# Reduction in CPI Commodity Substitution Bias by Using the Modified Lloyd–Moulton Index

**Jacek Białek** [1] | *University of Lodz, Lodz, Poland*

***Abstract***

The Consumer Price Index (CPI) is used as a basic measure of inflation. In practice, the Laspeyres price index is used to measure the CPI, although this formula does not take into account changes in the structure of consumption. The difference between the Laspeyres index and the superlative index should approximate the value of the commodity substitution bias. The Lloyd–Moulton price index does not make use of current-period expenditure data and, as it is commonly known, it allows to approximate superlative indices, in particular the Fisher price index (Von der Lippe, 2007). This is a very important property for the inflation measurement and the Consumer Price Index bias calculations. In this paper we verify the utility of the Lloyd–Moulton price index as the Fisher price index approximation. We propose a simple modification of that index and verify this modification for the real data set.

| ***Keywords*** | ***JEL code*** |
|---|---|
| *Price indices, Fisher index, Lloyd–Moulton index, Consumer Price Index* | *C43* |

## INTRODUCTION

The Consumer Price Index (CPI) is used as a basic measure of inflation. In practice, the Laspeyres price index is used to measure the CPI, although this formula does not take into account changes in the structure of consumption. The difference between the Laspeyres index and the superlative index should approximate the value of the commodity substitution bias. The Lloyd–Moulton price index does not make use of current-period expenditure data and, as it is commonly known, it allows to approximate superlative indices, in particular the Fisher price index (Von der Lippe, 2007). This is a very important property for the inflation measurement and the Consumer Price Index bias calculations. In this paper we verify the utility of the Lloyd–Moulton price index as the Fisher price index approximation. We propose a simple modification of that index and verify this modification for the real data set.

[1] University of Lodz, Department of Statistical Methods, 3/5 POW Street, 90-255 Lodz, Poland. E-mail: jbialek@uni.lodz.pl.

## INTRODUCTION

The Consumer Price Index (CPI) is used as a basic measure of inflation. The index approximates changes in the costs of household consumption assuming the constant utility (COLI, Cost of Living Index). In practice, the Laspeyres price index is used to measure the CPI (see White, 1999; Clements and Izan, 1987) although this formula does not take into account changes in the structure of consumption. It means that the Laspeyres index can be biased due to the commodity substitution because relative prices change over time and also consumers' preferences. Many economists consider the *superlative indices* (see Von der Lippe, 2007) to be the best approximation of COLI. The difference between the Laspeyres index and the superlative index should approximate the value of the commodity substitution bias. However there is a way to reduce that bias: using a Constant Elasticity of Substitution (CES) framework, a superlative price index can be approximated once we have estimated the elasticity of substitution. The Lloyd–Moulton price index (see Lloyd, 1975; Moutlon, 1996; Shapiro and Wilcox, 1997) does not make use of current-period expenditure data, so it is even possible to approximate a superlative index (like the *ideal* Fisher index) in real time and extrapolate the time series. In this paper we verify the applicability of the Lloyd–Moulton price index as the Fisher price index approximation. We propose a modification of the Lloyd–Moulton price index which facilities numerical calculations. We also examine the modified Lloyd–Moulton price index on the real data set for Poland.

## 1 SUPERLATIVE PRICE INDICES IN CPI BIAS MEASUREMENT

An interesting discussion on the theory of the COLI can be found in the following papers: Diewert (1993), Jorgenson and Slesnick (1983), Pollak (1989). Let $E(P,\bar{u}) = \min_Q\{P^T Q | U(Q) \geq \bar{u}$ be the expenditure function of a representative consumer which is dual to the utility function $U(Q)$. In other words it is the minimum expenditure necessary to achieve a reference level of utility $\bar{u}$ at vector of prices $P$. Then the Konüs cost of the living price index is defined as (see Von der Lippe, 2007):

$$P_K = \frac{E(P^t, \bar{u})}{E(P^s, \bar{u})}, \tag{1}$$

where $t$ denotes the current period, $s$ denotes the base period, and in general, the vector of $N$ considered prices at any moment $\tau$ is given by $P^\tau = [p_1^\tau, p_2^\tau, ..., p_N^\tau]'$. $P_K$ is the true cost of living index in which the commodity $Q$ changes together with the vector of prices facing the consumer changes. The CPI, in contrast, measures the change in the cost of purchasing a fixed basket of goods over the time interval, i.e. $Q^s = [q_1^s, q_2^s, ..., q_N^s]' = Q^t$. The CPI is a Laspeyres-type index defined by:

$$P_{La} = \frac{\sum_{i=1}^{N} q_i^s p_i^t}{\sum_{i=1}^{N} q_i^s p_i^s}, \tag{2}$$

so we assume here the constant consumption vector on the base period level. It can be shown (see Diewert, 1993) that under the assumption that the consumption vector $Q^t$ solves the period $t$ expenditure minimization problem, then

$$P_K = \frac{E(P^t, U(Q^s))}{E(P^s, U(Q^s))} \leq P_{La}, \tag{3}$$

and thus $P_{La} - P_K$ is the extent of the commodity substitution bias, where $P_K$ plays the role of the reference benchmark. In the so called economic price index approach the superlative price indices are treated as the best approximation of the $P_K$ index (see White, 1999).

We define a price index $P$ to be *exact* for a linearly homogeneous aggregator function $f$ (here a utility function), which has a dual unit cost function $c(\cdot)$ and it holds

$$P = \frac{c(P^t)}{c(P^s)}. \tag{4}$$

In other words, an *exact* price index is the one whose functional form is *exactly* equal to the ratio of cost functions for some underlying functional form representing preferences. The Fisher price index $P_F$ is exact for the linearly homogeneous quadratic aggregator function $f(x) = (x^T Ax)^{0.5}$, where $A$ is a symmetric and positive matrix of constants (Diewert, 1976). The quadratic function above is an example of a *flexible functional form* (i.e. a function that provides a second order approximation to an arbitrary twice continuously differentiable function). Since $P_F$ is exact for a flexible functional form, it is said to be a *superlative* index number. In Afriat (1972), Pollak (1971) and Samuelson-Swamy (1974) we can find other examples of exact index numbers as well as superlative index numbers. The Fisher price index is defined as a geometric mean of the Laspeyres and Paasche indices ($P_{Pa}$), where:

$$P_{Pa} = \frac{\sum_{i=1}^{N} q_i^t p_i^t}{\sum_{i=1}^{N} q_i^t p_i^s}. \tag{5}$$

Let us notice that the Fisher price index makes use of current-period expenditure data and its usefulness in CPI measurement is limited.

## 2 LLOYD–MOULTON PRICE INDEX

The quadratic mean of order $r$ price index was defined by Diewert (1976) as follows ($r \neq 0$):

$$P_{QM}(r) = \{[\sum_{i=1}^{N} w_i^s (\frac{p_i^t}{p_i^s})^{\frac{r}{2}}]^{\frac{2}{r}} [\sum_{i=1}^{N} w_i^t (\frac{p_i^t}{p_i^s})^{-\frac{r}{2}}]^{-\frac{2}{r}}\}^{\frac{1}{2}}, \tag{6}$$

where $w_i^s$ and $w_i^t$ denote the expenditure share of commodity $i$ in the base period $s$ and the current period $t$, respectively. It is a superlative price index. By setting $r = 2(1 - \sigma)$ expression (6) becomes:

$$P_{QM}(2(1-\sigma)) = \sqrt{P_{LM}(\sigma) \cdot P_{CW}(\sigma)}, \tag{7}$$

where $P_{LM}(\sigma)$ denotes the Lloyd-Moutlon price index defined as:

$$P_{LM}(\sigma) = \{[\sum_{i=1}^{N} w_i^s (\frac{p_i^t}{p_i^s})^{1-\sigma}]^{\frac{1}{1-\sigma}}, \tag{8}$$

and $P_{CW}(\sigma)$ denotes its "current weight (CW) counterpart" (see De Haan et al., 2000), i.e.:

$$P_{CW}(\sigma) = \{[\sum_{i=1}^{N} w_i^t (\frac{p_i^t}{p_i^s})^{-(1-\sigma)}]^{\frac{-1}{1-\sigma}}. \tag{9}$$

These formulas $P_{LM}(\sigma)$, $P_{CW}(\sigma)$ and $P_{QM}(\sigma)$ do not exists for, $\sigma = 1$, but for $\sigma \to 1$ these indices tend to Geometric Laspeyres, Geometric Paasche and Törnqvist price indices, respectively (De Haan et al., 2000). Since the price index $P_{LM}(\sigma)$ monotonically decreases and $P_{CW}(\sigma)$ monotonically increases as $\sigma$ increases (see Biggeri, Ferrari, 2010) we conclude that there exists a value $\sigma_0$ which satisfies:

$$P_{LM}(\sigma_0) = P_{CW}(\sigma_0) = P_{QM}(2(1-\sigma_0)). \tag{10}$$

Thus we get that for $\sigma = \sigma_0$ the Lloyd–Moulton index becomes superlative. The value $\sigma_0$ should be obtained numerically. Since the superlative Fisher price index satisfies the most important tests from the axiomatic price index theory (Fisher, 1922; Balk, 1995) the approximation $P_{LM}(\sigma_0) \approx P_F$ is desirable. In other words we are going to find such value $\sigma_0$ that minimizes the expression $|P_{LM}(\sigma) - P_F|$.

**Remark**

The numerical methods need some starting assumptions about the interval of possible values of the given parameter (or parameters). If the interval is wide the methods could be computationally inefficient. It is not convenient if we must establish the interval for numerical calculations each time we change the starting set of random variables. It would be ideal to have a fixed, narrow interval including the value of the parameter $\sigma_0$. In our opinion we can not recommend one general value of the parameter $\sigma$ or even the interval of its possible values (see Example) although some papers suggest it is a number between 0.7 and 1 (Shapiro, Wilcox, 1997; Biggeri, Ferrari, 2010). The value of the elasticity ($\sigma$) depends on the aggregation level: at a detailed levels of product aggregation the substitution elasticity could well be above 1 (see e.g. Balk, 2000). In this paper we propose some simple modification of the Lloyd–Moulton price index which makes that the estimated value of the parameter ($\phi$) is always in the interval (0, 1) and thus $\phi$ should not be treated as the elasticity of substitution.

Let us replace $1 - \sigma$ by $\frac{1}{ctg(\frac{\pi\phi}{2})}$ in expression (8) for the Lloyd-Moulton index.

We obtain:

$$P_{LM}(\phi) = \{[\sum_{i=1}^{N} w_i^s (\frac{p_i^t}{p_i^s})^{\frac{1}{ctg(\frac{\pi\phi}{2})}}]^{ctg(\frac{\pi\phi}{2})}. \tag{11}$$

Since the function $ctg(\pi\phi/2)$ is decreasing with respect to $\phi \in (0,1)$, $ctg(0^+) = \infty$ and $ctg(\frac{\pi}{2}) = 0$ we can consider only $\phi \in (0,1)$. In fact we have:

$$\lim_{\phi \to 1^-} P_{LM}(\phi) = \lim_{x \to 0^+} (\sum_{i=1}^{N} w_i^s (\frac{p_i^t}{p_i^s})^{\frac{1}{x}})^x = \lim_{x \to 0^+} \exp(x \ln(\sum_{i=1}^{N} w_i^s (\frac{p_i^t}{p_i^s})^{\frac{1}{x}})) \le$$

$$\le \lim_{x \to 0^+} \exp(x \ln(\max_i \frac{p_i^t}{p_i^s})^{\frac{1}{x}}) = \exp(\ln(\max_i \frac{p_i^t}{p_i^s})) = \max_i \frac{p_i^t}{p_i^s}. \tag{12}$$

We are going to find such value $\phi_0$ that minimizes the expression $|P_{LM}(\phi) - P_F|$. Finding the minimum of $|P_{LM}(\phi) - P_F|$ in Mathematica 6.0 the best estimated solution, with feasibility residual, Karush–Kuhn–Tucker (KKT) residual or complementary residual, is returned.

**Example**

Let us consider the case when vectors of prices and quantities of $N = 12$ commodities are described as follows:

$P^t = [a \cdot 700, a \cdot 1900, 400, 8, 120, 120, 1200, 1000, a \cdot 500, 5, 120, 2200]'$;

$P^s = [800, 1700, 300, 9, 130, 1300, 900, 1700, 560, 6, 135, 2300]'$;

$Q^t = [400, 200, 5000, 500, 340, 700, 800, 500, 3000, 500, 340, 700]'$;

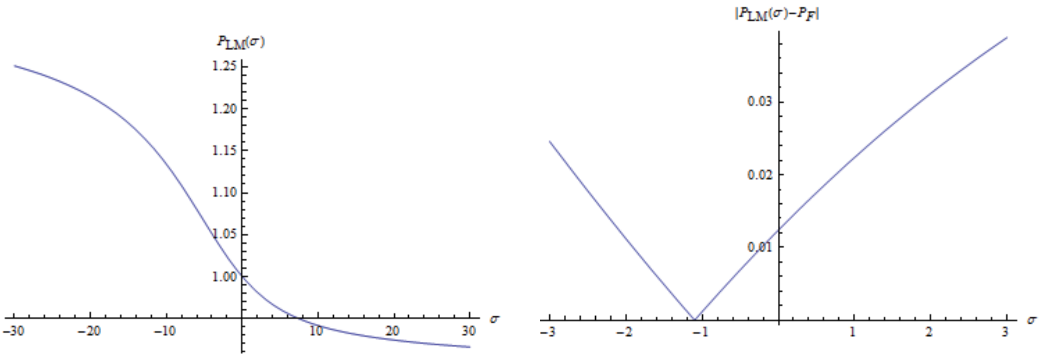
$Q^s = [350, 350, 4000, 800, 450, 700, 550, 400, 5000, 700, 250, 700]'$;

where $a$ is some positive parameter which influences on the sign of a difference $P_{La} - P_F$.

After calculations for $a = 1$ we obtain $P_{La} = 1.000$, $P_{Pa} = 1.025$ and $P_F = 1.012$. Functions $P_{LM}(\sigma)$ and $|P_{LM}(\sigma) - P_F|$ depending on $\sigma$ (still $a = 1$) are presented in Figure 1. In the same case functions $P_{LM}(\phi)$ and $|P_{LM}(\phi) - P_F|$ depending on $\phi$ are presented in Figure 2.

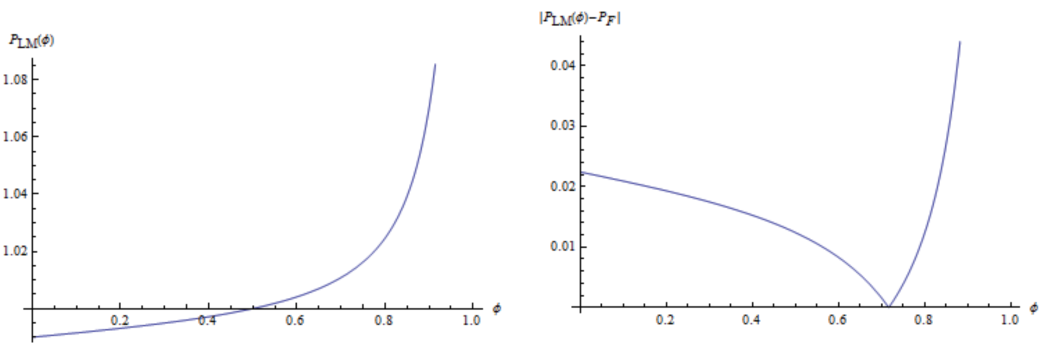
Values $\sigma_0$, $\phi_0$, $P_{La}$, $P_F$, $|P_{LM}(\sigma_0) - P_F|$ and $|P_{LM}(\phi_0) - P_F|$ for different values of $a$ are presented in Table 1. We observe $\phi \in (0,1)$ since $\sigma_0$ can be positive or negative.

**Figure 1** Values $P_{LM}(\sigma)$ and $|P_{LM}(\sigma) - P_F|$ depending on $\sigma$




**Source:** Mathematica 6.0

**Figure 2** Values $P_{LM}(\phi)$ and $|P_{LM}(\phi) - P_F|$ depending on $\phi$




**Source:** Mathematica 6.0

**Table 1** Values $\sigma_0$, $\phi_0$, $P_{La}$, $P_F$, $|P_{LM}(\sigma_0) - P_F|$ and $|P_{LM}(\phi_0) - P_F|$ for different values of $a$

| Parametr $a$ | $a$ = 0.8 | $a$ = 0.9 | $a$ = 1 | $a$ = 1.1 | $a$ = 1.2 | $a$ = 1.3 |
|---|---|---|---|---|---|---|
| $P_{La}$ | 0.921 | 0.960 | 1.000 | 1.039 | 1.078 | 1.118 |
| $P_F$ | 0.946 | 0.979 | 1.012 | 1.0456 | 1.078 | 1.111 |
| $\sigma_0$ | −1.106 | −1.192 | −1.098 | −0.674 | 0 | 0.529 |
| $\lvert P_{LM}(\sigma_0) - P_F\rvert$ | $2.633 . 10^{-6}$ | $1.575 . 10^{-7}$ | $3.6 . 10^{-8}$ | $1.317 . 10^{-8}$ | $6.677 . 10^{-5}$ | $1.307 . 10^{-8}$ |
| $\phi_0$ | 0.703 | 0.727 | 0.716 | 0.657 | 0.502 | 0.280 |
| $\lvert P_{LM}(\phi_0) - P_F\rvert$ | 0.0002 | $3.965 . 10^{-7}$ | $2.584 . 10^{-7}$ | $1.511 . 10^{-7}$ | $8.311 . 10^{-8}$ | $5.927 . 10^{-8}$ |

**Source:** Mathematica 6.0

## 3 EMPIRICAL STUDY

In our empirical illustration of the presented method of the CPI bias reduction we use monthly data[2] on price indices of consumer goods and services in Poland for the time period Jan. 2010–Jan. 2013 (36 observations). The weights $w_i^s$ and $w_{i,t}$ also are taken from data published by the Central Statistical Office.[3] Very low CPI commodity substitution bias was observed in Poland in the period under study (the largest for the data from the period of Jan. 2010–Jan. 2011, less than 0.034 percentage points, and the smallest for the data from the period of Jan. 2011–Jan. 2012, 0.013 percentage points – see Białek, 2014). This is in part due to the frequent annual, update of the weights in the CPI basket of goods in Poland. The results described in the study indicate that there is virtually no difference whether this bias is measured with the Fisher superlative index or the Törnqvist superlative index. In the period under study, although the CPI bias should be considered as small, it is positive for each year (relative to the Laspeyres index). This conclusion corresponds to the results of most studies in the world – similar results were observed in Germany (Hoffmann, 1999), Sweden (Dahlen, 1994), the Czech Republic (Filer, Hanousek, 2003), and Australia (Woolford, 1994). However, in some countries the CPI commodity substitution bias proved many times larger (e.g., in the US – see Boskin et al., 1996). Moreover, the size of the CPI substitution bias may be bigger if the system of weights is updated rarely. This is just the reason of our study where we intend to verify the scale of a reduction in CPI substitution bias by using the modified Lloyd–Moulton index (see formula 11) and under the consideration not only the frequent annual, update of the weights in the CPI basket of goods in Poland.

Let us notice that having expenditure shares of commodity $i$ in the base period $s$ and the current period $t$ we can express the Laspeyres and Paasche formulas as follows:

$$P_{La} = \sum_{k=1}^{N} w_k^s P_k^{s,t}, \tag{13}$$

$$P_{Pa} = \frac{1}{\sum_{k=1}^{N} \frac{w_k^t}{P_k^{s,t}}}, \tag{14}$$

where $P_k^{s,t}$ denotes the $k$ – th price relative (partial index) for the compared time moments $s$ and $t$, and it is obviously published by the Central Statistical Office. The first step of the study is to compare the reduced CPI substitution bias (i.e. $|P_{LM}(\phi_0) - P_F|$) for each yearly period of time, i.e. a) Jan. 2010–Jan. 2011 b) Jan. 2011–Jan. 2012 and c) Jan. 2012–Jan. 2013. Our results are presented in Table 2 and in Figure 3.

[2] We use highly-aggregated data taking into account price indices of the following group of consumer goods and services in Poland: food and non-alkoholic beverages (*X1*), alcoholic beverages, tobacco (*X2*), clothing and footwear (*X3*), housing, water, electricity, gas and other fuels (*X4*), furnishings, household equipment and routine maintenance of the house (*X5*), health (*X6*), transport (*X7*), communications (*X8*), recreation and culture (*X9*), education (*X10*), restaurants and hotels (*X11*) and miscellaneous goods and services (*X12*). The author is aware of the fact that the presented calculations play only a role of some illustration. Drawing any serious conclusions must be based on data from a lower level of aggregation, preferably at the 4-digit class level of the COICOP (Classification of Individuals Consumption according to Purpose).

[3] Główny Urząd Statystyczny (GUS) in Poland.

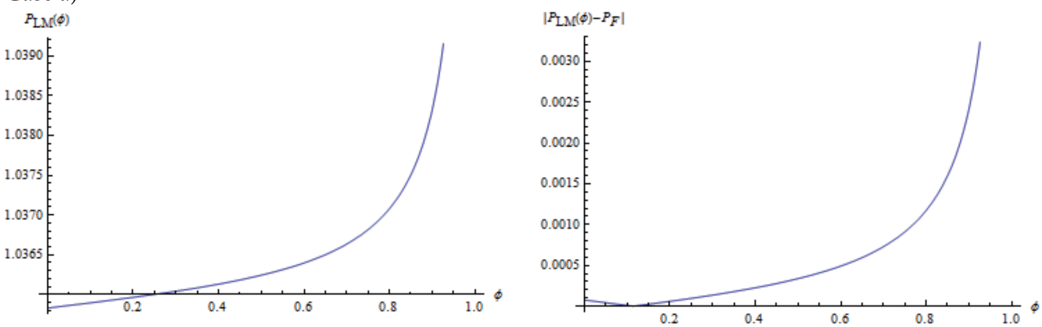
**Table 2** Values $P_{La}$, $P_F$, $\phi_0$ and $|P_{LM}(\phi_0) - P_F|$ for different time periods

| Time period | Jan. 2010–Jan. 2011 | Jan. 2011–Jan. 2012 | Jan. 2012–Jan. 2013 |
|---|---|---|---|
| $P_{La}$ | 1.0362 | 1.0389 | 1.0169 |
| $P_F$ | 1.0359 | 1.0397 | 1.0167 |
| $\phi_0$ | 0.1142 | 0.6075 | 0.0756 |
| $\|P_{LM}(\phi_0) - P_F\|$ | $1.89 \cdot 10^{-9}$ | $5.61 \cdot 10^{-9}$ | $1.002 \cdot 10^{-9}$ |

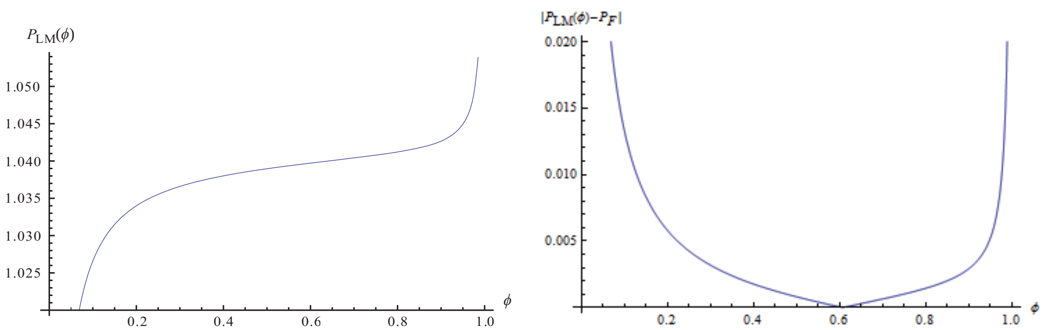
**Source:** Mathematica 6.0

**Figure 3** Values $P_{LM}(\phi)$ and $|P_{LM}(\phi) - P_F|$ depending on $\phi$ for different time periods

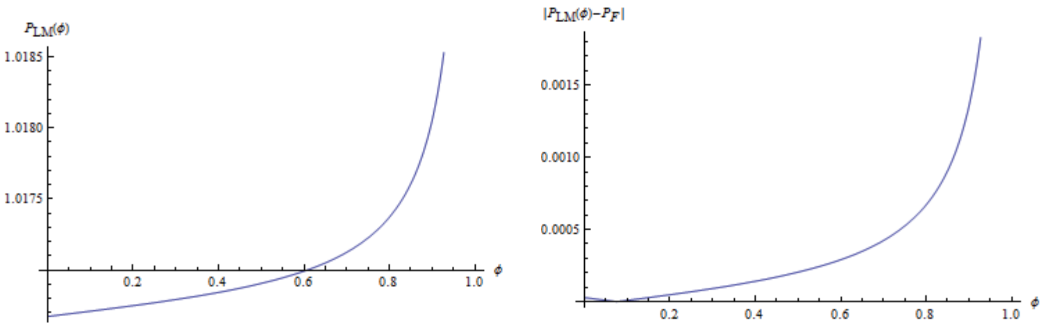
Case a)




Case b)




Case c)




**Source:** Mathematica 6.0

The next step of the study is verifying the hypothesis that we can eliminate the CPI substitution bias using the Lloyd–Moulton price index even if we do not update the weights in the CPI basket of goods each year. Only for the Lloyd–Moulton price index calculations we assume that weights in the CPI basket of goods are from Jan. 2010 (outdated) and we consider the CPI substitution bias for time periods: Jan. 2011–Jan. 2012 and Jan. 2012–Jan. 2013. The results are presented in Table 3 and in Figure 4.

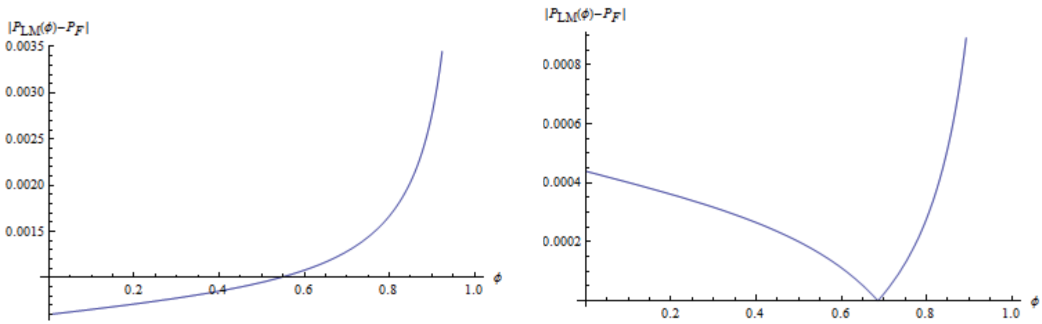
**Table 3** Values $P_{La}$, $P_F$, $\phi_0$ and $|P_{LM}(\phi_0) - P_F|$ for different time periods

| Time period | Jan. 2011–Jan. 2012 | Jan. 2012–Jan. 2013 |
|---|---|---|
| $P_F$ | 1.0397 | 1.0167 |
| $\phi_0$ | 0.550 | 0.685 |
| $\|P_{LM}(\phi_0) - P_F\|$ | $6 \cdot 10^{-8}$ | $4.6 \cdot 10^{-9}$ |

**Source:** Mathematica 6.0

**Figure 4** Values $|P_{LM}(\phi) - P_F|$ depending on $\phi$ for periods: a) Jan. 2011–Jan. 2012 and b) Jan. 2012–Jan. 2013

Case a) Case b)




**Source:** Mathematica 6.0

## CONCLUSIONS

The major advantage of using the modified Lloyd–Moulton price index (11) is that the value of the estimated parameter $\phi_0$ is always in the interval (0,1) and calculations are faster. As we can notice (see Table 1) this rule is not satisfied in the case of the parameter $\sigma_0$ from the original Lloyd–Moulton index defined in formula 8. The empirical study shows that using the modified Lloyd–Moulton price index we can approximate the superlative Fisher index with an excellent precision. Thus, if we used the modified (or original) Lloyd–Moulton index instead of the Laspeyres price index in CPI calculations we would almost eliminate the CPI commodity substitution bias. The empirical study shows additionally that we can strongly reduce the CPI substitution bias using the modified Lloyd–Moulton price index even if we do not update the weights in the CPI basket of goods each year (see Table 3).

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