises that received questionnaire where all questions were shown at once and negative pictures were included. The questionnaire version with all questions shown at once and no pictures achieved the highest response rate in simple limited liability enterprises.

Table 4 Survey response results according to number of questions shown per questionnaire screen and according to kind of pictures included in the questionnaire, enterprises observed according to their main activity

Number of questions shown	Category	Pictures			Total
		Without	Negative	Positive	
Industrial enterprises					
One question per screen	Contacted enterprises	1 254	1 252	1 252	3 758
	Response Rate 1 (in %)	2.07	2.16	1.84	2.02
Group of questions	Contacted enterprises	1 254	1 250	1 251	3 755
	Response Rate 1 (in %)	1.67	1.68	1.84	1.73
All questions	Contacted enterprises	1 254	1 251	1 251	3 756
	Response Rate 1 (in %)	1.52	2.56	1.76	1.94
Total	Contacted enterprises	3 762	3 753	3 754	11 269
	Response Rate 1 (in %)	1.75	2.13	1.81	1.90
Trade enterprises					
One question per screen	Contacted enterprises	1 067	1 068	1 065	3 200
	Response Rate 1 (in %)	1.69	2.25	1.88	1.94
Group of questions	Contacted enterprises	1 066	1 067	1 065	3 198
	Response Rate 1 (in %)	1.31	0.94	1.88	1.38
All questions	Contacted enterprises	1 064	1 066	1 066	3 196
	Response Rate 1 (in %)	2.07	1.78	1.13	1.66
Total	Contacted enterprises	3 197	3 201	3 196	9 594
	Response Rate 1 (in %)	1.69	1.66	1.63	1.66
Service enterprises					
One question per screen	Contacted enterprises	1 753	1 755	1 758	5 266
	Response Rate 1 (in %)	2.85	1.94	2.96	2.58
Group of questions	Contacted enterprises	1 752	1 753	1 755	5 260
	Response Rate 1 (in %)	2.00	1.71	2.45	2.05
All questions	Contacted enterprises	1 754	1 754	1 756	5 264
	Response Rate 1 (in %)	2.79	2.68	2.11	2.53
Total	Contacted enterprises	5 259	5 262	5 269	15 790
	Response Rate 1 (in %)	2.55	2.11	2.51	2.39
Other enterprises					
One question per screen	Contacted enterprises	133	134	133	400
	Response Rate 1 (in %)	4.51	2.24	3.76	3.50
Group of questions	Contacted enterprises	134	134	135	403
	Response Rate 1 (in %)	1.49	1.49	0.74	1.24
All questions	Contacted enterprises	134	133	132	399
	Response Rate 1 (in %)	4.48	3.01	0.76	2.76
Total	Contacted enterprises	401	401	400	1 202
	Response Rate 1 (in %)	3.49	2.24	1.75	2.50

Source: Own survey

The results from Table 4 show that service enterprises had the most completed questionnaires whereas the highest Response Rate 1 was achieved by other enterprises. The highest response rates are achieved when trade, service and other enterprises are observed and when one question per questionnaire screen is shown to the respondents. However, the trade enterprises show the highest response rate when questionnaire had negative pictures, service enterprises when positive pictures were included in the questionnaire whereas other enterprises had the highest response rate when no pictures had been included in the questionnaire. Industrial enterprises had the highest response rate when all questions were shown at once and when negative pictures were included.

Table 5 Survey response results according to number of questions shown per questionnaire screen and according to kind of pictures included in the questionnaire, enterprises observed according to the location of their headquarters

Number of questions shown	Category	Pictures			Total
		Without	Negative	Positive	
Adriatic Croatia					
One question per screen	Contacted enterprises	1 577	1 586	1 592	4 755
	Response Rate 1 (in %)	2.47	2.02	2.07	2.19
Group of questions	Contacted enterprises	1 618	1 557	1 632	4 807
	Response Rate 1 (in %)	1.79	1.48	1.96	1.75
All questions	Contacted enterprises	1 609	1 642	1 592	4 843
	Response Rate 1 (in %)	1.80	1.89	1.44	1.71
Total	Contacted enterprises	4 804	4 785	4 816	14 405
	Response Rate 1 (in %)	2.02	1.80	1.83	1.88
Continental Croatia					
One question per screen	Contacted enterprises	2 630	2 623	2 616	7 869
	Response Rate 1 (in %)	2.32	2.13	2.56	2.34
Group of questions	Contacted enterprises	2 588	2 647	2 574	7 809
	Response Rate 1 (in %)	1.66	1.51	2.14	1.77
All questions	Contacted enterprises	2 597	2 562	2 613	7 772
	Response Rate 1 (in %)	2.58	2.77	1.88	2.41
Total	Contacted enterprises	7 815	7 832	7 803	23 450
	Response Rate 1 (in %)	2.19	2.13	2.19	2.17

Source: Own survey

In Table 5 survey response results for enterprises according to the location of their headquarters are presented. It has been shown that enterprises from Continental Croatia have higher number of completed questionnaires and higher overall Response Rate 1 than enterprises from Adriatic Croatia. The enterprises from Adriatic Croatia achieved the highest response rate at questionnaires without pictures and where one question per questionnaire screen is shown. On the other hand, the enterprises with the headquarters in Continental Croatia reached the highest response rate at questionnaire version that includes negative pictures and where all questions are shown at once to the respondents.

The results given in Tables 1–5 show that there is no unique favourite questionnaire version that would result in the highest response rate in all observed stratifications of enterprises. Due to the fact that a sample of Croatian enterprises is observed, the inferential statistical approach is going to be used to detect one or more questionnaire designs which result in significant higher response than the other questionnaire designs. In the first step it is going to be investigated if there is a relationship between the number of questions shown per questionnaire screen and the kind of pictures which are (not) included in the questionnaire. In order to inspect this relationship Pearson's chi-square tests will be performed at overall level and at each strata sublevel. In the following step the impact of different questionnaire designs on the number of completed questionnaire is going to be quantified by using log-linear analysis. The log-linear analyses will be conducted on original numbers of completed questionnaires but also on corrected numbers of completed questionnaires where different number of contacted enterprises is going to be taken into account.

2 ANALYSIS OF NUMBERS OF QUESTIONS AND FIGURES IMPACT ON RESPONSE RATES

2.1 Determining Relationship between Numbers of Questions and Figures

The response data, given as the number of completed questionnaires, from contingency tables presented in Tables 1–5 are used in conducting Pearson's chi-square tests. The Pearson's chi-square test is used to test the relationship between two categorical variables. In the observed case the relationship between the number of questions shown per questionnaire screen and the kind of pictures included in the questionnaire is inspected. In Table 6 the results of conducted Pearson's chi-square tests are given.

Table 6 Pearson's chi-square tests and Cramer's V measure results, in the analysis used responses are measured as the numbers of completed questionnaires

Observation level of enterprises	Pearson's chi-square test			Cramer's V test statistic
	Test statistic	df	p-value	
All	10.249	4	0.036	0.081
Size				
Small	9.070	4	0.059	0.078
Legal form				
Limited liability	7.319	4	0.120	0.071
Main activity				
Industrial	2.587	4	0.634	0.078
Trade	7.347	4	0.119	0.152
Service	6.083	4	0.194	0.090
NUTS-2 region				
Adriatic Croatia	2.911	4	0.576	0.073
Continental Croatia	8.066	4	0.089	0.089

Source: Own survey

According to the results from Table 6, when all enterprises are observed together Pearson's chi-square test results lead to rejection of the null hypothesis at significance level of 5%. Thereby, the confidence that those variables are in some way related, considering the numbers of completed questionnaires, is gained. However, Cramer's V statistic of only 0.081 suggests that this association is very weak (Cramer, 1999).

If enterprises are observed according to one of their characteristic, the conclusion is opposite to the conclusion which has been brought when all enterprises are observed. For example, if only small enterprises are observed, at significance level of 5%, the null hypothesis that variables the number of questions shown per questionnaire screen and the kind of pictures included in the questionnaire are independent cannot be rejected. It has to be emphasized that at certain categories of enterprises the assumption of Pearson's chi-square test was not fulfilled and they were omitted from Table 6. The problem arose because there were some cells in contingency tables with expected values less than five (Cochran, 1952; Bolboaca et al., 2011).

2.2 Measuring Impact of Numbers of Questions and Figures on Response Rates

In the previous chapter the presence of relationship between the number of questions shown per questionnaire screen and the kind of pictures included in the questionnaire was tested. In this chapter the interaction effect size of these two variables on responses will be observed. In order to find out which combination of the observed values result in the highest number of completed questionnaires or responses, hierarchical log-linear analysis will be applied.

Log-linear analysis is a statistical technique used to examine the relationship between categorical variables (Howell, 2009). Because hierarchical log-linear analysis is used, the main effects and the interaction effects are included in the model (Howell, 2009). In order to reduce the impact of expected frequencies lower than five, the value of 0.5 has been added to all observed cells (Field, 2005). Furthermore, for each formed log-linear model it was inspected if the interaction effect was statistically significant by using backward elimination approach with significance level of 0.05. However, no matter whether the interaction effect appears to be or not to be statistically significant, the interaction effect size will be observed (George, Mallery, 2016). The interaction effect will be observed for all enterprises in total and at each strata level. In Table 7 log-linear model parameter estimates of the interaction effect are provided.

Table 7 Log-linear model parameter estimates of the interaction effect sizes of variables number of questions shown per questionnaire screen and kind of pictures included in the questionnaire, all enterprises

Number of questions shown	Pictures		
	Without	Negative	Positive
One question per screen	0.009	–0.052	0.043
Group of questions	–0.051	–0.118	**0.169**
All questions	0.042	**0.170**	–0.212

Source: Own survey

According to Table 7 the highest interaction effect sizes on the response rates are present in cases when all questions were shown immediately to the respondents with negative pictures included (effect = +0.170) and when questionnaire is divided into logical groups of questions with positive pictures (effect = +0.169). It is interesting to see that the lowest interaction effect size is present in questionnaires with all questions immediately shown but with included positive pictures (effect = –0.212). The backward elimination analysis has shown that in case when all enterprises are observed together the interaction effect is statistically significant at 5% significance level (p-value = 0.036).

Table 8 Log-linear model parameter estimates of the interaction effect sizes of variables number of questions shown per questionnaire screen and kind of pictures included in the questionnaire, enterprises observed according to their size

Size	Number of questions shown	Pictures		
		Without	Negative	Positive
Small	One question per screen	–0.006	–0.027	0.033
	Group of questions	–0.033	–0.135	**0.168**
	All questions	0.039	**0.162**	–0.201
Medium	One question per screen	**0.999**	–0.748	–0.251
	Group of questions	–0.899	0.398	0.501
	All questions	–0.100	0.350	–0.250
Large	One question per screen	–0.602	–0.065	0.667
	Group of questions	–0.236	0.301	–0.065
	All questions	**0.838**	–0.236	–0.602

Source: Own survey

According to Table 8 results, the highest impacts on response rates in small enterprises have the same questionnaire versions as at overall enterprises level. However, when medium enterprises are observed the highest interaction effect size on response rates has such questionnaire version in which only one question per questionnaire screen without pictures is shown to respondents (effect = +0.999) whereas the highest interaction effect size at large enterprises is shown in case when all questions are shown immediately to the respondents but also without pictures (effect = +0.838). However, the interaction effects in small (p-value = 0.058), medium (p-value = 0.070) and at large (p-value = 0.269) enterprises seemed not to be statistically significant at 5% significance level.

Table 9 Log-linear model parameter estimates of the interaction effect sizes of variables number of questions shown per questionnaire screen and kind of pictures included in the questionnaire, enterprises observed according to their legal form

Legal form	Number of questions shown	Pictures		
		Without	Negative	Positive
Joint stock	One question per screen	**0.695**	–0.940	0.245
	Group of questions	–0.574	0.499	0.075
	All questions	–0.121	0.441	–0.320
Limited liability	One question per screen	–0.020	0.005	0.015
	Group of questions	–0.010	–0.147	**0.157**
	All questions	0.030	**0.142**	–0.172
Simple limited liability	One question per screen	0.275	–0.674	0.399
	Group of questions	–0.763	0.206	**0.557**
	All questions	0.488	0.468	–0.956

Source: Own survey

In case of limited liability enterprises, results from Table 9 show that the highest impacts on response rates are present at the same questionnaire versions as at overall enterprises level and in small enterprises. On the other hand, joint stock enterprises show the highest interaction effect size at questionnaire version where just one question per screen is shown and where no pictures are included (effect = +0.695) whereas simple limited liability enterprises, similar as limited liability enterprises, have the highest interaction effect size at questionnaires where questions are given in logical groups and where positive pictures are provided (effect = +0.557). According to conducted backward elimination analysis the interaction effect is not statistically significant at significance level of 5% in joint stock enterprises (p-value = 0.250) and in limited liability enterprises (p-value = 0.117) but the interaction effect is statistically significant in simple limited liability enterprises (p-value = 0.022).

Table 10 Log-linear model parameter estimates of the interaction effect sizes of variables number of questions shown per questionnaire screen and kind of pictures included in the questionnaire, enterprises observed according to their main activity

Main activity	Number of questions shown	Pictures		
		Without	Negative	Positive
Industrial	One question per screen	0.101	0.044	–0.145
	Group of questions	–0.045	–0.139	**0.184**
	All questions	–0.056	0.095	–0.039
Trade	One question per screen	–0.164	–0.044	0.208
	Group of questions	0.179	–0.305	0.126
	All questions	–0.015	**0.349**	–0.334

Table 10 Log-linear model parameter estimates of the interaction effect sizes of variables number of questions shown per questionnaire screen and kind of pictures included in the questionnaire, enterprises observed according to their main activity (continuation)

Main activity	Number of questions shown	Pictures		
		Without	Negative	Positive
Service	One question per screen	0.046	–0.085	0.039
	Group of questions	–0.143	–0.045	**0.188**
	All questions	0.097	0.130	–0.227
Other	One question per screen	–0.086	–0.178	0.264
	Group of questions	–0.376	0.151	0.225
	All questions	**0.462**	0.027	–0.489

Source: Own survey

The results from Table 10 reveal that the highest interaction effect sizes in industrial (effect = +0.184) and at service (effect = +0.188) enterprises is present in case of questionnaire with logical questions groups with positive pictures. In case of trade enterprises, the highest interaction effect size is present at questionnaires with all questions shown at once with negative pictures (effect = +0.349). Similar to trade enterprise, in other enterprises the highest interaction effect is also present at questionnaire where all questions are shown at once to respondents but without pictures (effect = 0.462). Unfortunately, in industrial (p-value = 0.630), trade (p-value = 0.115), service (p-value = 0.188) and at other (p-value = 0.563) enterprises the interaction effect seems not to be statistically significant at significance level of 5%.

Table 11 Log-linear model parameter estimates of the interaction effect sizes of variables number of questions shown per questionnaire screen and kind of pictures included in the questionnaire, enterprises observed according to the location of their headquarters

NUTS-2 region	Number of questions shown	Pictures		
		Without	Negative	Positive
Adriatic Croatia	One question per screen	0.047	–0.029	–0.018
	Group of questions	–0.029	–0.137	**0.166**
	All questions	–0.018	**0.166**	–0.148
Continental Croatia	One question per screen	–0.011	–0.065	0.076
	Group of questions	–0.063	–0.104	**0.167**
	All questions	0.074	**0.169**	–0.243

Source: Own survey

According to Table 11, the highest interaction effect sizes at enterprises that have their headquarters in Adriatic and in Continental Croatia is achieved at questionnaires with logical group of questions and with positive pictures and at questionnaires where all questions are presented to respondents at once along with negative pictures. However, the backward elimination analysis has shown that in enterprises from Adriatic (p-value = 0.578) and from Continental Croatia (p-value = 0.085) the interaction effect is not statistically significant at significance level of 5%.

Except from original numbers of completed questionnaires, the log-linear analyses were conducted by taking into account different number of contacted enterprises. In Table 12 only the questionnaire versions with the highest interaction effects were emphasized.

Table 12 Questionnaire versions with the highest interaction effects size and interaction effects significance tests results, selection based on log-linear model parameter estimates of the interaction effect sizes of variables number of questions shown per questionnaire screen and kind of pictures included in the questionnaire where different number of contacted enterprises is taken into account

Observation level of enterprises	Questionnaire version	Effect size	Interaction effect significance test		
			Chi-square test statistic	df	p-value
All	Group of questions, positive pictures All questions, negative pictures	+0.169 +0.169	10.316	4	0.035
Size					
Small	Group of questions, positive pictures All questions, negative pictures	+0.168 +0.162	9.149	4	0.057
Medium	One question per screen, without pictures	+1.003	8.752	4	0.068
Large	All questions, without pictures	+0.831	5.100	4	0.277
Legal form					
Joint stock	One question per screen, without pictures	+0.702	5.450	4	0.244
Limited liability	Group of questions, positive pictures All questions, negative pictures	+0.157 +0.142	7.391	4	0.117
Simple limited liability	Group of questions, positive pictures	+0.558	11.548	4	0.021
Main activity					
Industrial	Group of questions, positive pictures	+0.184	2.585	4	0.629
Trade	All questions, negative pictures	+0.349	7.476	4	0.113
Service	Group of questions, positive pictures	+0.188	6.160	4	0.188
Other	All questions, without pictures	+0.464	3.008	4	0.557
NUTS-2 region					
Adriatic Croatia	Group of questions, positive pictures All questions, negative pictures	+0.146 +0.151	2.361	4	0.670
Continental Croatia	Group of questions, positive pictures All questions, negative pictures	+0.180 +0.179	9.170	4	0.057

Source: Own survey

If there are questionnaire versions with the highest interaction effects when the different number of contacted enterprises is not taken into account (Tables 7–11) and when it is (Table 12), it can be concluded that there are no differences in selected questionnaire versions. There are some differences in the interaction effect sizes but the differences can be considered as very small. In Table 12 interaction effects significance test results are also provided. The test results lead to the same conclusions about the interaction effect significances as in cases when the different number of contacted enterprises is not taken into account.

DISCUSSION AND CONCLUSION

The aim of the paper is to investigate which combination of questionnaire design characteristics would lead to the highest response rates in web business surveys. As questionnaire design characteristics the number of questions shown per questionnaire screen and kind of pictures included in the questionnaire are observed. Furthermore, here the combinations or interactions of these two characteristics were observed solely The main effects of the number of questions shown per questionnaire screen and kind of pictures included in the questionnaire on response rates were observed in Žmuk (2017, 2018).

In the first step, the relationship between the number of questions shown per questionnaire screen and the kind of pictures included in the questionnaire were inspected. If all enterprises in the sample are observed, it can be concluded that the relationship is statistically significant, at significance level of 5%, or, in other words, that some questionnaire versions with different questionnaire design characteristics can lead to considerably higher response rates than the other questionnaire versions. On that way, the presence of interaction effect was confirmed at overall enterprises level.

The presence of the relationship between the number of questions shown per questionnaire screen and the kind of pictures included in the questionnaire were inspected on some enterprises' sublevels. In order to do such analysis enterprises were stratified according to their size, legal form, main activity and location of their headquarters. Each of the defined stratification level was observed separately. Unfortunately, the stratification of enterprises leads to such enterprises split which resulted in too low enterprises number in certain strata. Consequently, the prerequisites of conducted Pearson's chi-square tests were not fulfilled completely. Because of that the test results should be taken with some precaution into account. However, even if that problem is neglected, the following problem appears. Namely, at significance level of 5%, only in simple limited liability enterprises statistically significant relationship between the number of questions shown per questionnaire screen and the kind of pictures included in the questionnaire was found. If the significance level is increased to 10%, the statistically significant relationship is found in small and medium enterprises as well.

Despite those devastating results on strata levels, in the second step the interaction effect sizes of variables number of questions shown per questionnaire screen and kind of pictures included in the questionnaire were measured by using log-linear model approach. The log-linear analysis results have shown that, when all enterprises are observed, questionnaires where questions are logically grouped with positive pictures and questionnaires where all questions with negative pictures are at once are shown to the respondents have the highest interaction effect size. Accordingly, these two questionnaire versions should result in the highest response rates at all enterprises level. Moreover, the backward elimination analysis has shown that the interaction effect is statistically significant at significance level of 5%.

If interaction effect sizes on different enterprises strata level are observed, different questionnaire versions are pointed out as the ones with the highest interaction effect size. However, in most cases the questionnaire version with logically grouped questions and positive pictures turned out to have the highest interaction effect size. Because of that, when no certain kind of enterprises is in the researchers' focus, it can be concluded that this questionnaire version should be used in web business surveys in order to achieve the highest response rate. If certain kind of enterprises is in the focus of researchers, it is recommended to check Table 12 for selecting the best questionnaire version for the observed kind of enterprises. Unfortunately, the backward elimination analyses have shown that the interaction effect in not statistically significant at significance level of 5% in majority of observed strata. Despite the fact that the given results of research, selected questionnaire versions and calculated interaction effect sizes, can be used as a good starting point for further research on this topic but also the results can be used in the following business web surveys conduction.

Response rates of web surveys tend to be very low with a trend of being even lower. Because of that something has to be done to increase response rates. There are different possibilities. In the paper the interaction effect of the number of questions shown per questionnaire screen to respondents and the kind of included pictures on response rate in business web surveys. In order to inspect the interaction effect overall nine questionnaire designs or questionnaire versions have been created. Some of questionnaire versions have not shown statistically significant interaction effect on response rates whereas some did. Despite that, the questionnaire versions with the highest interaction effects have been emphasized. On that way, at overall and at all observed strata levels of enterprises allocated are questionnaire versions that should result in the highest response rate among other defined questionnaire designs.

However, there are some drawbacks of the research. The questionnaire may also be too short to measure good effect of pictures on response rates. Also, the number of pictures introduced in the questionnaire could be seen as small. In the further research that should be improved. Furthermore, at some strata level the number of enterprises that participated in the survey is really low and in some cases it was equal to zero. More efforts should be invested to receive appropriate and needed number of responses from such enterprises for analyses purposes.

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Analysis of Data from Questionnaire Surveys – Book Review

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ŘEZANKOVÁ, H. *Analýza dat z dotazníkových šetření* (Analysis of Data from Questionnaire Surveys). 4th Ed. Prague: Professional Publishing, 2017, 225 p. ISBN 978-80-906594-8-3.

Questionnaire surveys are a source of statistical data in many application areas. Publications dealing with methods of analysing these data are very sought-after and demanded. Therefore, it is no surprise that Professional Publishing has released already the fourth, substantially revised edition of Hana Řezanková's book on this issue.

The book is divided into seven parts and an appendix. The basic difference from previous editions lies in data illustrating the presented methods. These data are inspired by a real questionnaire survey *REFLEX* that is repeatedly conducted among college graduates. IBM SPSS Statistics is used to make specific calculations in the book. Thus, the reader can follow the individual steps of the analysis and implement them analogously for his own data.

The data collection phase of the questionnaire survey, as well as the data preparation for processing, are discussed only marginally in the introductory part of the book. However, it is clear from the two relevant chapters that the type of variables in the analysed data files has to be taken into account and that adequate tools must be used for the analysis. Attention is rightly paid to the issue of missing data. These are very common in questionnaire surveys and correct treatment of them is necessary exactly already in the phase of data files preparation.

The book focuses logically on an analysis of data files with categorical variables. The specific tools applicable in this case to describe the obtained data can be found in the third chapter of the book. Their structure is explained, calculations of their values are made on the examples; the results are interpreted. At the end of the chapter, principles of statistical inference and statistical tests (that are the most frequent in the given area) are summarized.

Questionnaire surveys are often motivated by questions about relationships between variables. The reviewed publication provides an overview of various types of coefficients of association constructed on the basis of frequencies in a two-dimensional contingency table. Again, examples of their calculations are attached. The coefficients of association exist in a large amount and the software usually offers

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the calculation of many of them. The reader would probably like to get here a more detailed advice as to when and why choose a particular coefficient. The analysis options for multidimensional (three-dimensional) classification – on the example of several 2x2 tables in the layers – are mentioned at the end of the chapter.

The next part of the book deals with a file comparison. The choice of analytical instruments is primarily influenced by whether confronted files can be considered independent or dependent samples. Again, the nature of the analysed variables has to be taken into account and therefore the author applies even many of the non-parametric tests here.

Logistic regression deals with the modelling of asymmetric dependence when the response variable is categorical. The author devotes a separate chapter to its most widespread variant, when the dependent variable is only dichotomous. The practical application of the method and clarification of individual steps, the use of evaluation tools and interpretation of results are preferred in the text over the theoretical background of this method. Finally, the book summarizes, again with the support of examples, the principles of some methods of multidimensional statistics determined for analysing files with categorical variables, such as cluster analysis, multidimensional scaling or correspondence analysis.

As already mentioned, the book includes many examples. To explain the procedure, the solution is usually made without a computer (perhaps here and there in too detail). To describe the practical data analysis, the solution is then realized with the modules of IBM SPSS system. The way of working with data and the input of the analysis in the system itself are described, as well as the output components and the interpretation of the results. I consider this approach to be very helpful especially for readers, who are more interested in the practical side of data analysis than in its theoretical background.

The reviewed publication does not require from readers to have much theoretical knowledge; however, it assumes that they have already been educated in the field of statistical analysis. It contributes to developing readers' knowledge and their ability to apply it adequately.

The book is readable, clear, and I believe that it will find its readers and users again. (Many of them will appreciate even its soft bookbinding, which is handy to work with.) Moreover, the book might be for someone an invitation to study more.

ROBUST 2018 (Rybník) 20th Event of International Statistical Conference

Ondřej Vozár [1] | *Czech Statistical Office; University of Economics, Prague, Czech Republic*

The 20th event of the well-established biennial statistical conference *ROBUST 2018* took place in the *Rybník* Resort in the *Český Les* (Czech Republic) during 21–26 January 2018. It was organized by joint effort of the Expert Group of Computational Statistics of the Czech Mathematical Society (Section of the Union of Czech Mathematicians and Physicists), Department of Probability and Mathematical Statistics of the Faculty of Mathematics and Physics, Charles University, Prague and the Czech Statistical Society. In total, 86 participants from 5 countries presented and discussed contributions covering a broad spectrum ranging from theoretical statistics, probability and stochastic analysis, official statistics, and computer science to applied statistics in several fields, including insurance mathematics, medicine, sports, metrology, geology, and criminology. Foreign participants came from Slovakia (12), Germany (1), Switzerland (1), and Belgium (1). The idea behind the *ROBUST* conferences has always been to bring together statisticians of all generations and all fields from different Czech and Slovak institutions, Czech and Slovak experts living abroad to enable the exchange of ideas and to provide them with interdisciplinary insight into the research in statistics.

Six invited lectures were delivered. The leading expert in discrete mathematics and interval computation, Doc. Milan Hladík (Faculty of Mathematics and Physics, Charles University, Prague) delivered a two-hour opening lecture on interval robustness in linear programming. The interval-data approach is a statistical tool for dealing with, for example, rounded data. Doc. Ján Mačutek (Faculty of Mathematics, Physics and Informatics, Komenský University, Bratislava) presented an application of cluster analysis to the New Year and Christmas speeches of Czechoslovak and Czech Presidents. The classification of the speeches to the three clusters – the first Presidents (T. G. Masaryk, E. Beneš, E. Hácha), the Presidents of the communist regime, and the President of the democratic regime impressed the participants.

Dr. Miroslav Singer (ex-governor of the Czech National Bank, now in the Generali CEE Holding) delivered a lecture on practical experience in decision under uncertainty. It was focused on two topics – the monetary interventions of the Czech crown and the effects of relative exchange rates on the purchasing power of the Czechs. These issues were lively discussed by the audience. The lecture was supplemented by a short presentation delivered by Doc. Jaroslav Sixta (Director of the Macroeconomic Statistics Section of the Czech Statistical Office; Faculty of Mathematics and Statistics, University of Economics, Prague) on estimates of quarterly GDP and assessment of prediction by macroeconomic analysis.

Prof. Jan Hanousek (CERGE-EI, Charles University, Prague) delivered a presentation on endogeneity of variables in corporate finance application. These techniques enable to assess impacts of subsidies and government policies. He also moderated a one-hour discussion on assessment of scientific publications.

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Doc. Eva Fišerová (Faculty of Sciences, Palacký University, Olomouc) provided an interesting presentation on estimation of quadrics and its precision. Besides metrology, the topic is also vital in archaeology to estimate the size of burial chambers in excavations. Doc. Zbyněk Pawlas (Department of Probability and Mathematical Statistics, Faculty of Mathematics and Physics, Charles University, Prague) provided an overview of stochastic mosaics including applications in materials science.

This Conference was focused, as usually, on participation of doctoral and master degree students, who presented 25 posters. Prizes were awarded in two categories: Master and First-Year Doctoral Students and Advanced Doctoral Students. The prices were sponsored as usually by RSJ Securities, a.s., and RSJ Foundation. The honourable mentions – monographs and textbooks of the Matfyzpress publishing house were provided by anonymous sponsors. Conference fees for many participants (mostly master and first-year doctoral students) were covered as in the past conferences by RSJ and the Czech Statistical Society. The editor-in-chief of the *Statistika: Statistics and Economy Journal* kindly invited participants to submit relevant papers to the Journal.

Recent Publications and Events

New publications of the Czech Statistical Office

Direct Public Support for Research and Development in the Czech Republic 2016. Prague: CZSO, 2017.
Food consumption in 2016. Prague: CZSO, 2017.
Foreigners in the Czech Republic 2017. Prague: CZSO, 2017.
Generation, Recovery and Disposal of Waste. Prague: CZSO, 2017.
Small Lexicon of Municipalities of the Czech Republic 2017. Prague: CZSO, 2017.

Other selected publications

Development of the basic living standard indicators in the Czech Republic 1993–2016. Prague: MoLSA, 2017.
EUROSTAT. *Key figures on Europe*. 2017 Ed. Luxembourg: Publication Office of the European Union, 2017.

Conferences

The ***European Conference on Quality in Official Statistics (Q2018)*** will be held **in Kraków, Poland, during 26–29 June 2018**. More information available at: <*http://www.q2018.pl*>.

The ***21st AMSE 2018 Conference*** will take place in **Kutná Hora, Czech Republic, from 28th August to 2nd September 2018**. More information available at: <*http://www.amse-conference.eu*>.

The ***26th Interdisciplinary Information Management Talks (IDIMT 2018 Conference)*** will be held in **Kutná Hora, Czech Republic, during 5–7 September 2018**. More information available at: <*http://idimt.org*>.

At deep sorrow we shall notice to all friends, colleagues, and the public that

Prof. Iva Ritschelová
President of the Czech Statistical Office

passed away.

She deceased after a severe illness in late evening of 3rd December 2017 aged 53 years.

Prof. Iva Ritschelová was appointed to the title of the President of the Czech Statistical Office by the President of the Czech Republic, upon a proposal of the Government of the Czech Republic, on 1st September 2010. In her term of office, she promoted significant modernisation of statistics production processes, including the transition to the electronic data collection. She reduced total administrative burden on respondents by statistical surveys by over 35%. She also encouraged raising of statistical literacy of the general public.

As an active member of academic community she dealt with education and enlightenment. She had ample work experience from abroad. She was an author of tens of professional publications focused on environmental economy and environmental policy.

She was a patriot of North Bohemia region and a benefactor of the Children's Home in the City of Ústí nad Labem on the North Terrace.

Tribute to her memory!

With respect, Statistika journal editors.

Professor Lubomír Cyhelský has passed away

Richard Hindls [1] | *University of Economics, Prague, Czech Republic*

Professor Lubomír Cyhelský, doyen of the Czech statistics, died on 22 January 2018, at the age of 88. An exceptional and charismatic personality of the Czech academic milieu, tireless promoter of statistics and statistical science studies has gone. A man, who devoted his entire life to the development and propagation of statistics, has died.

The name of professor Cyhelský will always be connected mainly to the Department of Statistics at the University of Economics, Prague. He belonged to its founders and devoted the whole 50 years of his life to its development. He had been the head of the Department of Statistics of the University of Economics for almost 30 years (1962–1990) and he held the post with an empathic and human approach. Thanks to his managerial skills the department had gradually prepared a great number of excellent graduates who later left their imprints in the social life of the Czech Republic. Yet, he remained an unpretentious man full of humanity, humbleness to the university background and statistical science.

At the beginning of his professional career in 1951–1952 he worked at the former State Statistical Office, focusing on national economic balances. In the latter half of the 1960's he worked as a scientist in the Research Institute of Economic Planning, then in the Research Institute of Finance. In 1958, he got PhD in economic statistics at the University of Economics, Prague, and in 1961 he was awarded a degree of senior lecturer there. Seven years later he was appointed professor. In 1981, he became a Doctor of Science. He held also important academic posts at the university – in 1966–1970 he was a vice-dean at the Faculty of National Economy of the University of Economics, Prague, in 1976–1985 he was a vice-rector for pedagogy at the University of Economics, Prague.

In addition to his more than 60-year of pedagogic activity he contributed enormously to the development of the State Statistical Service. In 1965, professor Cyhelský co-initiated a two-year postgraduate statistical study for those who worked in the state and departmental statistics, later renamed to "Specialized statistical study" (which continues to exist). Professor Cyhelský also co-operated on and contributed to the birth and development of the Statistika journal and was a member of its editorial board. He also worked as a managing editor of eleven scientific journals of the Research Institute of Social Economic Information in Prague, called Statistical Review. He published over 200 works many of which contributed greatly to the successful promotion of statistics. He was reading his scientific lectures in the Czech Republic as well as abroad. He has become the first laureate ever awarded the Prize of the President of the Czech Statistical Office and of the rector of the University of Economics, Prague, for his contribution to statistics.

Professor Cyhelský's work was highly appreciated abroad, too. In 1979, he was elected a member of the International Statistical Institute (ISI) and he actively participated in several congresses held by the Institute. He also helped to found the Czech Statistical Society (1990) and was a member of its first committee.

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The life of professor Cyhelský came to an end and his work was finished. He left behind a long row of grateful graduates and colleagues, many outstanding works and human memories recalled by his followers. Words of gratitude cannot fill the gap his death left behind and all I can do is to express my appreciation and respect to his life work.

Thank you, professor!

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