> Analysis of Structural Differences and Asymmetry of Shocks between the Czech Economy and the Euro Area - Appendix

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## Appendix - Model

It is a New Keynesian model of two economies, originally presented in Kolasa (2009). The model assumes that there are only two economies in the world: domestic economy (indexed by H and represented by the Czech economy) and the foreign economy (indexed by F and represented by the Euro Area). Each economy is populated by a continuum of infinitely-lived inhabitants, in the domestic economy distributed over the interval $[0, n]$ and in the foreign economy over the interval $[n, 1]$. Both economies produce a continuum of differentiated tradable (non-tradable) goods that is also distributed over the interval $[0, n]$ in the domestic economy, and over the interval $[n, 1]$ in the foreign economy. The parameter $n$, therefore, represents the relative size of the domestic economy with respect to the foreign economy. Because both economies are modeled in the same way, the assumptions about representative agents as well as the parameters and variables of the model have the same interpretation in both economies. Moreover, derived equations describing the behavior of the economy have the same structural form in both economies. Therefore, I will describe the assumptions about agents and their optimization problems only in the domestic economy, knowing that the same optimization problems hold for the foreign economy. Parameters and variables in the foreign economy are distinguished from those in the domestic economy by an asterisk and for distinguishing tradable goods produced in the domestic economy and foreign economy I employ the subscripts " H " and "F". For example, $C_{H}^{*}$ denotes foreign consumption of goods produced in the domestic economy (i.e. Czech export of consumption goods), while $C_{F}$ denotes domestic consumption of goods produced in the foreign economy (i.e. Czech import of consumption goods).

## Households

Households in a given economy are assumed to be homogenous. Households consume tradable and non-tradable goods produced by firms and make their intertemporal decisions about consumption by trading bonds. Households also supply labor and rent capital to firms. A typical household $j$ in a domestic economy seeks to maximize its life-time utility function, which is function of household's consumption $C_{t}(j)$ and labor effort $L_{t}(j)$. The utility function is in the form CRRA function (constant relative risk aversion function)

$$
\begin{equation*}
U_{t}(j)=E_{t} \sum_{k=0}^{\infty} \beta^{k}\left[\frac{\varepsilon_{d, t+k}}{1-\sigma}\left(C_{t+k}(j)-H_{t+k}\right)^{1-\sigma}-\frac{\varepsilon_{l, t+k}}{1+\phi} L_{t+k}(j)^{1+\phi}\right] \tag{1}
\end{equation*}
$$

where $E_{t}$ denotes expectations in the period $t, \beta$ is a discount factor, $\sigma$ is an inverse elasticity of intertemporal substitution in consumption, $H_{t}=h C_{t-1}$ is an external habit taken by the household as exogenous, $h$ is a parameter of habit formation in consumption, $C_{t}$ is a composite consumption index (to be defined later), $\phi$ is an inverse elasticity of labor supply, $\varepsilon_{d, t}$ is a preference shock in the period $t$, which influences intertemporal decisions about consumption and $\varepsilon_{l, t}$ is a labor supply shock in the period $t$.

Maximization of the utility function (1) is subject to a set of flow budget constraints given by

$$
\begin{array}{r}
P_{C, t} C_{t}(j)+P_{I, t} I_{t}(j)+E_{t}\left\{\Upsilon_{t, t+1} B_{t+1}(j)\right\}=B_{t}(j)+W_{t}(j) L_{t}(j) \\
+R_{K, t} K_{t}(j)+\Pi_{H, t}(j)+\Pi_{N, t}(j)+T_{t}(j), \quad \text { for } t=0,1,2 \ldots, \tag{2}
\end{array}
$$

where $P_{C, t}$ denotes the price of the consumption $C_{t}, P_{I, t}$ is the price of investment goods $I_{t}, B_{t+1}$ is the nominal payoff in period $t+1$ of the portfolio held at the end of period $t, W_{t}$ is the nominal wage, $R_{K, t}$ denotes income of households achieved from renting capital $K_{t}, \Pi_{H, t}$ and $\Pi_{N, t}$ are dividends from tradable and non-tradable goods producers and $T_{t}$ denotes lump sum government transfers net of lump sum taxes. $\Upsilon_{t, t+1}$ is the stochastic discount factor for nominal payoffs, such that $E_{t} \Upsilon_{t, t+1}=R_{t}^{-1}$, where $R_{t}$ is the gross return on a riskless one-period bond.

## Consumption Choice

First order condition of optimality for intertemporal decisions about consumption is in the form of a standard Euler equation

$$
\begin{equation*}
\beta R_{t} E_{t}\left\{\frac{\varepsilon_{d, t+1}}{\varepsilon_{d, t}}\left(\frac{C_{t+1}-h C_{t}}{C_{t}-h C_{t-1}}\right)^{-\sigma} \frac{P_{C, t}}{P_{C, t+1}}\right\}=1 . \tag{3}
\end{equation*}
$$

Consumption index $C_{t}$ consists of final tradable goods index $C_{T, t}$ and nontradable goods index $C_{N, t}$ which are aggregated according to

$$
C_{t}=\frac{C_{T, t}^{\gamma_{c}} l_{N, t}^{1-\gamma_{c}}}{\gamma_{c}^{\gamma_{c}}\left(1-\gamma_{c}\right)^{1-\gamma_{c}},}
$$

where $\gamma_{c}$ denotes share of final tradable goods in consumption of households. Following Burstein et al. (2003) and Corsetti and Dedola (2005), it is assumed that consumption of a final tradable good requires $\omega$ units of distribution services $Y_{D, t}$, which implies

$$
\begin{equation*}
C_{T, t}=\min \left\{C_{R, t} ; \omega^{-1} Y_{D, t}\right\} . \tag{4}
\end{equation*}
$$

The consumption index of raw tradable goods is defined as

$$
C_{R, t}=\frac{C_{H, t}^{\alpha} C_{F, t}^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}},
$$

where $\alpha$ denotes share of domestic goods in the domestic basket of raw tradable goods ${ }^{1}, C_{H, t}$ is an index of home-made raw tradable goods and $C_{F, t}$ is

[^0]an index of foreign-made raw tradable goods, both consumed in the domestic economy and defined as
\[

$$
\begin{aligned}
& C_{H, t}=\left[\left(\frac{1}{n}\right)^{\frac{1}{\phi_{H}}} \int_{0}^{n} C_{t}\left(z_{H}\right)^{\frac{\phi_{H}-1}{\phi_{H}}} d z_{H}\right]^{\frac{\phi_{H}}{\phi_{H}-1}}, \\
& C_{F, t}=\left[\left(\frac{1}{1-n}\right)^{\frac{1}{\phi_{F}}} \int_{n}^{1} C_{t}\left(z_{F}\right)^{\frac{\phi_{F}-1}{\phi_{F}}} d z_{F}\right]^{\frac{\phi_{F}}{\phi_{F}-1}},
\end{aligned}
$$
\]

where $\phi_{H}\left(\phi_{F}\right)$ is an elasticity of substitution between domestic (foreign) raw tradable goods, consumed in the domestic economy. Analogously, the consumption index of non-tradable goods is defined as

$$
C_{N, t}=\left[\left(\frac{1}{n}\right)^{\frac{1}{\phi_{N}}} \int_{0}^{n} C_{t}\left(z_{N}\right)^{\frac{\phi_{N}-1}{\phi_{N}}} d z_{N}\right]^{\frac{\phi_{N}}{\phi_{N}-1}},
$$

where $\phi_{N}$ is an elasticity of substitution between domestic non-tradable goods.

Let us now discuss intratemporal decisions households make about consumption. First of all, households have to choose how many tradable goods and non-tradable goods they want to consume. Formally, households want to maximize consumption ${ }^{2}$

$$
\begin{equation*}
C_{t}=\frac{C_{T, t}^{\gamma_{c}} C_{N, t}^{1-\gamma_{c}}}{\gamma_{c}^{\gamma_{c}}\left(1-\gamma_{c}\right)^{1-\gamma_{c}}}, \tag{5}
\end{equation*}
$$

conditionally on their consumption expenditures

$$
P_{C, t} C_{t}=P_{T, t} C_{T, t}+P_{N, t} C_{N, t} .
$$

same. The reason why I use the modified specification is as follows: In my opinion, the modified definition of $\alpha^{*}$ better corresponds to the definition of its counterpart in the domestic economy $\alpha$.
${ }^{2}$ Equivalently, we can think about households wanting to minimize their consumption expenditures for a given level of their consumption.

The first order conditions for an optimal allocation of consumption expenditures between tradable and non-tradable goods imply that

$$
\begin{align*}
C_{N, t} & =\left(1-\gamma_{c}\right)\left(\frac{P_{N, t}}{P_{C, t}}\right)^{-1} C_{t}  \tag{6}\\
C_{T, t} & =\gamma_{c}\left(\frac{P_{T, t}}{P_{C, t}}\right)^{-1} C_{t} . \tag{7}
\end{align*}
$$

After substituting these allocation functions, i.e. (6) and (7), into the composite consumption index (5), we get a corresponding composite price index in the form

$$
\begin{equation*}
P_{C, t}=P_{T, t}^{\gamma_{c}} P_{N, t}^{1-\gamma_{c}} . \tag{8}
\end{equation*}
$$

Consequently, household have to make a choice between home-made tradable goods and foreign-made tradable goods. As mentioned above, price of tradable goods $P_{T, t}$ depends on the price of raw tradable goods $P_{R, t}$ and also on the price of non-tradable distribution services $P_{N, t}$. Formally,

$$
\begin{equation*}
P_{T, t}=P_{R, t}+\omega P_{N, t} . \tag{9}
\end{equation*}
$$

The price of distribution services is the same for both home-made tradable goods and foreign-made tradable goods, so it does not influence households' choice between them. Therefore, it is correct to assume that households want to maximize consumption of raw tradable goods

$$
\begin{equation*}
C_{R, t}=\frac{C_{H, t}^{\alpha} C_{F, t}^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \tag{10}
\end{equation*}
$$

conditional on their expenditures on raw tradable goods

$$
\begin{equation*}
P_{R, t} C_{R, t}=P_{H, t} C_{H, t}+P_{F, t} C_{F, t} . \tag{11}
\end{equation*}
$$

First order conditions of this maximization problem require

$$
\begin{align*}
C_{H, t} & =\alpha \gamma_{c}\left(\frac{P_{H, t}}{P_{R, t}}\right)^{-1}\left(\frac{P_{T, t}}{P_{C, t}}\right)^{-1} C_{t}  \tag{12}\\
C_{F, t} & =(1-\alpha) \gamma_{c}\left(\frac{P_{F, t}}{P_{R, t}}\right)^{-1}\left(\frac{P_{T, t}}{P_{C, t}}\right)^{-1} C_{t} \tag{13}
\end{align*}
$$

where I use the equilibrium condition $C_{R, t}=C_{T, t}$ and FOC condition (7). ${ }^{3}$ After substituting these FOCs, i.e. (12) and (13), into the consumption index of raw tradable goods (10), we get a corresponding price index of raw tradable goods in the form

$$
\begin{equation*}
P_{R, t}=P_{H, t}^{\alpha} P_{F, t}^{1-\alpha} \tag{14}
\end{equation*}
$$

Finally, households have to choose which particular goods they want to consume. I show their optimization problem only for non-tradable goods as these optimization problems are analogous for home-made tradable goods and foreign-made tradable goods. Households want to minimize their expenditures on non-tradable goods

$$
\int_{0}^{n} P_{t}\left(z_{N}\right) C_{t}\left(z_{N}\right) d z_{N}
$$

conditional on their consumption level of non-tradable goods

$$
C_{N, t}=\left[\frac{1}{n} \int_{0}^{n} P_{t}\left(z_{N}\right)^{1-\phi_{N}} d z_{N}\right]^{\frac{1}{1-\phi_{N}}}
$$

All these three minimization problems lead to first order conditions in the

[^1]form
\[

$$
\begin{aligned}
& C_{t}\left(z_{N}\right)=\frac{1}{n}\left(1-\gamma_{c}\right)\left(\frac{P_{t}\left(z_{N}\right)}{P_{N, t}}\right)^{-\phi_{N}}\left(\frac{P_{N, t}}{P_{C, t}}\right)^{-1} C_{t} \\
& C_{t}\left(z_{H}\right)=\frac{1}{n} \gamma_{c} \alpha\left(\frac{P_{t}\left(z_{H}\right)}{P_{H, t}}\right)^{-\phi_{H}}\left(\frac{P_{H, t}}{P_{R, t}}\right)^{-1}\left(\frac{P_{T, t}}{P_{C, t}}\right)^{-1} C_{t} \\
& C_{t}\left(z_{F}\right)=\frac{1}{1-n} \gamma_{c}(1-\alpha)\left(\frac{P_{t}\left(z_{F}\right)}{P_{F, t}}\right)^{-\phi_{F}}\left(\frac{P_{F, t}}{P_{R, t}}\right)^{-1}\left(\frac{P_{T, t}}{P_{C, t}}\right)^{-1} C_{t} .
\end{aligned}
$$
\]

Corresponding price indices are in the form

$$
\begin{aligned}
P_{N, t} & =\left[\frac{1}{n} \int_{0}^{n} P_{t}\left(z_{N}\right)^{1-\phi_{N}} d z_{N}\right]^{\frac{1}{1-\phi_{N}}} \\
P_{H, t} & =\left[\frac{1}{n} \int_{0}^{n} P_{t}\left(z_{H}\right)^{1-\phi_{H}} d z_{H}\right]^{\frac{1}{1-\phi_{H}}} \\
P_{F, t} & =\left[\frac{1}{1-n} \int_{n}^{1} P_{t}\left(z_{F}\right)^{1-\phi_{F}} d z_{F}\right]^{\frac{1}{1-\phi_{F}}}
\end{aligned}
$$

Similar optimization problems and resulting optimality conditions hold also for the foreign economy and are distinguished from those in the domestic economy by superscript "*".

## Investment Decisions

Households use part of their income to accumulate capital $K_{t}$, assumed to be homogenous, which they rent to firms. Capital is accumulated according to the formula

$$
\begin{equation*}
K_{t+1}=(1-\tau) K_{t}+\varepsilon_{i, t}\left(1-S\left(\frac{I_{t}}{I_{t-1}}\right)\right) I_{t} \tag{15}
\end{equation*}
$$

where $\tau$ is a depreciation rate of capital and $I_{t}$ denotes investment in the period $t$. Following Christiano et. al. (2005), capital accumulation is subject to investment specific technological shock $\varepsilon_{i, t}$ and adjustment costs repre-
sented by function $S(\cdot) .{ }^{4}$ This function has to satisfy following properties $S(1)=S^{\prime}(1)=0$ and $S^{\prime \prime}(\cdot)=S^{\prime \prime}>0$.

In order to decide how much capital would a household accumulate, it is again necessary to solve the optimization problem. Household wants to maximize its utility expressed by (1), which is subject to the budget constraint (2) and to the formula for capital accumulation (15). First order conditions corresponding to capital $K_{t}$ and investment $I_{t}$ imply the following equations

$$
\begin{gather*}
\frac{P_{I, t}}{P_{C, t}}=\varepsilon_{i, t}\left(1-S\left(\frac{I_{t}}{I_{t-1}}\right)-\frac{I_{t}}{I_{t-1}} S^{\prime}\left(\frac{I_{t}}{I_{t-1}}\right)\right) Q_{T, t}+ \\
+E_{t}\left\{\frac{P_{C, t+1}}{P_{C, t} R_{t}} \varepsilon_{i, t+1}\left(\frac{I_{t+1}}{I_{t}}\right)^{2} S^{\prime}\left(\frac{I_{t+1}}{I_{t}}\right) Q_{T, t+1}\right\},  \tag{16}\\
Q_{T, t}=  \tag{17}\\
E_{t}\left\{\frac{R_{K, t+1}}{P_{C, t+1}} \frac{P_{C, t+1}}{P_{C, t} R_{t}}\right\}+(1-\tau) E_{t}\left\{\frac{P_{C, t+1}}{P_{C, t} R_{t}} Q_{T, t+1}\right\} .
\end{gather*}
$$

The equation (16) represents the demand for investment and the equation (17) determines a relative price of installed capital (known as Tobin's Q) which is defined as

$$
Q_{T, t}=\frac{\lambda_{K, t}}{\lambda_{C, t} P_{C, t}},
$$

where $\lambda_{C, t}$ is a marginal utility of nominal income (it is also a Lagrange multiplier on households' budget constraint) and $\lambda_{K, t}$ is a Lagrange multiplier on the law of motion for capital.

Homogenous investment goods are produced in a similar way as the final consumption goods, except that there are no distribution costs associated

[^2]with using tradable investment goods, ${ }^{5}$ which implies the following definitions
\[

$$
\begin{aligned}
I_{t} & =\frac{I_{R, t}^{\gamma_{i}} I_{N, t}^{1-\gamma_{i}}}{\gamma_{i}^{\gamma_{i}}\left(1-\gamma_{1}\right)^{1-\gamma_{i}}}, \\
I_{R, t} & =\frac{I_{H, t}^{\alpha} 1-\alpha}{\alpha_{F, t}^{\alpha}(1-\alpha)^{1-\alpha}}, \\
P_{I, t} & =P_{R, t}^{\gamma_{i}} P_{N, t}^{1-\gamma_{i}}
\end{aligned}
$$
\]

It is assumed that a composition of consumption and investment basket in a given economy can differ, i.e. parameters $\gamma_{c}$ and $\gamma_{i}$ can be different, and that composition of tradable baskets is identical, i.e. parameter $\alpha$ is the same for both tradable consumption goods and tradable investment goods in the given economy.

## Wage Setting

Each household is specialized in a different type of labor $L_{t}(j)$, which it supplies in a monopolistically competitive labor market. All supplied labor types are aggregated into homogenous labor input $L_{t}$ according to the formula

$$
L_{t}=\left[\left(\frac{1}{n}\right)^{\frac{1}{\phi_{W}}} \int_{0}^{n} L_{t}(j)^{\frac{\phi_{W}-1}{\phi_{W}}} d j\right]^{\frac{\phi_{W}}{\phi_{W}-1}}
$$

where $\phi_{W}$ is the elasticity of substitution between different labor types. A corresponding aggregate wage index is then defined as

$$
W_{t}=\left[\frac{1}{n} \int_{0}^{n} W_{t}(j)^{1-\phi_{W}} d j\right]^{\frac{1}{1-\phi_{W}}},
$$

where $W_{t}(j)$ denotes a wage of the household $j$. Cost minimization of firms implies the following demand schedules for each labor type $L_{t}(j)$ in the form

$$
\begin{equation*}
L_{t}(j)=\frac{1}{n}\left(\frac{W_{t}(j)}{W_{t}}\right)^{-\theta_{W}} L_{t} \tag{18}
\end{equation*}
$$

[^3]Following Erceg, Henderson and Levin (2000), a wage setting mechanism a-la Calvo in a modified version with partial wage indexation is assumed. According to this set-up, every period only $1-\theta_{W}$ portion of households (randomly chosen) can reset their wages optimally, while the remaining portion of households $\theta_{W}$ adjust their wages according to the indexation rule

$$
W_{t}(j)=W_{t-1}(j)\left(\frac{P_{C, t-1}}{P_{C, t-2}}\right)^{\delta_{W}}
$$

where $\delta_{W} \in(0,1)$ is a parameter of wage indexation. If I set $\delta_{W}=0$, I get the original Calvo wage setting mechanism, where all households which can not reoptimize their wages leave their wages unchanged. By setting $\delta_{W}=1$, I get the Calvo wage setting mechanism with full wage indexation, where all households which can not reoptimize their wages fully adjust their wages according to the past inflation.

Households, which are allowed to reset their wages optimally, want to maximize their utility represented by the utility function (1), subject to the set of budget constraints (2) and labor demand constraints (18), taking into account the Calvo constraint that they can not always reset their wages. Formally, households want to maximize

$$
E_{t} \sum_{k=0}^{\infty} \theta_{W}^{k} \beta^{k}\left[-\frac{\varepsilon_{l, t+k}}{1+\phi} L_{t+k}(j)^{1+\phi}+\lambda_{C, t+k} W_{t}(j)\left(\frac{P_{C, t+k-1}}{P_{C, t-1}}\right)^{\delta_{W}} L_{t+k}(j)\right]
$$

which is subject to the following constraint

$$
L_{t+k}(j)=\frac{1}{n}\left[\frac{W_{t}(j)}{W_{t+k}}\left(\frac{P_{C, t+k-1}}{P_{C, t-1}}\right)^{\delta_{W}}\right]^{-\phi_{W}} L_{t+k}
$$

First order condition of this optimization problem is in the form

$$
\begin{aligned}
E_{t} \sum_{k=0}^{\infty} \theta_{W}^{k} \beta^{k} & {\left[\frac{W_{t}(j)}{P_{C, t+k}}\left(\frac{P_{C, t+k-1}}{P_{C, t-1}}\right)^{\delta_{W}}-\frac{\phi_{W}}{\phi_{W}-1} M R S_{t+k}(j)\right] . } \\
& \cdot \varepsilon_{d, t+k}\left(C_{t+k}(j)-h C_{t+k-1}\right)^{-\sigma} L_{t+k}(j)=0,
\end{aligned}
$$

where $\frac{\phi_{W}}{\phi_{W}-1}$ represents gross wage mark-up resulting from certain monopoly power of the household, $M R S_{t}(j)$ is the marginal rate of substitution between labor and consumption of household $j$, defined as

$$
M R S_{t}(j)=\frac{\varepsilon_{l, t} L_{t}(j)^{\phi}}{\varepsilon_{d, t}\left(C_{t}(j)-h C_{t-1}\right)^{-\sigma}} .
$$

Since all households face the same optimization problem, they all set the same optimal wage $\tilde{W}_{t}$. Therefore, the aggregate wage index is then defined as a weighted average of optimally set wages, and wages which are partially adjusted according to the past inflation. Formally,

$$
W_{t}=\left[\theta_{W}\left(W_{t-1}\left(\frac{P_{C, t-1}}{P_{C, t-2}}\right)^{\delta_{W}}\right)^{1-\phi_{W}}+\left(1-\theta_{W}\right) \tilde{W}_{t}^{1-\phi_{W}}\right]^{\frac{1}{1-\phi_{W}}}
$$

Similar conditions and formulas hold also for the foreign economy. It is allowed for parameters governing the wage setting of households to differ between countries.

## Firms

## Production Technology

There is a continuum of homogenous, monopolistic competitive firms in the tradable and non-tradable sectors of the domestic economy. The production functions of firms are represented by Cobb-Douglas functions, homogenous in labor and capital of degree one (i.e. with constant returns to scale)

$$
\begin{aligned}
& Y_{t}\left(z_{N}\right)=\varepsilon_{a^{N}, t} L_{t}\left(z_{N}\right)^{1-\eta} K_{t}\left(z_{N}\right)^{\eta}, \\
& Y_{t}\left(z_{H}\right)=\varepsilon_{a^{H}, t} L_{t}\left(z_{H}\right)^{1-\eta} K_{t}\left(z_{H}\right)^{\eta},
\end{aligned}
$$

where $\eta$ is the elasticity of output with respect to capital (common to both sectors, but potentially different in individual countries), and $\varepsilon_{a^{H}, t}\left(\varepsilon_{a^{N}, t}\right)$ is a productivity shock in the tradable (non-tradable) sector. The index of
output in each sector is given by Dixit-Stiglitz aggregator

$$
\begin{aligned}
& Y_{N, t}=\left[\left(\frac{1}{n}\right)^{\frac{1}{\phi_{N}}} \int_{0}^{n} Y_{t}\left(z_{N}\right)^{\frac{\phi_{N}-1}{\phi_{N}}} d z_{N}\right]^{\frac{\phi_{N}}{\phi_{N}-1}} \\
& Y_{H, t}=\left[\left(\frac{1}{n}\right)^{\frac{1}{\phi_{H}}} \int_{0}^{n} Y_{t}\left(z_{H}\right)^{\frac{\phi_{H}-1}{\phi_{H}}} d z_{H}\right]^{\frac{\phi_{H}}{\phi_{H}-1}} .
\end{aligned}
$$

All firms try to minimize their costs for a given level of production. Formally, firms try to minimize

$$
\begin{equation*}
\min _{\substack{L_{t}\left(z_{i}\right) \\ K_{t}\left(z_{i}\right)}} W_{t} L_{t}\left(z_{i}\right)+R_{K, t} K_{t}\left(z_{i}\right)+\lambda_{i, t}\left(Y_{t}\left(z_{i}\right)-\varepsilon_{a^{i}, t} L_{t}\left(z_{i}\right)^{1-\eta} K_{t}\left(z_{i}\right)^{\eta}\right) \tag{19}
\end{equation*}
$$

for $i=N, H$. Since all firms have the same technology and face the same prices of inputs, cost minimization (19) requires the same ratio of capital and labor for all domestic firms

$$
\frac{W_{t} L_{t}}{R_{K, t} K_{t}}=\frac{1-\eta}{\eta}
$$

Lagrange multiplier $\lambda_{i, t}$ can be interpreted as nominal marginal costs. Therefore, the real marginal costs, identical for all firms in the given sector, are defined by the formula

$$
\begin{equation*}
M C_{i, t}=\frac{\lambda_{i, t}}{P_{i, t}}=\frac{1}{P_{i, t} \varepsilon_{a^{i}, t}}\left(\frac{W_{t}}{1-\eta}\right)^{1-\eta}\left(\frac{R_{K, t}}{\eta}\right)^{\eta}, \quad \text { for } i=N, H \tag{20}
\end{equation*}
$$

## Price Setting

In this section I shall describe price setting problem of firms in the domestic non-tradable sector. Price setting of foreign firms as well as firms in the tradable sector is defined analogously.

Firms in the non-tradable sector set their prices in order to maximize their profits. It is assumed that firms face modified Calvo restriction with partial indexation on the frequency of price adjustment. According to this restriction, every period only $1-\theta_{N}$ portion of firms in non-tradable sector
(randomly chosen) can reset their prices optimally, while $\theta_{N}$ portion of firms in non-tradable sector partially adjust their prices according to the past inflation, following the indexation rule

$$
P_{t}\left(z_{N}\right)=P_{t-1}\left(z_{N}\right)\left(\frac{P_{N, t-1}}{P_{N, t-2}}\right)^{\delta_{N}}
$$

where $\delta_{N}$ is a parameter of price indexation. Setting $\delta_{N}=0$, I get the original Calvo constraint, as suggested by Calvo (1983). By setting $\delta_{N}=1$, I get the Calvo constraint with full price indexation, where all firms which can not reoptimize their prices fully adjust their prices according to the past inflation.

A firm $j$, which is allowed to reset its price, chooses the price $P_{t}\left(z_{N}\right)$ in order to maximize current market value of profits generated until the firm can again reoptimize its price. Formally, firms solve maximization problem

$$
E_{t} \sum_{k=0}^{\infty} \theta_{N}^{k} \beta^{k} \lambda_{C, t+k} Y_{t+k}\left(z_{N}\right)\left[P_{t}\left(z_{N}\right)\left(\frac{P_{N, t+k-1}}{P_{N, t-1}}\right)^{\delta_{N}}-P_{N, t+k} M C_{N, t+k}\right]
$$

taking into account the sequence of demand constraints

$$
Y_{t+k}\left(z_{N}\right)=\frac{1}{n}\left[\frac{P_{t}\left(z_{N}\right)}{P_{N, t+k}}\left(\frac{P_{N, t+k-1}}{P_{N, t-1}}\right)^{\delta_{N}}\right]^{-\phi_{N}} Y_{N, t+k}
$$

where $\lambda_{C, t}$ is the marginal utility of households' nominal income in period $t$, considered by firms as exogenous, and $M C_{N, t}$ is the real marginal costs in the period $t$, defined in (20). The first order condition of the maximization problem of firms implies

$$
\begin{array}{r}
E_{t} \sum_{k=0}^{\infty} \theta_{N}^{k} \beta^{k} \lambda_{C, t+k} Y_{t+k}\left(z_{N}\right)\left[P_{t}\left(z_{N}\right)\left(\frac{P_{N, t+k-1}}{P_{N, t-1}}\right)^{\delta_{N}}-\right. \\
\left.-\frac{\phi_{N}}{\phi_{N}-1} P_{N, t+k} M C_{N, t+k}\right]=0 \tag{21}
\end{array}
$$

Firms do not face any firm-specific shocks, so all firms in the given sector
choose the same optimal price $\tilde{P}_{N, t}$. Hence, it is possible to express the aggregate price index of non-tradable goods as a weighted average of the optimizing firms' price $\tilde{P}_{N, t}$, and the price of firms which adjust their price to the previous inflation

$$
\begin{equation*}
P_{N, t}=\left[\theta_{N}\left(P_{N, t-1}\left(\frac{P_{N, t-1}}{P_{N, t-2}}\right)^{\delta_{N}}\right)^{1-\phi_{N}}+\left(1-\theta_{N}\right) \tilde{P}_{N, t}^{1-\phi_{N}}\right]^{\frac{1}{1-\phi_{N}}} . \tag{22}
\end{equation*}
$$

Foreign firms and domestic firms in the tradable sector deal with analogous maximization problems. Therefore, first order conditions and resulting price indices associated with maximization problems of foreign firms and domestic firms in tradable sector are analogous to those expressed in equations (21) and (22). It is assumed that structural parameters of price stickiness $\theta$ and price indexation $\delta$ as well as stochastic properties of shocks in productivity can differ among countries and sectors.

It is assumed that prices are set in the producer's currency and that international law of one price holds for intermediate tradable goods. Thus, prices of domestic goods sold in the foreign economy and prices of foreign goods sold in the domestic economy are given by formulas

$$
P_{t}^{*}\left(z_{H}\right)=E R_{t}^{-1} P_{t}\left(z_{H}\right) \quad P_{t}\left(z_{F}\right)=E R_{t} P_{t}^{*}\left(z_{F}\right),
$$

where $E R_{t}$ is the nominal exchange rate expressed as units of domestic currency per one unit of foreign currency.

## International Risk Sharing

The assumption of complete financial markets implies the perfect risk-sharing condition. Loosely speaking, this condition requires that prices of similar bonds must be the same in the domestic as well as in the foreign economy.

This condition can be expressed using gross returns on these bonds as

$$
R_{t}=R_{t}^{*} E_{t}\left\{\frac{E R_{t+1}}{E R_{t}}\right\}
$$

This formula requires that gross returns on domestic bonds must be the same as gross returns on foreign bonds adjusted by expected appreciation (depreciation) of the foreign currency. By substituting for the gross returns on domestic and foreign bonds from the Euler equation (3) and after subsequent mathematical manipulation we get the formula

$$
\begin{equation*}
Q_{t}=\kappa \frac{\varepsilon_{d, t}^{*}}{\varepsilon_{d, t}} \frac{\left(C_{t}^{*}-h^{*} C_{t-1}^{*}\right)^{-\sigma^{*}}}{\left(C_{t}-h C_{t-1}\right)^{-\sigma}}, \tag{23}
\end{equation*}
$$

where

$$
\kappa=E_{t}\left\{Q_{t+1} \frac{\varepsilon_{d, t+1}\left(C_{t+1}-h C_{t}\right)^{-\sigma}}{\varepsilon_{d, t+1}^{*}\left(C_{t+1}^{*}-h^{*} C_{t}^{*}\right)^{-\sigma^{*}}}\right\}
$$

is regarded as a constant, which (using iterations) depends on initial conditions and $Q_{t}$ is a real exchange rate defined as

$$
\begin{equation*}
Q_{t}=\frac{E R_{t} P_{C, t}^{*}}{P_{C, t}} \tag{24}
\end{equation*}
$$

Formula (23) implies that the real exchange rate is proportional to the ratio of marginal utility of consumption between domestic and foreign households.

The real exchange rate can deviate from purchasing power parity (PPP) because of changes in relative prices of tradable and non-tradable goods, changes in relative distribution costs and changes in terms of trade, as long as there is a difference between household preferences among countries, i.e. $\alpha \neq 1-\alpha^{*}$. This can be demonstrated this by substituting for the price indices in the definition of real exchange rate (24) from definitions of these price indices (8), (9) and (14). After some mathematical manipulation we obtain

$$
Q_{t}=S_{t}^{\alpha+\alpha^{*}-1} \frac{1+\omega^{*} D_{t}^{*}}{1+\omega D_{t}} \frac{X_{t}^{* 1-\gamma_{c}^{*}}}{X_{t}^{1-\gamma_{c}}}
$$

where $S_{t}$ are terms of trade defined as domestic import prices relative to domestic export prices ${ }^{6}$

$$
S_{t}=\frac{E R_{t} P_{F, t}^{*}}{P_{H, t}}
$$

$X_{t}$ and $X_{t}^{*}$ are internal exchange rates defined as prices of non-tradable goods relative to prices of tradable goods

$$
X_{t}=\frac{P_{N, t}}{P_{T, t}} \quad X_{t}^{*}=\frac{P_{N, t}^{*}}{P_{T, t}^{*}}
$$

and $D_{t}$ and $D_{t}^{*}$ are relative distribution costs, defined as prices of nontradable goods relative to prices of raw tradable goods

$$
D_{t}=\frac{P_{N, t}}{P_{R, t}} \quad D_{t}^{*}=\frac{P_{N, t}^{*}}{P_{R, t}^{*}} .
$$

## Monetary and Fiscal Authorities

The behavior of central bank is described by a variant of Taylor rule. ${ }^{7}$

$$
R_{t}=R_{t-1}^{\rho}\left[E_{t}\left\{\left(\frac{Y_{t+1}}{\bar{Y}}\right)^{\phi_{y}}\left(\frac{P_{C, t+1}}{(1+\bar{\pi}) P_{C, t}}\right)^{\phi_{\pi}}\right\}\right]^{1-\rho} \varepsilon_{m, t},
$$

where $\rho$ is a parameter of interest rate smoothing, $Y_{t}$ is a total output in the economy, $\bar{Y}$ denotes a steady state level of this output, $\bar{\pi}$ is a steady state level of inflation, $\phi_{y}$ is an elasticity of the interest rate to the output, $\phi_{\pi}$ is an elasticity of the interest rate to inflation and $\varepsilon_{m, t}$ is a monetary policy shock.

Fiscal policy is modeled in a very simple fashion. Government expenditures and transfers to households are fully financed by lump-sum taxes so

[^4]that state budget is balanced every period. Government expenditures consist only of non-tradable domestic goods and are modeled as a stochastic process $\varepsilon_{g, t}$. Given the assumptions about households, Ricardian equivalence holds in this model.

## Market Clearing Conditions

The model is closed by satisfying the market clearing conditions. Goods market clearing requires that output of each firm producing non-tradable goods is either consumed by households in the domestic economy, spent on investment, used for distribution services or purchased by the government. Similarly, output of firms producing tradable goods is either consumed or invested in the domestic or foreign economy. Formally

$$
\begin{align*}
& Y_{N, t}=C_{N, t}+I_{N, t}+Y_{D, t}+G_{t}  \tag{25}\\
& Y_{H, t}=C_{H, t}+C_{H, t}^{*}+I_{H, t}+I_{H, t}^{*} . \tag{26}
\end{align*}
$$

By plugging in allocation functions (6), (7), and (12) together with analogous allocation functions for investment and their foreign counterparts into the goods market clearing conditions (25) and (26), the aggregate output in both domestic sectors can be rewritten as

$$
\begin{gather*}
Y_{N, t}=\left(1-\gamma_{c}\right)\left(\frac{P_{N, t}}{P_{C, t}}\right)^{-1} C_{t}+\omega \gamma_{c}\left(\frac{P_{T, t}}{P_{C, t}}\right)^{-1} C_{t}+ \\
+\left(1-\gamma_{i}\right)\left(\frac{P_{N, t}}{P_{I, t}}\right)^{-1} I_{t}+G_{t}  \tag{27}\\
Y_{H, t}=\alpha \gamma_{c}\left(\frac{P_{H, t}}{P_{R, t}}\right)^{-1}\left(\frac{P_{T, t}}{P_{C, t}}\right)^{-1} C_{t}+\frac{1-n}{n}\left(1-\alpha^{*}\right) \gamma_{c}^{*}\left(\frac{P_{H, t}^{*}}{P_{R, t}^{*}}\right)^{-1}\left(\frac{P_{T, t}^{*}}{P_{C, t}^{*}}\right)^{-1} C_{t}^{*} \\
+\alpha \gamma_{i}\left(\frac{P_{H, t}}{P_{R, t}}\right)^{-1}\left(\frac{P_{R, t}}{P_{I, t}}\right)^{-1} I_{t}+\frac{1-n}{n}\left(1-\alpha^{*}\right) \gamma_{i}^{*}\left(\frac{P_{H, t}^{*}}{P_{R, t}^{*}}\right)^{-1}\left(\frac{P_{R, t}^{*}}{P_{I, t}^{*}}\right)^{-1} I_{t}^{*},
\end{gather*}
$$

where in (27) I use the following condition of optimality

$$
Y_{D, t}=\omega C_{T, t},
$$

which links distribution services with tradable consumption goods. The total output in the economy is given by the sum of output in tradable and nontradable sectors

$$
Y_{t}=Y_{N, t}+Y_{H, t} .
$$

Finally, market clearing conditions for factor markets requires

$$
\begin{aligned}
L_{t} & =\int_{0}^{n} L_{t}\left(z_{N}\right) d z_{N}+\int_{0}^{n} L_{t}\left(z_{H}\right) d z_{H} \\
K_{t} & =\int_{0}^{n} K_{t}\left(z_{N}\right) d z_{N}+\int_{0}^{n} K_{t}\left(z_{H}\right) d z_{H}
\end{aligned}
$$

Analogous market clearing conditions hold for the foreign economy, too.

## Exogenous Shocks

Behavior of the model is driven by seven structural shocks in each economy: productivity shocks in tradable sector ( $\varepsilon_{a^{H}, t}$ and $\varepsilon_{a^{F}, t}^{*}$ ), productivity shocks in non-tradable sector $\left(\varepsilon_{a^{N}, t}\right.$ and $\left.\varepsilon_{a^{N}, t}^{*}\right)$, labor supply shocks $\left(\varepsilon_{l, t}\right.$ and $\left.\varepsilon_{l, t}^{*}\right)$, investment efficiency shocks ( $\varepsilon_{i, t}$ and $\varepsilon_{i, t}^{*}$ ), consumption preference shocks ( $\varepsilon_{d, t}$ and $\varepsilon_{d, t}^{*}$ ), government spending shocks ( $\varepsilon_{g, t}$ and $\varepsilon_{g, t}^{*}$ ) and monetary policy shocks $\left(\varepsilon_{m, t}\right.$ and $\left.\varepsilon_{m, t}^{*}\right)$. Except for monetary policy shocks, all other shocks are represented by AR1 processes in the log-linearised version of the model, see (68) - (79). Monetary policy shocks are represented by IID processes in the log-linearised version of the model. ${ }^{8}$ I also allow for correlations between innovations in corresponding domestic and foreign shocks.

[^5]
## Log-linearised Model

The model presented above is highly non-linear and does not have any analytical solution. A log-linear approximation around the non-stochastic steady state is employed for the purposes of empirical analysis. For details about methods of log-linear approximation see Uhlig (1995). Nice introduction to methods used for log-linearising around the steady state is provided by $\mathrm{Zi}-$ etz (2006). In this section I present a log-linearised form of the model. All variables of the model are in the form of log-deviations from their respective steady state. Formally, $x_{t}=\log X_{t}-\log \bar{X}$, where $\bar{X}$ is a steady state value.

The model is formed by 40 equations describing endogenous variables (from equation (28) to equation (67)) and by 12 equations for exogenous shocks (from equation (68) to equation (79)). An interpretation of the model variables is presented in Table 1. Interpretation of the structural parameters and the parameters related to shocks is presented in Tables 2 and 3.

## Market Clearing Conditions:

$$
\begin{align*}
y_{H, t}= & \frac{\bar{C}}{\bar{Y}_{H}} \frac{\gamma_{c} \alpha}{1+\omega}\left(c_{t}+\left(1-\gamma_{c}\right) x_{t}+(1-\alpha) s_{t}\right) \\
& +\frac{\bar{C}^{*}}{\bar{Y}_{H}} \frac{1-n}{n} \frac{\gamma_{c}^{*}\left(1-\alpha^{*}\right)}{1+\omega^{*}}\left(c_{t}^{*}+\left(1-\gamma_{c}^{*}\right) x_{t}^{*}+\alpha^{*} s_{t}\right)  \tag{28}\\
& +\frac{\bar{I}}{\bar{Y}_{H}} \gamma_{i} \alpha\left(i_{t}+\left(1-\gamma_{i}\right)(1+\omega) x_{t}+(1-\alpha) s_{t}\right) \\
& +\frac{\bar{I}^{*}}{\bar{Y}_{H}} \frac{1-n}{n} \gamma_{i}^{*}\left(1-\alpha^{*}\right)\left(i_{t}^{*}+\left(1-\gamma_{i}^{*}\right)(1+\omega) x_{t}^{*}+\alpha^{*} s_{t}\right)
\end{align*}
$$

$$
\begin{align*}
& y_{F, t}^{*}= \frac{\bar{C}^{*}}{\bar{Y}_{F}^{*}} \frac{\gamma_{c}^{*} \alpha^{*}}{1+\omega^{*}}\left(c_{t}^{*}+\left(1-\gamma_{c}^{*}\right) x_{t}^{*}-\left(1-\alpha^{*}\right) s_{t}\right) \\
&+\frac{\bar{C}}{\bar{Y}_{F}^{*}} \frac{n}{1-n} \frac{\gamma_{c}(1-\alpha)}{1+\omega}\left(c_{t}+\left(1-\gamma_{c}\right) x_{t}-\alpha s_{t}\right)  \tag{29}\\
&+\frac{\bar{I}^{*}}{\bar{Y}_{F}^{*}} \gamma_{i}^{*} \alpha^{*}\left(i_{t}^{*}+\left(1-\gamma_{i}^{*}\right)\left(1+\omega^{*}\right) x_{t}^{*}-\left(1-\alpha^{*}\right) s_{t}\right) \\
&+\frac{\bar{I}}{\bar{Y}_{F}^{*}} \frac{n}{1-n} \gamma_{i}(1-\alpha)\left(i_{t}+\left(1-\gamma_{i}\right)(1+\omega) x_{t}-\alpha s_{t}\right) \\
& y_{N, t}= \overline{\bar{C}}\left(\left(1-\gamma_{c}\right)\left(c_{t}-\gamma_{c} x_{t}\right)+\frac{\gamma_{c} \omega}{1+\omega}\left(c_{t}+\left(1-\gamma_{c}\right) x_{t}\right)\right) \\
&+\frac{\bar{I}}{\bar{Y}_{N}}\left(1-\gamma_{i}\right)\left(i_{t}-\gamma_{i}(1+\omega) x_{t}\right)+\frac{\bar{G}}{\overline{Y_{N}}} \varepsilon_{g, t}  \tag{30}\\
& y_{N, t}^{*}= \bar{C}^{*} \\
& \bar{Y}_{N}^{*}\left.\left(1-\gamma_{c}^{*}\right)\left(c_{t}^{*}-\gamma_{c}^{*} x_{t}^{*}\right)+\frac{\gamma_{c}^{*} \omega^{*}}{1+\omega^{*}}\left(c_{t}^{*}+\left(1-\gamma_{c}^{*}\right) x_{t}^{*}\right)\right)  \tag{31}\\
&+\frac{\bar{I}^{*}}{\bar{Y}_{N}^{*}}\left(1-\gamma_{i}^{*}\right)\left(i_{t}^{*}-\gamma_{i}^{*}\left(1+\omega^{*}\right) x_{t}^{*}\right)+\frac{\bar{G}^{*}}{\bar{Y}_{N}^{*}} \varepsilon_{g, t}^{*}  \tag{32}\\
& y_{t}=\frac{\bar{Y}_{H}}{\bar{Y}} y_{H, t}+\frac{\bar{Y}_{N}}{\bar{Y}} y_{N, t}  \tag{33}\\
& y_{t}^{*}=\frac{\bar{Y}_{F}^{*}}{\bar{Y}^{*}} y_{F, t}^{*}+\frac{\bar{Y}_{N}^{*}}{\bar{Y}_{N, t}^{*}}
\end{align*}
$$

## Euler Equation:

$$
\begin{align*}
c_{t}-h c_{t-1} & =E_{t}\left(c_{t+1}-h c_{t}\right)-\frac{1-h}{\sigma} E_{t}\left(r_{t}-\pi_{t+1}+\varepsilon_{d, t+1}-\varepsilon_{d, t}\right)  \tag{34}\\
c_{t}^{*}-h^{*} c_{t-1}^{*} & =E_{t}\left(c_{t+1}^{*}-h^{*} c_{t}^{*}\right)-\frac{1-h^{*}}{\sigma^{*}} E_{t}\left(r_{t}^{*}-\pi_{t+1}^{*}+\varepsilon_{d, t+1}^{*}-\varepsilon_{d, t}^{*}\right) \tag{35}
\end{align*}
$$

## International Risk Sharing Condition:

$$
\begin{equation*}
q_{t}=\varepsilon_{d, t}^{*}-\varepsilon_{d, t}-\frac{\sigma^{*}}{1-h^{*}}\left(c_{t}^{*}-h^{*} c_{t-1}^{*}\right)+\frac{\sigma}{1-h}\left(c_{t}-h c_{t-1}\right) \tag{36}
\end{equation*}
$$

## Capital Accumulation:

$$
\begin{align*}
& k_{t+1}=(1-\tau) k_{t}+\tau\left(i_{t}+\varepsilon_{i, t}\right)  \tag{37}\\
& k_{t+1}^{*}=\left(1-\tau^{*}\right) k_{t}^{*}+\tau^{*}\left(i_{t}^{*}+\varepsilon_{i, t}^{*}\right) \tag{38}
\end{align*}
$$

## Real Costs of Capital:

$$
\begin{align*}
& r_{K, t}=w_{t}+l_{t}-k_{t}  \tag{39}\\
& r_{K, t}^{*}=w_{t}^{*}+l_{t}^{*}-k_{t}^{*} \tag{40}
\end{align*}
$$

## Investment Demand:

$$
\begin{align*}
& i_{t}-i_{t-1}=\beta E_{t}\left(i_{t+1}-i_{t}\right)+\frac{1}{S^{\prime \prime \prime}}\left(q_{T, t}+\varepsilon_{i, t}\right)-\frac{\gamma_{i}(1+\omega)-\gamma_{c}}{S^{\prime \prime}} x_{t}  \tag{41}\\
& i_{t}^{*}-i_{t-1}^{*}=\beta^{*} E_{t}\left(i_{t+1}^{*}-i_{t}^{*}\right)+\frac{1}{S^{\prime \prime *}}\left(q_{T, t}^{*}+\varepsilon_{i, t}^{*}\right)-\frac{\gamma_{i}^{*}\left(1+\omega^{*}\right)-\gamma_{c}^{*}}{S^{\prime \prime *}} x_{t}^{*} \tag{42}
\end{align*}
$$

## Price of Installed Capital:

$$
\begin{align*}
& q_{T, t}=\beta(1-\tau) E_{t} q_{T, t+1}-\left(r_{t}-E_{t} \pi_{t+1}\right)+(1-\beta(1-\tau)) E_{t} r_{K, t+1}  \tag{43}\\
& q_{T, t}^{*}=\beta^{*}\left(1-\tau^{*}\right) E_{t} q_{T, t+1}^{*}-\left(r_{t}^{*}-E_{t} \pi_{t+1}^{*}\right)+\left(1-\beta^{*}\left(1-\tau^{*}\right)\right) E_{t} r_{K, t+1}^{*} \tag{44}
\end{align*}
$$

## Labor Input:

$$
\begin{align*}
l_{t} & =\eta\left(r_{K, t}-w_{t}\right)+\frac{\bar{Y}_{H}}{\bar{Y}}\left(y_{H, t}-\varepsilon_{a^{H}, t}\right)+\frac{\bar{Y}_{N}}{\bar{Y}}\left(y_{N, t}-\varepsilon_{a^{N}, t}\right)  \tag{45}\\
l_{t}^{*} & =\eta^{*}\left(r_{K, t}^{*}-w_{t}^{*}\right)+\frac{\overline{Y_{F}^{*}}}{\bar{Y}^{*}}\left(y_{F, t}^{*}-\varepsilon_{a^{F}, t}^{*}\right)+\frac{\bar{Y}_{N}^{*}}{\bar{Y}^{*}}\left(y_{N, t}^{*}-\varepsilon_{a^{N}, t}^{*}\right) \tag{46}
\end{align*}
$$

## Real Wage:

$$
\begin{align*}
w_{t}-w_{t-1}= & \frac{\left(1-\theta_{W}\right)\left(1-\beta \theta_{W}\right)}{\theta_{W}\left(1+\phi_{W} \phi\right)}\left(m r s_{t}-w_{t}\right)+\beta E_{t}\left(w_{t+1}-w_{t}\right)  \tag{47}\\
& +\beta E_{t}\left(\pi_{t+1}-\delta_{W} \pi_{t}\right)-\left(\pi_{t}-\delta_{W} \pi_{t-1}\right)
\end{align*}
$$

$$
\begin{align*}
w_{t}^{*}-w_{t-1}^{*}= & \frac{\left(1-\theta_{W}^{*}\right)\left(1-\beta^{*} \theta_{W}^{*}\right)}{\theta_{W}^{*}\left(1+\phi_{W}^{*} \phi^{*}\right)}\left(m r s_{t}^{*}-w_{t}^{*}\right)+\beta^{*} E_{t}\left(w_{t+1}^{*}-w_{t}^{*}\right)  \tag{48}\\
& +\beta^{*} E_{t}\left(\pi_{t+1}^{*}-\delta_{W}^{*} \pi_{t}^{*}\right)-\left(\pi_{t}^{*}-\delta_{W}^{*} \pi_{t-1}^{*}\right)
\end{align*}
$$

## Marginal Rate of Substitution:

$$
\begin{align*}
m r s_{t} & =\varepsilon_{l, t}+\phi l_{t}-\varepsilon_{d, t}+\frac{\sigma}{1-h}\left(c_{t}-h c_{t-1}\right)  \tag{49}\\
m r s_{t}^{*} & =\varepsilon_{l, t}^{*}+\phi^{*} l_{t}^{*}-\varepsilon_{d, t}^{*}+\frac{\sigma^{*}}{1-h^{*}}\left(c_{t}^{*}-h^{*} c_{t-1}^{*}\right) \tag{50}
\end{align*}
$$

Phillips Curve for Tradable Sector:

$$
\begin{align*}
\pi_{H, t}-\delta_{H} \pi_{H, t-1} & =\beta E_{t}\left(\pi_{H, t+1}-\delta_{H} \pi_{H, t}\right)+\frac{\left(1-\theta_{H}\right)\left(1-\beta \theta_{H}\right)}{\theta_{H}} m c_{H, t}  \tag{51}\\
\pi_{F, t}^{*}-\delta_{F}^{*} \pi_{F, t-1}^{*} & =\beta^{*} E_{t}\left(\pi_{F, t+1}^{*}-\delta_{F}^{*} \pi_{F, t}^{*}\right)+\frac{\left(1-\theta_{F}^{*}\right)\left(1-\beta^{*} \theta_{F}^{*}\right)}{\theta_{F}^{*}} m c_{F, t}^{*} \tag{52}
\end{align*}
$$

Phillips Curve for Non-tradable Sector:

$$
\begin{align*}
\pi_{N, t}-\delta_{N} \pi_{N, t-1}=\beta E_{t}\left(\pi_{N, t+1}\right. & \left.-\delta_{N} \pi_{N, t}\right)+\frac{\left(1-\theta_{N}\right)\left(1-\beta \theta_{N}\right)}{\theta_{N}} m c_{N, t}  \tag{53}\\
\pi_{N, t}^{*}-\delta_{N}^{*} \pi_{N, t-1}^{*} & =\beta^{*} E_{t}\left(\pi_{N, t+1}^{*}-\delta_{N}^{*} \pi_{N, t}^{*}\right)+ \\
& +\frac{\left(1-\theta_{N}^{*}\right)\left(1-\beta^{*} \theta_{N}^{*}\right)}{\theta_{N}^{*}} m c_{N, t}^{*} \tag{54}
\end{align*}
$$

## Real Marginal Costs in Tradable Sector:

$$
\begin{align*}
m c_{H, t} & =(1-\eta) w_{t}+\eta r_{K, t}-\varepsilon_{a^{H}, t}+(1-\alpha) s_{t}+\left(1-\gamma_{c}+\omega\right) x_{t}  \tag{55}\\
m c_{F, t}^{*} & =\left(1-\eta^{*}\right) w_{t}^{*}+\eta^{*} r_{K, t}^{*}-\varepsilon_{a^{F}, t}^{*}+\left(1-\alpha^{*}\right) s_{t}+\left(1-\gamma_{c}^{*}+\omega^{*}\right) x_{t}^{*} \tag{56}
\end{align*}
$$

Real Marginal Costs in Non-tradable Sector:

$$
\begin{align*}
m c_{N, t} & =(1-\eta) w_{t}+\eta r_{K, t}-\varepsilon_{a^{N}, t}-\gamma_{c} x_{t}  \tag{57}\\
m c_{N, t}^{*} & =\left(1-\eta^{*}\right) w_{t}^{*}+\eta^{*} r_{K, t}^{*}-\varepsilon_{a^{N}, t}^{*}-\gamma_{c}^{*} x_{t}^{*} \tag{58}
\end{align*}
$$

Relative Price of Non-tradable Goods:

$$
\begin{align*}
x_{t}-x_{t-1} & =\pi_{N, t}-\pi_{T, t}  \tag{59}\\
x_{t}^{*}-x_{t-1}^{*} & =\pi_{N, t}^{*}-\pi_{T, t}^{*} \tag{60}
\end{align*}
$$

Inflation of Tradable Goods:

$$
\begin{align*}
& \pi_{T, t}=\frac{1}{1+\omega}\left(\pi_{H, t}+(1-\alpha) \Delta s_{t}+\omega \pi_{N, t}\right)  \tag{61}\\
& \pi_{T, t}^{*}=\frac{1}{1+\omega^{*}}\left(\pi_{F, t}^{*}+\left(1-\alpha^{*}\right) \Delta s_{t}+\omega^{*} \pi_{N, t}^{*}\right) \tag{62}
\end{align*}
$$

## Overall Inflation:

$$
\begin{align*}
\pi_{t} & =\gamma_{c} \pi_{T, t}+\left(1-\gamma_{c}\right) \pi_{N, t}  \tag{63}\\
\pi_{t}^{*} & =\gamma_{c}^{*} \pi_{T, t}^{*}+\left(1-\gamma_{c}^{*}\right) \pi_{N, t}^{*} \tag{64}
\end{align*}
$$

## Real Exchange Rate:

$$
\begin{equation*}
q_{t}=\left(\alpha+\alpha^{*}-1\right) s_{t}+\left(1-\gamma_{c}^{*}+\omega^{*}\right) x_{t}^{*}-\left(1-\gamma_{c}+\omega\right) x_{t} \tag{65}
\end{equation*}
$$

Monetary Policy Rule:

$$
\begin{align*}
& r_{t}=\rho r_{t-1}+(1-\rho)\left(\psi_{y} E_{t}\left\{y_{t+1}\right\}+\psi_{\pi} E_{t}\left\{\pi_{t+1}\right\}\right)+\varepsilon_{m, t}  \tag{66}\\
& r_{t}^{*}=\rho^{*} r_{t-1}^{*}+\left(1-\rho^{*}\right)\left(\psi_{y}^{*} E_{t}\left\{y_{t+1}^{*}\right\}+\psi_{\pi}^{*} E_{t}\left\{\pi_{t+1}^{*}\right\}\right)+\varepsilon_{m, t}^{*} \tag{67}
\end{align*}
$$

## Productivity Shock in Tradable Sector:

$$
\begin{align*}
\varepsilon_{a^{H}, t} & =\rho_{a^{H}} \varepsilon_{a^{H}, t-1}+\mu_{a^{H}, t}  \tag{68}\\
\varepsilon_{a^{F}, t}^{*} & =\rho_{a^{F}}^{*} \varepsilon_{a^{F}, t-1}^{*}+\mu_{a^{F}, t}^{*} \tag{69}
\end{align*}
$$

Productivity Shock in Non-tradable Sector:

$$
\begin{align*}
& \varepsilon_{a^{N}, t}=\rho_{a^{N}} \varepsilon_{a^{N}, t-1}+\mu_{a^{N}, t}  \tag{70}\\
& \varepsilon_{a^{N}, t}^{*}=\rho_{a^{N}}^{*} \varepsilon_{a^{N}, t-1}^{*}+\mu_{a^{N}, t}^{*} \tag{71}
\end{align*}
$$

## Preference Shock:

$$
\begin{align*}
& \varepsilon_{d, t}=\rho_{d} \varepsilon_{d, t-1}+\mu_{d, t}  \tag{72}\\
& \varepsilon_{d, t}^{*}=\rho_{d}^{*} \varepsilon_{d, t-1}^{*}+\mu_{d, t}^{*} \tag{73}
\end{align*}
$$

Labor Supply Shock:

$$
\begin{align*}
& \varepsilon_{l, t}=\rho_{l} \varepsilon_{l, t-1}+\mu_{l, t}  \tag{74}\\
& \varepsilon_{l, t}^{*}=\rho_{l}^{*} \varepsilon_{l, t-1}^{*}+\mu_{l, t}^{*} \tag{75}
\end{align*}
$$

Shock in Government Expenditures:

$$
\begin{align*}
& \varepsilon_{g, t}=\rho_{g} \varepsilon_{g, t-1}+\mu_{g, t}  \tag{76}\\
& \varepsilon_{g, t}^{*}=\rho_{g}^{*} \varepsilon_{g, t-1}^{*}+\mu_{g, t}^{*} \tag{77}
\end{align*}
$$

Shock in Investment Efficiency:

$$
\begin{align*}
& \varepsilon_{i, t}=\rho_{i} \varepsilon_{i, t-1}+\mu_{i, t}  \tag{78}\\
& \varepsilon_{i, t}^{*}=\rho_{i}^{*} \varepsilon_{i, t-1}^{*}+\mu_{i, t}^{*} \tag{79}
\end{align*}
$$

Table 1: Interpretation of Variables

| variable | interpretation |
| :---: | :---: |
| $c_{t}, c_{t}^{*}$ | consumption |
| $i_{t}, i_{t}^{*}$ | investment |
| $y_{t}, y_{t}^{*}$ | total output |
| $y_{H, t}, y_{F, t}^{*}$ | output in tradable sector |
| $y_{N, t}, y_{N, t}^{*}$ | output in non-tradable sector |
| $x_{t}, x_{t}^{*}$ | internal exchange rates |
| $s_{t}$ | terms of trade |
| $r_{t}, r_{t}^{*}$ | nominal interest rate |
| $q_{t}$ | real exchange rate |
| $k_{t}, k_{t}^{*}$ | capital |
| $r_{K, t}, r_{K, t}^{*}$ | payoff from renting capital |
| $w_{t}, w_{t}^{*}$ | real wage |
| $q_{T, t}, q_{T, t}^{*}$ | price of installed capital (Tobin's Q) |
| $l_{t}, l_{t}^{*}$ | labor |
| $m r s_{t}, m r s_{t}^{*}$ | marginal rate of substitution |
| $\pi_{t}, \pi_{t}^{*}$ | inflation |
| $\pi_{T, t}, \pi_{T, t}^{*}$ | inflation of tradable goods |
| $\pi_{H, t}, \pi_{F, t}^{*}$ | inflation in tradable sector |
| $\pi_{N, t}, \pi_{N, t}^{*}$ | inflation of non-tradable goods |
| $m c_{H, t}, m c_{F, t}^{*}$ | real marginal costs in tradable sector |
| $m c_{N, t}, m c_{N, t}^{*}$ | real marginal costs in non-tradable sector |
| $\varepsilon_{a^{H}, t}, \varepsilon_{a^{F}, t}^{*}$ | productivity shock in tradable sector |
| $\varepsilon_{a^{N}, t}, \varepsilon_{a^{N}, t}^{*}$ | productivity shock in non-tradable sector |
| $\varepsilon_{d, t}, \varepsilon_{d, t}^{*}$ | preference shock |
| $\varepsilon_{l, t}, \varepsilon_{l, t}^{*}$ | labor supply shock |
| $\varepsilon_{g, t} \varepsilon_{g, t}^{*}$ | government expenditures shock |
| $\varepsilon_{i, t}, \varepsilon_{i, t}^{*}$ | investment efficiency shock |
| $\varepsilon_{m, t}, \varepsilon_{m, t}^{*}$ | monetary policy shock |

Table 2: Interpretation of Structural Parameters

| parameter | interpretation | domain |
| :--- | :--- | :--- |
| $n$ | relative size of the domestic economy | $\langle 0,1\rangle$ |
| $\beta, \beta^{*}$ | discount factor | $\langle 0,1\rangle$ |
| $h, h^{*}$ | habit formation in consumption | $\langle 0,1\rangle$ |
| $\sigma, \sigma^{*}$ | inv. elast. of intertemporal substitution | $\langle 0, \infty)$ |
| $\phi, \phi^{*}$ | inv. elast. of labor supply | $\langle 0, \infty)$ |
| $\phi_{H}, \phi_{F}$ | elast. of subst. among tradable goods | $\langle 1, \infty)$ |
| $\phi_{N}, \phi_{N}^{*}$ | elast. of subst. among non-tradable goods | $\langle 1, \infty)$ |
| $\phi_{W}, \phi_{W}^{*}$ | elast. of subst. among labor types | $\langle 1, \infty)$ |
| $\gamma_{c}, \gamma_{c}^{*}$ | share of tradable goods in consumption | $\langle 0,1\rangle$ |
| $\gamma_{i}, \gamma_{i}^{*}$ | share of tradable goods in investment | $\langle 0,1\rangle$ |
| $\alpha, \alpha^{*}$ | share of domestic tradable goods | $\langle 0,1\rangle$ |
| $\omega, \omega^{*}$ | distribution costs | $\langle 0, \infty)$ |
| $\tau, \tau^{*}$ | capital depreciation rate | $\langle 0,1\rangle$ |
| $S^{\prime \prime}, S^{\prime \prime}$ | adjustment costs of capital | $\langle 0, \infty)$ |
| $\eta, \eta^{*}$ | elasticity of output with respect to capital | $\langle 0,1\rangle$ |
| $\theta_{H}, \theta_{F}^{*}$ | Calvo parameter for tradable sector | $\langle 0,1\rangle$ |
| $\theta_{N}, \theta_{N}^{*}$ | Calvo parameter for non-tradable sector | $\langle 0,1\rangle$ |
| $\theta_{W}, \theta_{W}^{*}$ | Calvo parameter for households | $\langle 0,1\rangle$ |
| $\delta_{H}, \delta_{F}^{*}$ | indexation in tradable sector | $\langle 0,1\rangle$ |
| $\delta_{N}, \delta_{N}^{*}$ | indexation in non-tradable sector | $\langle 0,1\rangle$ |
| $\delta_{W}, \delta_{W}^{*}$ | indexation of households | $\langle 0,1\rangle$ |
| $\rho_{i}, \rho_{i}^{*}$ | interest rate smoothing | $\langle 0,1\rangle$ |
| $\psi_{\pi}, \psi_{\pi}^{*}$ | elasticity of interest rate to inflation | $\langle 0, \infty)$ |
| $\psi_{y}, \psi_{y}^{*}$ | elasticity of interest rate to output | $\langle 0, \infty)$ |

Table 3: Interpretation of Parameters related to Shocks

| parameter | interpretation | domain |
| :--- | :--- | :--- |
| $\rho_{a^{H},}, \rho_{a^{F}}^{*}$ | persistence of productivity shocks - tradables | $\langle 0,1\rangle$ |
| $\rho_{a^{N}}, \rho_{a^{N}}^{*}$ | persistence of productivity shocks - non-tradables | $\langle 0,1\rangle$ |
| $\rho_{d}, \rho_{d}^{*}$ | persistence of preference shocks | $\langle 0,1\rangle$ |
| $\rho_{l}, \rho_{l}^{*}$ | persistence of labor supply shocks | $\langle 0,1\rangle$ |
| $\rho_{g}, \rho_{g}^{*}$ | persistence of shocks in government expenditures | $\langle 0,1\rangle$ |
| $\rho_{i}, \rho_{i}^{*}$ | persistence of shocks in investment efficiency | $\langle 0,1\rangle$ |
| $\sigma_{a^{H},}, \sigma_{a^{F}}^{*}$ | std. dev. of productivity shocks - tradables | $\langle 0, \infty)$ |
| $\sigma_{a^{N},}, \sigma_{a^{N}}^{*}$ | std. dev. of productivity shocks - non-tradables | $\langle 0, \infty)$ |
| $\sigma_{d}, \sigma_{d}^{*}$ | std. dev. of preference shocks | $\langle 0, \infty)$ |
| $\sigma_{l}, \sigma_{l}^{*}$ | std. dev. of labor supply shocks | $\langle 0, \infty)$ |
| $\sigma_{g}, \sigma_{g}^{*}$ | std. dev. of shocks in government expenditures | $\langle 0, \infty)$ |
| $\sigma_{i}, \sigma_{i}^{*}$ | std. dev. of shocks in investment efficiency | $\langle 0, \infty)$ |
| $\sigma_{m}, \sigma_{m}^{*}$ | std. dev. of monetary shocks | $\langle 0, \infty)$ |
| $\operatorname{cor}_{a^{H}, a^{F *}}$ | correlation of productivity shocks - tradables | $\langle-1,1\rangle$ |
| $\operatorname{cor}_{a^{N}, a^{N *}}$ | correlation of productivity shocks - non-tradables | $\langle-1,1\rangle$ |
| $\operatorname{cor}_{d, d^{*}}$ | correlation of preference shocks | $\langle-1,1\rangle$ |
| $\operatorname{cor}_{l, l^{*}}$ | correlation of labor supply shocks | $\langle-1,1\rangle$ |
| $\operatorname{cor}_{g_{, a^{*}}}$ | correlation of shocks in government expenditures | $\langle-1,1\rangle$ |
| $\operatorname{cor}_{i, i^{*}}$ | correlation of shocks in investment efficiency | $\langle-1,1\rangle$ |
| $\operatorname{cor}_{m, m *}$ | correlation of shocks in investment efficiency | $\langle-1,1\rangle$ |

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## Appendix - Software

## Dynare

The model is estimated using the programme Dynare, version 4.2.4. It is a free toolbox for Matlab and it was designed for solving and estimating a wide class of economic models, especially those with rational expectations. It is a very suitable tool for handling DSGE models. Dynare offers two approaches to the estimation of the model: (i) maximal likelihood method and (ii) Random Walk Chain Metropolis-Hastings algorithm. Beside the estimation, it is also able to produce many useful statistics, such as convergence diagnostics of the MH algorithm, checkplots, impulse-response functions, variance decomposition, shock decomposition, conditional and unconditional forecasts, etc. All versions of Dynare toolbox and Dynare manuals are available on the website http://www.dynare.org/.

## Demetra

Seasonal adjustment of the HICP and its components was performed in the Demetra programme. Demetra is a free software designed for seasonal adjustment of time series, developed by researchers from Eurostat and the National Bank of Belgium. Demetra offers several specifications of TRAMO/SEATS and X12 methods for seasonal adjustment of time series. It also performs many statistical tests focusing on evaluation of the quality of seasonal adjustment. All versions of the programme Demetra as well as various manuals and guidelines are available on http://www.cros-portal.eu/page/demetra

## Appendix - Data

Real GDP, consumption and investment are measured as "Millions of euro, chain-linked volumes, reference year 2005 (at 2005 exchange rates), Seasonally adjusted and adjusted data by working days". Real GDP is given by Gross domestic product at market prices. Consumption is given by "Household and NPISH final consumption expenditures". ${ }^{9}$ Investment is given by "Gross fixed capital formation".

Prices are measured by the "HICP, Index, $2005=100$, All-items HICP". Real wage is given by the "Labour Cost Index - Wages and salaries (total), Nominal value, Business economy, Index, 2008=100, Seasonally adjusted and adjusted data by working days", which is divided by HICP in each period. Short-term interest rate is given by the "Money market interest rates, 3month rates".

Internal exchange rate defined as prices of non-tradable goods relative to prices of tradable goods is calculated from the components of HICP, where "Services (overall index excluding goods)" and "Energy" are regarded as nontradable goods, while "Non-energy industrial goods" and "Food including alcohol and tobacco" are regarded as tradable goods.

Except for the nominal interest rates, all observed variables are seasonally adjusted. If it was possible, I used official seasonal adjusted series from the web database of Eurostat. However, seasonally adjusted versions of HICP and its components are not available there, therefore I had to adjust them by myself. I used TRAMO/SEATS algorithm for seasonal adjustment of

[^6]these time series and took advantage of the Demetra software developed for seasonal adjustment of time series.

Except for the nominal interest rates, all observed variables are expressed as demeaned $100^{*} \log$ differences. Nominal interest rates are demeaned and expressed as quarterly rates per cent. The following formulas show how are transformed observed variables linked to the model variables.

|  | CZ: | EA: |
| :---: | :---: | :---: |
| consumption: | $C_{t}^{\text {obs }}=c_{t}-c_{t-1}$ | $C_{t}^{\text {obs* }}=c_{t}^{*}-c_{t-1}^{*}$ |
| investment: | $I_{t}^{o b s}=i_{t}-i_{t-1}$ | $I_{t}^{o b s *}=i_{t}^{*}-i_{t-1}^{*}$ |
| GDP: | $Y_{t}^{\text {obs }}=y_{t}-y_{t-1}$ | $Y_{t}^{o b s *}=y_{t}^{*}-y_{t-1}^{*}$ |
| prices: | $H I C P_{t}^{\text {obs }}=\pi_{t}$ | $\mathrm{HICP}_{t}^{\text {obs* }}=\pi_{t}^{*}$ |
| int. exchange rate: | $X_{t}^{\text {obs }}=x_{t}-x_{t-1}$ | $X_{t}^{\text {obs* }}=x_{t}^{*}-x_{t-1}^{*}$ |
| real wage: | $W_{t}^{\text {obs }}=w_{t}-w_{t-1}$ | $W_{t}^{\text {obs* }}=w_{t}^{*}-w_{t-1}^{*}$ |
| interest rate: | $R_{t}^{\text {obs }}=r_{t}$ | $R_{t}^{o b s *}=r_{t}^{*}$ |

Figure 1 displays transformed data which enter the estimation.

Figure 1: Data for Estimation




int. exchange rate $C Z$ - demeaned $100^{*} \log$ differences










## Appendix - Estimation

Table 4: Priors for Estimated Prameters

| parameter | prior mean | prior std. dev. | distribution |
| :---: | :---: | :---: | :---: |
| $h, h^{*}$ | 0.7 | 0.1 | Beta |
| $\sigma, \sigma^{*}$ | 1.0 | 0.7 | Gamma |
| $\phi, \phi^{*}$ | 1.0 | 0.7 | Gamma |
| $S^{\prime \prime}, S^{\prime \prime} *$ | 4.0 | 1.5 | Normal |
| $\theta_{H}, \theta_{F}^{*}$ | 0.7 | 0.05 | Beta |
| $\theta_{N}, \theta_{N}^{*}$ | 0.7 | 0.05 | Beta |
| $\theta_{W}, \theta_{W}^{*}$ | 0.7 | 0.05 | Beta |
| $\rho, \rho^{*}$ | 0.7 | 0.15 | Beta |
| $\psi_{\pi}, \psi_{\pi}^{*}$ | 1.3 | 0.15 | Gamma |
| $\psi_{y}, \psi_{y}^{*}$ | 0.25 | 0.1 | Gamma |
| $\rho_{a^{H}}, \rho_{a^{F}}^{*}$ | 0.7 | 0.1 | Beta |
| $\rho_{a^{N}}, \rho_{a^{N}}^{*}$ | 0.7 | 0.1 | Beta |
| $\rho_{d}, \rho_{d}^{*}$ | 0.7 | 0.1 | Beta |
| $\rho_{l}, \rho_{l}^{*}$ | 0.7 | 0.1 | Beta |
| $\rho_{g}, \rho_{g}^{*}$ | 0.7 | 0.1 | Beta |
| $\rho_{i}, \rho_{i}^{*}$ | 0.7 | 0.1 | Beta |
| $\sigma_{a^{H}}, \sigma_{a^{F}}^{*}$ | 2 | $\infty$ | Inv. Gamma |
| $\sigma_{a^{N},}, \sigma_{a^{N}}^{*}$ | 2 | $\infty$ | Inv. Gamma |
| $\sigma_{d}, \sigma_{d}^{*}$ | 6 | $\infty$ | Inv. Gamma |
| $\sigma_{l}, \sigma_{l}^{*}$ | 10 | $\infty$ | Inv. Gamma |
| $\sigma_{g}, \sigma_{g}^{*}$ | 3 | $\infty$ | Inv. Gamma |
| $\sigma_{i}, \sigma_{i}^{*}$ | 6 | $\infty$ | Inv. Gamma |
| $\sigma_{m}, \sigma_{m}^{*}$ | 0.3 | $\infty$ | Inv. Gamma |
| $\operatorname{cor}_{a^{H}, a^{F *}}$ | 0 | 0.4 | Normal |
| $\operatorname{cor}_{a^{N,}, a^{N *}}$ | 0 | 0.4 | Normal |
| $\operatorname{cor}_{d, d^{*}}$ | 0 | 0.4 | Normal |
| $\operatorname{cor}_{l, l^{*}}$ | 0 | 0.4 | Normal |
| $\operatorname{cor}_{g, g^{*}}$ | 0 | 0.4 | Normal |
| $\operatorname{cor}_{i, i^{*}}$ | 0 | 0.4 | Normal |
| $\operatorname{cor}_{m, m *}$ | 0 | 0.4 | Normal |

Table 5: Estimated Values

| parameter | posterior <br> mean CZ | $90 \%$ <br> int. CZ | posterior <br> mean EA | $90 \%$ credible <br> int. EA |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $h, h^{*}$ | 0.68 | 0.53 | 0.83 | 0.74 | 0.62 | 0.86 |
| $\sigma, \sigma^{*}$ | 1.53 | 0.69 | 2.39 | 2.79 | 1.40 | 4.14 |
| $\phi, \phi^{*}$ | 0.35 | 0.01 | 0.69 | 0.96 | 0.24 | 1.67 |
| $S^{\prime \prime}, S^{\prime \prime} *$ | 3.75 | 1.70 | 5.71 | 5.12 | 3.27 | 6.91 |
| $\theta_{H}, \theta_{F}^{*}$ | 0.76 | 0.70 | 0.82 | 0.73 | 0.68 | 0.79 |
| $\theta_{N}, \theta_{N}^{*}$ | 0.77 | 0.72 | 0.82 | 0.64 | 0.57 | 0.70 |
| $\theta_{W}, \theta_{W}^{*}$ | 0.71 | 0.65 | 0.78 | 0.79 | 0.74 | 0.84 |
| $\rho, \rho^{*}$ | 0.86 | 0.84 | 0.89 | 0.84 | 0.81 | 0.88 |
| $\psi_{\pi}, \psi_{\pi}^{*}$ | 1.37 | 1.19 | 1.56 | 1.42 | 1.22 | 1.62 |
| $\psi_{y}, \psi_{y}^{*}$ | 0.08 | 0.05 | 0.10 | 0.13 | 0.08 | 0.17 |
| $\rho_{a^{H},}, \rho_{a^{F}}^{*}$ | 0.91 | 0.88 | 0.96 | 0.67 | 0.53 | 0.82 |
| $\rho_{a^{N},}, \rho_{a^{N}}$ | 0.44 | 0.33 | 0.56 | 0.60 | 0.47 | 0.72 |
| $\rho_{d}, \rho_{d}^{*}$ | 0.74 | 0.62 | 0.87 | 0.72 | 0.60 | 0.84 |
| $\rho_{l}, \rho_{l}^{*}$ | 0.45 | 0.32 | 0.59 | 0.52 | 0.37 | 0.68 |
| $\rho_{g}, \rho_{g}^{*}$ | 0.80 | 0.73 | 0.88 | 0.81 | 0.74 | 0.89 |
| $\rho_{i}, \rho_{i}^{*}$ | 0.63 | 0.50 | 0.77 | 0.74 | 0.66 | 0.83 |
| $\sigma_{a^{H},}, \sigma_{a^{F}}^{*}$ | 4.64 | 2.96 | 6.32 | 2.32 | 1.42 | 3.21 |
| $\sigma_{a^{N},}, \sigma_{a^{N}}^{*}$ | 8.50 | 4.58 | 12.32 | 2.26 | 1.44 | 3.05 |
| $\sigma_{d}, \sigma_{d}^{*}$ | 4.95 | 2.12 | 7.87 | 4.46 | 2.33 | 6.58 |
| $\sigma_{l}, \sigma_{l}^{*}$ | 29.81 | 9.20 | 51.39 | 16.41 | 5.31 | 27.23 |
| $\sigma_{g}, \sigma_{g}^{*}$ | 3.29 | 2.72 | 3.86 | 1.31 | 1.11 | 1.51 |
| $\sigma_{i}, \sigma_{i}^{*}$ | 8.33 | 3.63 | 12.90 | 3.48 | 2.34 | 4.57 |
| $\sigma_{m}, \sigma_{m}^{*}$ | 0.07 | 0.06 | 0.09 | 0.09 | 0.08 | 0.11 |
| $\operatorname{cor}_{a^{H}, a^{F *}}$ | -0.01 | -0.22 | 0.19 |  |  |  |
| $\operatorname{cor}_{a^{N}, a^{N *}}$ | 0.25 | 0.05 | 0.45 |  |  |  |
| $\operatorname{cor}_{d, d^{*}}$ | 0.22 | 0.01 | 0.43 |  |  |  |
| $\operatorname{cor}_{,, l^{*}}$ | 0.10 | -0.10 | 0.31 |  |  |  |
| $\operatorname{cor}_{g, g^{*}}$ | 0.14 | -0.06 | 0.35 |  |  |  |
| $\operatorname{cor}_{i, i^{*}}$ | 0.18 | -0.02 | 0.38 |  |  |  |
| $\operatorname{cor}_{m, m^{*}}$ | 0.67 | 0.53 | 0.81 |  |  |  |

## Appendix - Measures of Posterior Differences

In this chapter I formally define five criteria for evaluation of differences between two posterior distributions. The possible values of all five criteria range from zero to one, where zero represents absolute asymmetry in the parameter values while unity represents absolute symmetry in the parameter values.

The benchmark criterion is the overlapping area of normalized posterior densities, henceforth denoted as area, formally:

$$
\text { area }=p(\theta \mid y) \cap p\left(\theta^{*} \mid y\right),
$$

where $\theta, \theta^{*}$ are two parameters of interest, $y$ denotes the data, and $p(\theta \mid y)$ is a posterior density of the parameter $\theta$.

The next four criteria use credible intervals in various specifications. The criterion $s \_2 S$ is based on two-sided probability band and measures the lowest level of significance at which two-sided probability bands do not overlap, formally:

$$
s_{-} 2 S=\min \alpha, \quad \text { w.r.t. } \quad 2 S_{\alpha}(\theta) \cap 2 S_{\alpha}\left(\theta^{*}\right)=0,
$$

where $2 S_{\alpha}(\theta)$ denotes two-sided probability interval of the parameter $\theta$ on the significance level $\alpha$.

Similarly, the criterion $s \_H P D$ denotes the lowest level of significance at which highest posterior density intervals do not overlap, formally:

$$
s_{-} H P D=\min \alpha, \quad \text { w.r.t. } \quad H P D_{\alpha}(\theta) \cap H P D_{\alpha}\left(\theta^{*}\right)=0,
$$

where $H P D_{\alpha}(\theta)$ denotes highest posterior density interval of the parameter $\theta$ on the significance level $\alpha$.

The last two criteria are based on point estimates. The criterion s_med denotes the lowest level of significance at which posterior median is out of two-sided probability bands of its counterpart, formally:

$$
\text { s_med }=\min \alpha, \quad \text { w.r.t. } \quad \operatorname{median}(p(\theta \mid y)) \notin 2 S_{\alpha}\left(\theta^{*}\right),
$$

where median $(p(\theta \mid y))$ denotes median of a posterior distribution of the parameter $\theta$.

Similarly, the criterion s_mod denotes the lowest level of significance at which posterior mode is out of HPD interval bands of its counterpart, formally:

$$
s_{-} \bmod =\min \alpha, \quad \text { w.r.t. } \quad \operatorname{mode}(p(\theta \mid y)) \notin H P D_{\alpha}\left(\theta^{*}\right),
$$

where $\operatorname{mode}(p(\theta \mid y))$ denotes mode of a posterior distribution of the parameter $\theta$.

## Appendix - MCMC Convergence Diagnostics

Figures 2-20 depict convergence diagnostics of the Metropolis-Hastings algorithm developed by Brooks and Gelman (1998). Each subplot contains a red and a blue line. Let's now explain how are these lines constructed, what they imply, and how they ideally should look like. Let's denote

- $\Psi_{i j}$ - the $i^{\text {th }}$ draw (out of $I$, in our case $I=2000000$ ) in the $j^{\text {th }}$ sequence (out of $J$, in our case $J=4$ )
- $\bar{\Psi}_{. j}$ - the mean of $j^{\text {th }}$ sequence
- $\bar{\Psi}_{\text {.. }}$ - the mean across all available data.
$\widehat{B}$ defined as

$$
\widehat{B}=\frac{1}{J-1} \sum_{j=1}^{J}\left(\bar{\Psi}_{. j}-\bar{\Psi}_{. .}\right)^{2}
$$

is an an estimate of the variance of the mean $\left(\sigma^{2} / I\right)$, and $B=\widehat{B} I$ is therefore an estimate of the variance. Other estimates of the variance are

$$
\widehat{W}=\frac{1}{J} \sum_{j=1}^{J} \frac{1}{I} \sum_{t=1}^{I}\left(\Psi_{t j}-\bar{\Psi}_{. j}\right)^{2}
$$

and

$$
W=\frac{1}{J} \sum_{j=1}^{J} \frac{1}{I-1} \sum_{t=1}^{I}\left(\Psi_{t j}-\bar{\Psi}_{. j}\right)^{2} .
$$

Ideally, we would like to achieve such a result that the variance between streams should go to zero, i.e. $\lim _{I \rightarrow \infty} \widehat{B} \rightarrow 0$, and the variance within stream should settle down, i.e. $\lim _{I \rightarrow \infty} \widehat{W} \rightarrow$ constant. If we plot $W$ (red line) and $\widehat{W}+\widehat{B}$ (blue line), then the previous proposition about ideal result for the variance between and within streams can be reformulated so that the red and blue lines should get close to each other, and that both of them should remain constant after a certain amount of draws. We can see that in general the reported plots have the required form.

Figure 2: MCMC Convergence Diagnostics 1


Figure 3: MCMC Convergence Diagnostics 2


Figure 4: MCMC Convergence Diagnostics 3


SE_mu_d (Interval)


SE_mu_d_star (Interval)



SE_mu_d (m2)


SE_mu_d_star (m2)



Figure 5: MCMC Convergence Diagnostics 4


Figure 6: MCMC Convergence Diagnostics 5


SE_mu_i_star (Interval)



SE_mu_i (m2)


SE_mu_i_star (m2)



SE_mu_i_star (m3)


Figure 7：MCMC Convergence Diagnostics 6


Figure 8：MCMC Convergence Diagnostics 7


Figure 9: MCMC Convergence Diagnostics 8




Figure 10: MCMC Convergence Diagnostics 9


Figure 11: MCMC Convergence Diagnostics 10


Figure 12: MCMC Convergence Diagnostics 11


Figure 13: MCMC Convergence Diagnostics 12


Figure 14: MCMC Convergence Diagnostics 13


Figure 15: MCMC Convergence Diagnostics 14


Figure 16: MCMC Convergence Diagnostics 15


Figure 17: MCMC Convergence Diagnostics 16


Figure 18: MCMC Convergence Diagnostics 17

## 










Figure 19: MCMC Convergence Diagnostics 18


Figure 20: MCMC Convergence Diagnostics 19


## References

[1] Brooks SP, Gelman A (1998): General Methods for Monitoring Convergence of Iterative Simulations. Journal of Computational and Graphical Statistics, 7(4):434-455.

## Appendix - Forecast

## Performance

Quality of the model performance can be evaluated by comparison of the one-step-ahead forecast of the observed variables with the actual realization of the observed variables. Figure 21 displays one-step-ahead forecasts (green line) and the observed values (blue line) for each observed variable.

It is possible to compare the one-step-ahead predictions obtained from DSGE model with the predictions obtained from VAR1 model and with the naïve predictions. ${ }^{10}$. I can calculate the measure of fit of the predictions as the Root Mean Square Error ( $R M S E$ )

$$
R M S E=\sqrt{\frac{\sum_{t=2}^{T}\left(x_{t}^{f}-x_{t}^{o b s}\right)^{2}}{T-1}}
$$

where $T$ is the number of observations, $x_{t}^{f}$ is the one-step-ahead forecast for time $t, x_{t}^{o b s}$ is the observed value in time $t$, and RMSE stand for Root Mean Square Error of the one-step-ahead forecasts.

Table 6 displays calculated $R M S E$ for the DSGE model, VAR1 model, and for the naïve forecasts. We can see that except for the domestic output the DSGE model always outperforms the naïve forecasts. We can also see that except for foreign internal exchange rate the DSGE model is always outperformed by VAR1 model.

[^7]Figure 21: Observed Variables and One-step-ahead Forecasts, green line -one-step-ahead forecast, blue line - observations


Table 6: Forecast Performance of the Observed Variables

|  | RMSE |  |  |
| :--- | :---: | :---: | :---: |
| observable | DSGE | VAR1 | NAIVE |
| output CZ | 0.94 | 0.52 | 0.78 |
| output EA | 0.52 | 0.38 | 0.53 |
| consumption CZ | 0.96 | 0.59 | 1.14 |
| consumption EA | 0.35 | 0.26 | 0.39 |
| investment CZ | 3.72 | 2.28 | 4.64 |
| investment EA | 1.22 | 0.86 | 1.30 |
| interest rate CZ | 0.07 | 0.05 | 0.08 |
| interest rate EA | 0.09 | 0.06 | 0.10 |
| inflation CZ | 0.65 | 0.49 | 0.68 |
| inflation EA | 0.35 | 0.24 | 0.36 |
| real wage CZ | 1.86 | 1.29 | 2.48 |
| real wage EA | 0.52 | 0.32 | 0.64 |
| int. exchange rate EA | 0.71 | 0.92 | 0.95 |
| int. exchange rate CZ | 1.02 | 0.50 | 1.19 |

## Appendix - Dynare Code

```
//Kolasa - demeaned log differences
//CZ and EA17, 2Q 2000 - 1Q 2014
```

clc;
close all;
var //definition of variables
c, c_star, //consumption
x,x_star, //internal terms of trade
s, //external terms of trade
i,i_star, //investment
y_H,y_F_star, //product in tradable sector
y_N,y_N_star, //product in non-tradable sector
y,y_star, //product
r,r_star, //nominal interest rate
q, //real exchange rate
pi,pi_star, //inflation
pi_T,pi_T_star, //inflation of tradable goods
k,k_star, //capital
r_K,r_K_star, //rental rate of capital
w,w_star, //real wages
q_T,q_T_star, //tobin's Q
l,l_star, //labor
mrs,mrs_star, //marginal rate of substitution
pi_H,pi_F_star, //inflation of raw tradable goods

```
mc_H,mc_F_star, //real marginal costs in tradable sector
pi_N,pi_N_star, //inflation of non-tradable goods
mc_N,mc_N_star, //real marginal costs in non-tradable sector
//Observables
y_obs, y_star_obs, //observed output
c_obs, c_star_obs, //observed consumption
i_obs, i_star_obs, //observed investment
r_obs, r_star_obs, //observed int. rate
pi_obs, pi_star_obs, //observed inflation
w_obs, w_star_obs, //observed real wage
x_obs, x_star_obs, //observed int. terms of trade
//AR processes for shocks
epsilon_g,epsilon_g_star, //shock in government expenditures
epsilon_d,epsilon_d_star, //shock in consumption preferences
epsilon_i,epsilon_i_star, //shock in investment efficiency
epsilon_a_H,epsilon_a_F_star, //productivity shock - tradables
epsilon_a_N,epsilon_a_N_star, //productivity shock - nontradables
epsilon_l,epsilon_l_star; //labor supply shock
varexo //shocks - innovations
epsilon_m,epsilon_m_star, //monetary policy shock
mu_g,mu_g_star, //innovation in government expenditures
mu_d,mu_d_star, //innovation in consumption preferences
mu_i,mu_i_star, //innovation in investment efficiency
mu_a_H,mu_a_F_star, //innovation in productivity - tradables
mu_a_N,mu_a_N_star, //innovation in productivity - nontradables
mu_l,mu_l_star; //innovation in labor supply
parameters
beta, beta_star, //discount factor
gamma_c,gamma_c_star, //share of tradable goods in consumption
```

```
gamma_i,gamma_i_star, //share of tradable goods in investment
alpha,alpha_star, //share of domestic tradable goods
omega,omega_star, //share of distribution costs
n,
h,h_star, //habit formation
sigma,sigma_star, //inv. elasticity of intertemporal subs.
tau,tau_star, //depreciation rate
S_prime,S_prime_star, //adjustment costs
eta,eta_star, //elasticity of output wrt capital
theta_W,theta_W_star, //Calvo parameters for households
delta_W,delta_W_star, //wage indexation
phi_W,phi_W_star, //elasticity of subst. among labor types
phi,phi_star, //inv. elasticity of labor supply
delta_H,delta_F_star, //indexation of tradables
theta_H,theta_F_star, //Calvo parameters for tradables
delta_N,delta_N_star, //indexation of non-tradables
theta_N,theta_N_star, //Calvo parameters for non-tradables
rho,rho_star, //interest rate smoothing
psi_y,psi_y_star, //elasticity of interest rate to output
psi_pi,psi_pi_star, //elast. of interest rate to inflation
//persistence of shocks
rho_a_H, rho_a_F_star, //per. of productivity shock - tradables
rho_a_N, rho_a_N_star, //per. of productivity shock - nontradables
rho_d, rho_d_star, //per. of consumption preference shock
rho_l, rho_l_star, //per. of labor supply shock
rho_g, rho_g_star, //per. of shock in government expenditures
rho_i, rho_i_star, //per. of shock in investment efficiency
//shares of variables in steady-state
ss_C_Y, //share of dom. consumption on dom. output
ss_C_star_Y_star, //share of for. consumption on for. output
ss_I_Y, //share of dom. investment on dom. output
```

ss_I_star_Y_star, //share of for. investment on for. output ss_G_Y, //share of dom. gov. exp. on dom. output ss_G_star_Y_star, //share of for. gov. exp. on for. output ss_Y_N_Y, //share of dom. nontradables on dom. output ss_Y_N_star_Y_star,//share of for. nontradables on for. output ss_Y_H_Y, //share of dom. tradables on dom. output ss_Y_F_star_Y_star;//share of for. tradables on for. output
//calibration of the model
//calibrated parameters
beta $=0.9975 ;$ beta_star $=0.9975$;
gamma_c $=0.5384 ;$ gamma_c_star $=0.4953$;
gamma_i $=0.5006$; gamma_i_star $=0.4257$;
alpha $=0.28 ;$ alpha_star $=0.989$;
//omega = 1; omega_star = 1;
omega $=0 ;$ omega_star $=0$;
$\mathrm{n}=0.0138$;
tau $=0.025$; tau_star $=0.025$;
phi_W = 3; phi_W_star = 3;
eta $=0.4160 ;$ eta_star $=0.3618$;
delta_H = 0; delta_F_star = 0;
delta_N = 0; delta_N_star = 0;
delta_W = 0; delta_W_star = 0;
//calibrated shares of variables in steady-state
ss_C_Y = 0.4929;
ss_C_star_Y_star $=0.5681$;
ss_I_Y = 0.2590;
ss_I_star_Y_star = 0.1999;
ss_G_Y = 1 - ss_C_Y - ss_I_Y;
ss_G_star_Y_star = 1 - ss_C_star_Y_star - ss_I_star_Y_star;

```
ss_Y_N_Y = ss_C_Y * (1 + omega - gamma_c)/(1 + omega) +
    ss_G_Y + ss_I_Y * (1 - gamma_i);
ss_Y_N_star_Y_star = ss_C_star_Y_star * (1 + omega_star -
    gamma_c_star)/(1 + omega_star) +
    ss_G_star_Y_star + ss_I_star_Y_star *
    (1 - gamma_i_star);
```

ss_Y_H_Y = 1 - ss_Y_N_Y;
ss_Y_F_star_Y_star = 1 - ss_Y_N_star_Y_star;
model(linear);
// 1.-6. Market Clearing Conditions
y_H = ss_C_Y/ss_Y_H_Y*gamma_c*alpha/(1+omega) $*\left(c+\left(1-g a m m a \_c\right) * x+\right.$
(1-alpha) *s) +ss_C_star_Y_star/ss_Y_H_Y*(1-n)/n*gamma_c_star
*(1-alpha_star)/(1+omega_star)*(c_star+(1-gamma_c_star)*
x_star+alpha_star*s) + ss_I_Y/ss_Y_H_Y * gamma_i*alpha *
(i + (1-gamma_i) $*(1+o m e g a) * x+(1-a l p h a) * s)+s s_{\_} I_{-} s t a r_{-} Y \_s t a r /$
ss_Y_H_Y*(1-n)/n*gamma_i_star*(1-alpha_star)*(i_star +
(1-gamma_i_star) $*(1+$ omega_star $) * x_{\text {_ }}$ star + alpha_star*s) ;
y_F_star = ss_C_star_Y_star/ss_Y_F_star_Y_star*gamma_c_star*
alpha_star/(1+omega_star)*(c_star+(1-gamma_c_star)*x_star -

gamma_c*(1-alpha)/(1+omega)*(c+(1-gamma_c)*x - alpha*s) +
ss_I_star_Y_star/ss_Y_F_star_Y_star*gamma_i_star*alpha_star
*(i_star + (1-gamma_i_star)*(1+omega_star) *x_star -
$\left.\left(1-a l p h a_{-} s t a r\right) * s\right)+s s_{-} I_{-} Y / s s_{-} Y \_F_{-} s t a r_{-} Y \_s t a r * n /(1-n) *$
gamma_i*(1-alpha) * (i + (1-gamma_i)*(1+omega)*x -alpha*s);
y_N $=$ ss_C_Y/ss_Y_N_Y*((1-gamma_c)*(c-gamma_c*x)+gamma_c*omega/
$(1+o m e g a) *(c+(1-$ gamma_c $) * x))+s s_{-} I_{-} Y / s s_{-} Y \_N \_Y *\left(1-g a m m a_{-} i\right) *$
(i-gamma_i*(1+omega)*x) + ss_G_Y/ss_Y_N_Y * epsilon_g;

```
y_N_star=ss_C_star_Y_star/ss_Y_N_star_Y_star*((1-gamma_c_star)
    *(c_star-gamma_c_star*x_star)+gamma_c_star*omega_star/
    (1+omega_star)*(c_star+(1-gamma_c_star)*x_star))+
    ss_I_star_Y_star/ss_Y_N_star_Y_star*(1-gamma_i_star)*
    (i_star-gamma_i_star*(1+omega_star)*x_star)+
    ss_G_star_Y_star/ss_Y_N_star_Y_star * epsilon_g_star;
y = ss_Y_H_Y * y_H + ss_Y_N_Y * y_N;
y_star = ss_Y_F_star_Y_star*y_F_star+
    ss_Y_N_star_Y_star*y_N_star;
// 7.-8. Euler Equations
c-h*c(-1) = c(+1) - h*c - (1-h)/sigma * (r - pi(+1)) +
    (1-h)/sigma * (epsilon_d - epsilon_d(+1));
c_star-h_star*c_star(-1)=c_star(+1)-h_star*c_star-(1-h_star)/
    sigma_star*(r_star - pi_star(+1)) + (1-h_star)/sigma_star *
    (epsilon_d_star - epsilon_d_star(+1));
// 9. International Risk Sharing Condition
q = epsilon_d_star - epsilon_d - sigma_star/(1-h_star)*
    (c_star - h_star * c_star(-1)) + sigma/(1-h)*(c - h*c(-1));
// 10.-11. Law for Capital Acumulation
k = (1-tau)*k(-1) + tau*(i + epsilon_i);
k_star=(1-tau_star)*k_star(-1)+tau_star*(i_star +
    epsilon_i_star);
// 12.-13. Real Marginal Costs
r_K = w + l - k(-1);
```

```
r_K_star = w_star + l_star - k_star(-1);
// 14.-15. Investment Demand
i - i(-1) = beta *(i(+1)-i) + 1/S_prime *(q_T + epsilon_i) -
    (gamma_i*(1+omega)-gamma_c)/S_prime * x;
i_star - i_star(-1) = beta_star*(i_star(+1)-i_star)+
    1/S_prime_star*(q_T_star+epsilon_i_star)-
    (gamma_i_star*(1+omega_star)-gamma_c_star)/
    S_prime_star * x_star;
// 16.-17. Price of the Capital
q_T = beta*(1-tau)*q_T(+1)-(r - pi(+1)) +
    (1-beta*(1-tau))*r_K(+1);
q_T_star = beta_star *(1-tau_star) * q_T_star(+1) - (r_star -
    pi_star(+1)) + (1-beta_star*(1-tau_star))* r_K_star(+1);
// 18.-19. Labor Input
l = eta * (r_K - w) + ss_Y_H_Y * (y_H - epsilon_a_H) +
    ss_Y_N_Y * (y_N - epsilon_a_N);
l_star = eta_star * (r_K_star - w_star) + ss_Y_F_star_Y_star *
    (y_F_star - epsilon_a_F_star) + ss_Y_N_star_Y_star *
    (y_N_star - epsilon_a_N_star);
//20.-21. Real Wage Rate
w-w(-1)=(1-theta_W)*(1-beta*theta_W)/(theta_W*(1+phi_W*phi))*
    (mrs-w)+ beta*(w(+1)-w)+beta*(pi(+1)-delta_W*pi) -
    (pi - delta_W*pi(-1));
w_star-w_star(-1)=(1-theta_W_star)*(1-beta_star*theta_W_star)/
```

```
(theta_W_star*(1+phi_W_star*phi_star))*(mrs_star-w_star)+
beta_star*(w_star(+1)-w_star)+beta_star*(pi_star(+1)-
delta_W_star*pi_star)-(pi_star-delta_W_star*pi_star(-1));
//22.-23. Marginal Rate of Substitution
mrs = epsilon_l + phi*l - epsilon_d + sigma/(1-h)*(c-h*c(-1));
mrs_star = epsilon_l_star + phi_star*l_star - epsilon_d_star +
    sigma_star/(1-h_star)*(c_star - h_star*c_star(-1));
//24.-25. PC for Raw Tradables
pi_H - delta_H*pi_H(-1) = (1-theta_H)*(1-beta*theta_H)/theta_H*
    mc_H+beta *(pi_H(+1) - delta_H*pi_H);
pi_F_star - delta_F_star*pi_F_star(-1) = (1-theta_F_star)*
    (1-beta_star*theta_F_star)/theta_F_star * mc_F_star +
    beta_star *(pi_F_star(+1) - delta_F_star*pi_F_star);
//26.-27. PC for Non-tradables
pi_N - delta_N*pi_N(-1) = (1-theta_N)*(1-beta*theta_N)/theta_N*
    mc_N + beta * (pi_N(+1)- delta_N * pi_N);
pi_N_star-delta_N_star*pi_N_star(-1)=(1-theta_N_star)*
    (1-beta_star*theta_N_star)/theta_N_star * mc_N_star +
    beta_star * (pi_N_star(+1)- delta_N_star * pi_N_star);
//28.-29. Real Marginal Costs in Tradable Sector
mc_H = (1-eta)*w + eta * r_K - epsilon_a_H + (1-alpha)*s +
    (1+omega-gamma_c)*x;
mc_F_star=(1-eta_star)*w_star+eta_star*r_K_star-
    epsilon_a_F_star-(1-alpha_star)*s+
    (1+omega_star-gamma_c_star)*x_star;
```

```
//30.-31. Real Marginal Costs in Non-tradable Sector
mc_N = (1-eta)*w + eta * r_K - epsilon_a_N - gamma_c*x;
mc_N_star = (1-eta_star)*w_star + eta_star * r_K_star -
    epsilon_a_N_star - gamma_c_star*x_star;
//32.-33. Internal Exchange Rate
x - x(-1) = pi_N - pi_T;
x_star - x_star(-1) = pi_N_star - pi_T_star;
//34.-35. Inflation of Tradables
pi_T = 1/(1+omega)*(pi_H+(1-alpha)*(s-s(-1))+omega*pi_N);
pi_T_star = 1/(1+omega_star)*(pi_F_star-(1-alpha_star)*
    (s - s(-1))+omega_star * pi_N_star);
//36.-37. CPI Inflation
pi = gamma_c * pi_T + (1 - gamma_c) * pi_N;
pi_star = gamma_c_star*pi_T_star + (1-gamma_c_star)*pi_N_star;
//38. Real Exchange Rate
q = (alpha + alpha_star-1)*s + (1 + omega_star - gamma_c_star)*
    x_star - (1 + omega - gamma_c) * x;
//39.-40. Monetary Policy Rule
r = rho*r(-1)+(1-rho)*(psi_y*y(+1)+psi_pi*pi(+1))+epsilon_m;
r_star=rho_star*r_star(-1)+(1-rho_star)*(psi_y_star*y_star(+1)+
    psi_pi_star * pi_star(+1)) + epsilon_m_star;
```

```
//41.-42. Productivity Shock in Tradable Sector
epsilon_a_H = rho_a_H * epsilon_a_H(-1) + mu_a_H;
epsilon_a_F_star=rho_a_F_star*epsilon_a_F_star(-1)+mu_a_F_star;
//43.-44. Productivity Shock in Non-tradable Sector
epsilon_a_N = rho_a_N * epsilon_a_N(-1) + mu_a_N;
epsilon_a_N_star=rho_a_N_star*epsilon_a_N_star(-1)+mu_a_N_star;
//45.-46. Preference Shock
epsilon_d = rho_d * epsilon_d(-1) + mu_d;
epsilon_d_star=rho_d_star*epsilon_d_star(-1)+mu_d_star;
//47.-48. Labor Supply Shock
epsilon_l = rho_l * epsilon_l(-1) + mu_l;
epsilon_l_star=rho_l_star*epsilon_l_star(-1)+mu_l_star;
//49.-50. Shock in Government Expenditures
epsilon_g = rho_g * epsilon_g(-1) + mu_g;
epsilon_g_star=rho_g_star*epsilon_g_star(-1)+mu_g_star;
//51.-52. Investment Efficiency Shock
epsilon_i = rho_i * epsilon_i(-1) + mu_i;
epsilon_i_star=rho_i_star*epsilon_i_star(-1)+mu_i_star;
//Linking Observables to Model Variables
y_obs = y - y(-1);
y_star_obs = y_star - y_star(-1);
```

```
pi_obs = pi;
pi_star_obs = pi_star;
x_obs = x - x(-1);
x_star_obs = x_star - x_star(-1) ;
r_obs = r;
r_star_obs = r_star;
W_obs = w - w(-1);
w_star_obs = w_star - w_star(-1) ;
c_obs = c - c(-1);
c_star_obs = c_star - c_star(-1);
i_obs = i - i(-1);
i_star_obs = i_star - i_star(-1);
end;
```

//Model is in the gap form, therefore steady state for all
//variables is 0.
initval;
$c=0 ; c_{-}$star $=0$;
$\mathrm{x}=0$; x _star $=0$;
$\mathrm{s}=0 ;$
i $=0$; i_star $=0$;
y_H = 0; y_F_star = 0;
y_N = 0; y_N_star = 0;
y = 0; y_star = 0;
$r=0 ; r_{\text {_ star }}=0$;
$\mathrm{q}=0 ;$
pi $=0 ;$ pi_star $=0$;

```
k = 0; k_star = 0;
r_K = 0; r_K_star = 0;
W = 0; w_star = 0;
q_T = 0; q_T_star = 0;
l = 0; l_star = 0;
mrs = 0; mrs_star = 0;
pi_H = 0; pi_F_star = 0;
mc_H = 0; mc_F_star = 0;
pi_N = 0; pi_N_star = 0;
mc_N = 0; mc_N_star = 0;
epsilon_g = 0; epsilon_g_star = 0;
epsilon_d = 0; epsilon_d_star = 0;
epsilon_i = 0; epsilon_i_star = 0;
epsilon_a_H = 0; epsilon_a_F_star = 0;
epsilon_a_N = 0; epsilon_a_N_star = 0;
epsilon_l = 0; epsilon_l_star = 0;
end;
```

//Estimated Parameters and their Priors estimated_params;
h, $\quad 0.7,1 \mathrm{E}-5,0.9999$, beta_pdf,0.7,0.1;
h_star, $0.7,1 \mathrm{E}-5,0.9999$, beta_pdf,0.7,0.1;
sigma, $\quad 1.0,1 \mathrm{E}-5,10, \quad$ gamma_pdf,1,0.7;
sigma_star, $\quad 1.0,1 \mathrm{E}-5,10$, gamma_pdf, 1, 0.7;
phi, $1.0,1 \mathrm{E}-5,10$, gamma_pdf,1,0.7;
phi_star, $1.0,1 \mathrm{E}-5,10$, gamma_pdf,1,0.7;
S_prime, $\quad 4.0,1 \mathrm{E}-5,10$, normal_pdf, 4,1.5;
S_prime_star, $4.0,1 \mathrm{E}-5,10$ normal_pdf, 4,1.5;
//delta_H, $0.5,1 \mathrm{E}-5,0.9999$, beta_pdf, 0.5,0.2;
//delta_F_star, 0.5, 1E-5, 0.9999, beta_pdf,0.5,0.2;
//delta_N, 0.5, 1E-5, 0.9999, beta_pdf,0.5,0.2;
//delta_N_star, 0.5, 1E-5, 0.9999, beta_pdf,0.5,0.2;
//delta_W, $\quad 0.5,1 \mathrm{E}-5,0.9999$, beta_pdf, 0.5,0.2;
//delta_W_star, 0.5, 1E-5, 0.9999, beta_pdf,0.5,0.2;
theta_H, $\quad 0.7,1 \mathrm{E}-5,0.9999$, beta_pdf,0.7,0.05;
theta_F_star, 0.7, 1E-5, 0.9999, beta_pdf,0.7,0.05;
theta_N, $\quad 0.7,1 \mathrm{E}-5,0.9999$, beta_pdf,0.7,0.05;
theta_N_star, $0.7,1 \mathrm{E}-5,0.9999$, beta_pdf, 0.7,0.05;
theta_W, $0.7,1 \mathrm{E}-5,0.9999$, beta_pdf,0.7,0.05;
theta_W_star, 0.7, 1E-5, 0.9999, beta_pdf,0.7,0.05;
rho, $\quad 0.7,1 \mathrm{E}-5,0.9999$, beta_pdf,0.7,0.15;
rho_star, $0.7,1 \mathrm{E}-5,0.9999$, beta_pdf,0.7,0.15;
psi_y, $\quad 0.25,1 \mathrm{E}-5,2$, gamma_pdf,0.25,0.1;
psi_y_star, $\quad 0.25,1 \mathrm{E}-5,2$, gamma_pdf, $0.25,0.1$;
psi_pi, 1.3, 1E-5, 5, gamma_pdf,1.3,0.15;
psi_pi_star, 1.3, 1E-5, 5, gamma_pdf,1.3,0.15;
rho_a_H, $\quad 0.7,1 \mathrm{E}-5,0.9999$, beta_pdf,0.7,0.1;
rho_a_F_star, 0.7, 1E-5, 0.9999, beta_pdf,0.7,0.1;
rho_a_N, 0.7, 1E-5, 0.9999, beta_pdf,0.7,0.1;
rho_a_N_star, $0.7,1 \mathrm{E}-5,0.9999$, beta_pdf,0.7,0.1;
rho_d, $\quad 0.7,1 \mathrm{E}-5,0.9999$, beta_pdf,0.7,0.1;
rho_d_star, 0.7, 1E-5, 0.9999, beta_pdf,0.7,0.1;
rho_l, $\quad 0.7,1 \mathrm{E}-5,0.9999$, beta_pdf,0.7,0.1;
rho_l_star, $0.7,1 \mathrm{E}-5,0.9999$, beta_pdf,0.7,0.1;
rho_g, $\quad 0.7,1 \mathrm{E}-5,0.9999$, beta_pdf,0.7,0.1;
rho_g_star, $0.7,1 \mathrm{E}-5,0.9999$, beta_pdf,0.7,0.1;
rho_i, $\quad 0.7,1 \mathrm{E}-5,0.9999$, beta_pdf,0.7,0.1;
rho_i_star, $0.7,1 \mathrm{E}-5,0.9999$, beta_pdf,0.7,0.1;
stderr mu_a_H
stderr mu_a_F_star,
stderr mu_a_N,
stderr mu_a_N_star,

```
inv_gamma_pdf,2,inf;
inv_gamma_pdf,2,inf;
inv_gamma_pdf,2,inf;
inv_gamma_pdf,2,inf;
```

stderr mu_d,
stderr mu_d_star, stderr mu_l,
stderr mu_l_star, stderr mu_g,
stderr mu_g_star, stderr mu_i,
stderr mu_i_star, stderr epsilon_m, stderr epsilon_m_star,
inv_gamma_pdf,6,inf;
inv_gamma_pdf,6,inf;
inv_gamma_pdf, 10,inf;
inv_gamma_pdf, 10,inf;
inv_gamma_pdf, $3, i n f ;$
inv_gamma_pdf, $3, i n f ;$
inv_gamma_pdf,6,inf;
inv_gamma_pdf, 6,inf;
inv_gamma_pdf,0.3,inf;
inv_gamma_pdf,0.3,inf;
corr mu_a_H, mu_a_F_star, corr mu_a_N, mu_a_N_star, corr mu_d, mu_d_star,
corr mu_l, mu_l_star, corr mu_g, mu_g_star, corr mu_i, mu_i_star,
$0,-1,1$, normal_pdf,0,0.4;
0, -1, 1, normal_pdf,0,0.4;
0, -1, 1, normal_pdf,0,0.4;
0, -1, 1, normal_pdf,0,0.4;
$0,-1,1$, normal_pdf, 0,0.4;
0, -1, 1, normal_pdf,0,0.4;
corr epsilon_m, epsilon_m_star, 0, -1, 1, normal_pdf,0,0.4;
end;
varobs y_obs, y_star_obs, c_obs, c_star_obs, i_obs, i_star_obs, r_obs, r_star_obs, pi_obs, pi_star_obs, w_obs, w_star_obs, x_obs, x_star_obs;
estimation(datafile=data_cz_eu_2Q2000_1Q2014, mode_compute=4, mh_replic=2000000, mh_nblocks=2, mh_drop=0.75, mh_jscale=0.25, mode_check, smoother, nograph, bayesian_irf, filtered_vars, forecast=20);


[^0]:    ${ }^{1}$ Here I depart from the original specification of the model. Following Herber (2010) and Herber and Němec (2012) I am using a modified version of the model. Besides correcting several obvious typos, the modification is based on a different definition of the parameter $\alpha^{*}$. In the original specification this parameter would be defined as a share of the Czech tradable goods in the overall index of the tradable goods in the Euro Area, while in the modified specification this parameter is defined as a share of the tradable goods produced in the Euro Area in the overall index of tradable goods in the Euro Area. It implies that the parameter $\alpha^{*}$ in the original specification is equal to $1-\alpha^{*}$ in the modified specification, which results in different structural forms of several equations. However, after substituting the actual calibrated values of the parameter $\alpha^{*}$ into the equations and correcting two obvious typos, we can see that the equations in both specifications are the

[^1]:    ${ }^{3}$ Condition $C_{R, t}=C_{T, t}$ is implied by (4), where the other potential condition $C_{T, t}=$ $\omega^{-1} Y_{D, t}$ is not stable, because it would suggest that there are not enough distribution services to satisfy demand of households for tradable goods.

[^2]:    ${ }^{4}$ It is not important to know the exact form of this function, because a log-linearised form of the model contains only a second derivative of the function $S^{\prime \prime}$ (regarded as unknown parameter to be estimated).

[^3]:    ${ }^{5}$ Following Burstein et al. (2003).

[^4]:    ${ }^{6}$ The assumption of law of one price for tradable goods implies $S_{t}^{*}=S_{t}^{-1}$.
    ${ }^{7}$ Here I depart from the original specification of the model. I changed the specification of the interest rate rules. In the original model, interest rates depend on current inflation and output, while in my specification interest rates depend on expected inflation and expected output. This, in my view, better corresponds with the actual behavior of central banks in both economies.

[^5]:    ${ }^{8}$ IID - identically and independently distributed

[^6]:    ${ }^{9}$ The data series labeled as "Final consumption expenditures of households" are not available for the Euro Area 17, which is why I use "Household and NPISH final consumption expenditures". However, values of "Final consumption expenditure of NPISH" are so negligible that it does not make make any significant difference.

[^7]:    ${ }^{10}$ Naïve prediction means that the prediction is equal to the last observed value

