Pilot Application of the Dynamic Input-Output Model. Case Study of the Czech Republic 2005–2013

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Abstract

The aim of this article is to provide the very first analysis in the field of dynamic Input-Output (I-O) models for the Czech Republic. This study examines the practicality of production dynamic equations for an estimation of future production enhanced for gross fixed capital formation. The principal construction element of dynamic I-O models rests on a technical capital matrix illustrating a stock of gross fixed capital in an economy. The lack of available data for this matrix challenges this study to analyze two possible computation procedures. Namely, I examine extrapolation method and method based on a transformation from matrix classification by type of fixed assets (AN) to classification by product (CPA). The results of the application part indicate notable differences between both ways of calculation. Final prognosis of the structure of production exhibits 11 to 21% deviations from the real structure of production in the five-year period and thus significantly diverges from reality. Potential sources of these problems and their solutions are discussed in the conclusion of this study.

Keywords

Dynamic Input-Output model, Input-Output analysis, matrix of technical capital, production equation

JEL code

C67, D24, D57

INTRODUCTION

Although, one might nowadays regard the elementary static Input-Output models as an outdated concept for modelling structural relationships in an economy, advanced macroeconomic models commonly base their assumptions on the equations and relations stemming from these I-O models. Examples of such macroeconomic models comprise models constructed on the basis of dynamic equations such

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as INFORUM (Inforum, 2015), INFORGE (Lutz, Ch. et al., 2003), or DSGE models enhanced for Input-Output data (Bouakez et al., 2005, 2009) or for elementary relationship expressed by Leontief matrix. A relatively wide range of models then seems to utilize especially production side of Input-Output models.

Dynamic Input-Output models enlarge basic static ones and diverge from static approach in time. Next, they contain additional information, namely, the information about technical capital. This capital constitutes the growth part of the model, which critically establishes the direction of evolution for total production.

While, the intermediate consumption matrix was in the center of attention for static I-O analysis, the dynamic version partially diverts its scrutiny to the technical capital matrix **B**. Most of the research papers (Inforum, 2015) examining I-O models devote a substantial part of their studies to this matrix for several reasons. First motivation is the problem of interpretation and uncertainty about the appropriate content of this matrix (Diaz, Carvajal, 2002). Second reason origins in commonly not favorable structure of the matrix, irregularity (Miller, Blair, 2009). The third important inquiry is the depreciation time of the capital and its varying influence on individual industries. Some attempts to solve this latter problem comprise of studies such as Idenburg, Wilting (2000) or Inforum (2015). I find the last and one of the main obstacles in the method of matrix development. Most countries do not publish matrices in classification by product or industry.

The technical capital matrix **B** is from the construction viewpoint of dynamic (not static) model more important than the matrix of technical coefficients **A** for the subsequent arguments. Apart from the above mentioned reasons the technical capital matrix determines the stability of economic system along with the growth potential of production (Díaz, Carvajal, 2002).

The results of those several models will be compared with forecast of total production for a basic static model and reality. In order to control for several factors originating in final consumption, I will substitute estimates of the model for real measured data. The resulting ex-post calculation will reveal prediction capability of dynamic models in context of static ones for total production in relation to final consumption.

The first chapter sums up the current state of knowledge regarding Input-Output dynamic methods. This part precedes a chapter summarizing methodological and theoretical characteristics of dynamic models – static foundations, dynamic analysis and derivation of technical capital matrices. Next, I concentrate on the problem of data sources. The results of application of these models on the data of the Czech Republic follow. The context of my results is discussed in the end of this paper.

1 LITERATURE REVIEW

Static Input-Output models find an implementation mostly in structural impact evaluation of interventions into economics with an emphasis on inter-industrial linkages. The contemporaneous Czech studies of such merit consist of VICERRO (2013) or the Ministry of the Environment of the Czech Republic (2014). The construction of static models resembles Keynesian ideas (Goga, 2009, pp. 26–32). Dynamic Input-Output models transmit the ideas further, allowing thus to incorporate long-run effects and trends across and between industries. The critics most commonly denounce the dynamic Input-Output models for an attempt to capture a dynamic non-static process (Lee, 2005) as snap shot of an economy at given time (Murray, 2011). Despite this rather negative evaluation, a broad area of application exists for a dynamic model in context of various research questions. For example consider Model DIMITRI (Idenburg, Wilting, 2000) scrutinizing the impact of interrelationship between economy, technology and the environment or environmental dynamic I-O models (Yokoyama, Kagawa, 2006; Dobos, Tallos, 2013).

The dynamic I-O models provide analysis ranging from topics such as inflation studies caused by national currency devaluation (Katsinos, Mariolis, 2012) to models combining Input-Output dynamic methods with so-called Grey system theory (Li, 2009). Other models completely forward the idea of structural analysis of I-O models into the context of DSGE models to develop detailed DSGE model based on dynamic I-O model (Bouakez et al., 2005, 2009) and capital matrix of I-O model.

One of the most important parts of the dynamic Input-Output models is the technical capital matrix. The article of Díaz, Carvajal (2002) provides an interesting study encapsulating diverse theoretical case studies in the context of dynamic Input-Output model and the technical capital matrix. The authors in their paper examine an effect of the technical capital matrix in specific situations such as in the case of zero willingness to invest or in the presence of lack of alternations of consumption or production in an economy. My study (Šafr, 2014) describes some of those cases in bigger detail.

Despite the long-lasting economic discussion regarding the technical capital in macroeconomic models in general (OECD, 2009), the effect of the technical capital matrix on the dynamic Input-Output model is an under-researched area (Pauliuk, Wood, Hertwich, 2015, p. 105; Leontief, 2007a, 2007b; Raa, 1986) summarize the basic understanding of the issue of capital inclusion into the production function of dynamic model and explain its context.

The matter of construction of states of the capital consists of four points. First task concerns the way of matrix construction. The second one questions the included variables. The third task pertains to the effects of the matrix on dynamic I-O model and the fourth problem covers singularity issue.

Mathematics enables an evasion of the fourth difficulty with the help of pseudo-inverse methods. Such methods can result in unstable outcomes as other authors indicate (Miller, Blair, 2009; or Šafr, 2014). Some authors solve this situation by an enlargement of specific methods or by refinement of contemporary pseudo-inverse methods (Jódar, Merello, 2010) or for example by succeeding calculation to obtain an invertible matrix (Sharp, Perkins, 1973). Subsequently, it is possible to solve this obstacle by combining dynamic I-O models with other approach (Zhang, 2000).

The second problem is even more complicated than the previous one concerning matrix singularity. National accounting quantifies the volume of gross fixed capital formation (GFCF) denoted as item P.51 in SIOT tables. The GFCF demonstrates the pure acquisition of fixed capital regardless its depreciation (Hronová et al., 2009). Although other studies favor additional inclusion of human capital next to the fixed capital into macroeconomic models (Zhang, 2008), in respect to the structural analysis in this study, our I-O model will not incorporate human capital. The following part examines the methodology for the I-O model outlined in this paper.

Regardless the latter problem, this study mainly inspects the first and the third point. These issues will be analyzed with help of variable for fixed capital.

2 METHODS A METHODOLOGY

2.1 Dynamic Input-Output model

2.1.1 Basic static I-O approach

SIOT tables usually represent the dataset for Input-Output models. Elementary static Input-Output model is based on linear relationships between production flow (intermediate consumption) from individual industry (i) to other industry (inputs-j) and between creation of production and production of a particular industry as whole. One can illustrate this link as (Goga, 2011, p. 75):

$$x_{i,j} = f_j(x_j), \qquad i, j = 1,2,3 \ldots\ldots n, \tag{1}$$

where $x_{i,j}$ stands for the flow of production from industry i to industry j. Variable x_j represents total production in the industry j. I assume a linearly definable relationship between x_j and $x_{i,j}$ and stable fixed ratio between $x_{i,j}$ and x_j in the long run.

Given this relationship one can define the elementary linkages of Input-Output models as:

$$\sum_{j=1}^{n} a_{ij} x_j + y_i = x_{i,} \text{, matrix form: } \mathbf{Ax} + \mathbf{y} = \mathbf{x} \,. \tag{2}$$

Principal matrix representation of the model:

$$(\mathbf{I} - \mathbf{A})^{-1}\mathbf{y} = \mathbf{x}\,, \tag{3}$$

illustrates the link between final use (**y**) and total production (**x**) to satisfy the final use for the entire economy. Variable $a_{i,j}$ stands for the elements of the matrix **A** of a dimension n x n and illustrates technical coefficients of a production function. In other words, these elements symbolize the ratio between the input flow into a industry for creation of product and total production.

One can depict this relation in the following manner:

$$\text{Matrix: } \mathbf{A} = (a_{ij})_{n\times n}, \qquad \text{with elements: } a_{ij} = \frac{x_{ij}}{x_j}, \tag{4}$$

for which: $0 \leq a_{ij} \leq 1$, $\quad i, j = 1,2, \ldots\ldots, n$.

Technical coefficients of the matrix **A** are assumed to be stable in the long run. This supposition appears in the production function form of this and subsequent period as:

$$f_j^t(x_j) \approx f_j^{t+p}(x_j). \tag{5}$$

Primary assumption about long-run stability of production function leads then to long-run stability of individual technical coefficients (a_{ij}).

This basic Input-Output model offers a wide range of potential applications. It is most often used for an analysis of structural linkages in an economy and impact evaluation of predominantly multiplication effects.

2.1.2 Dynamic I-O approach

Dynamic models with help of difference and differential equations extend the basic static Input-Output model for capital-flow matrix. This matrix aims to capture the influence of realized investments to technical capital on the growth of an economy. Next, its goal is to elude principal obstacles of static analysis (Goga, 2009, p. 103) considers a link between calculated parameters of one period in relation to exogenous parameters of the next one as the most crucial problem concerning the static model. The model thus neglects the impact of capital investments, such as purchases of new machineries or capacity expansions.

The core topic of static I-O analysis covers the above mentioned technical coefficients matrix (**A**). The attention in dynamic models partially focuses on the dataset illustrating investments into technical capital. Therefore, I expand the basic model for the fixed capital stock matrix (matrix **F**) and coefficients of capital intensity (matrix **B**). Matrix $\mathbf{I}^F$ is the next considered matrix depicting the difference of matrix **F** elements in time *t+1* and in time *t*. I will discuss these matrices in more detail in chapter 2.3 of this paper.

Dynamic I-O models are characteristic for their endogenization of exogenous variables of the static I-O model. Fundamental dynamic model endogenizes the above-discussed influence of investments into technical capital. In contrast to the static version, such information now enters the production equations of the model itself. More complex models then endogenize wider scale of parameters, which should assist analysis of monetary and fiscal effects.

Significant attribute of dynamic I-O models is their transition from purely structural models to structural-growth models due to their extension for investments into technical capital. Eurostat (Eurostat, 2008, p. 517) denotes these models as "multiplier-accelerator models". These models serve especially for an examination of structural relationships within an economy. Next, they also embrace different long-run relationship between industries deforming own structure of an economy as defined in the model. This model should then result in more exact outcome in comparison to the basic static model, which neglects these factors.

Dynamic models generally expand and optionally modify elementary set of static I-O assumptions. Laščiak (1985, p. 132) states the principal assumptions as:

- Linear relationship between coefficients.
- Each industry has a firmly defined structure of inputs.
- No option of substitution between inputs and outputs.
- Capacity and work norms are exogenously defined.
- The model abstracts from the influence of foreign trade.
- Final consumption is generated as a residual variable.
- Entrance of all inputs of the model is smooth and continuous.
- The model is built upon a uniform cost structure or structure of production.

Fecanin (1985, p. 64) completes these assumptions for:

- an assumption of a unique source of production capacity growth in a form of investments into technical fixed capital;
- calculation of investments into a non-profit sector in final consumption matrix instead of investment matrix. This notion depends on a particular model and approach; it only pertains to basic dynamic model.

I can therefore define the basic dynamic I-O model as one encompassing direct but also indirect links between production and capital (see Goga, 2009, p. 103).

It is noteworthy to mention the absence of impact of the basic dynamic Input-Output model on the structure of simulated economy; it does not modify the ratio between inputs and outputs of individual industries. Hence, the model does not reflect the possible variability of multipliers caused by alteration of matrix **A** structure.

The dynamic model can be represented with help of difference as well as differential equations. Production function of the model appears as:

$$\sum x_i(t) = \sum\sum a_{ij}x_j(t) + \sum\sum b^i_{ij}\left[x_j(t+1) - x_j(t)\right] + \sum y(t). \tag{6}$$

In a matrix form (Eurostat, 2008, p. 517):

$$\mathbf{x}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\left[\mathbf{x}(t+1) - \mathbf{x}(t)\right] + \mathbf{y}(t). \tag{7}$$

The resulting form of the model is:

$$\mathbf{x}(t+1) = \mathbf{B}^{-1}\left[\mathbf{I} - \mathbf{A} + \mathbf{B})\mathbf{x}(t) - \mathbf{y}(t)\right]. \tag{8}$$

To compare such equation with the classical Input-Output model:

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y}. \tag{9}$$

Closed I-O model takes a form in case of difference equations:

$$\mathbf{B}\mathbf{x}(t+1) - (\mathbf{I} - \mathbf{A} + \mathbf{B})\mathbf{x}(t) = 0. \tag{10}$$

Final set of equations in case of differential equations (For proof, see: Diaz, Carvajal, 2002):

$$x(t) = e^{Mt}X(0) + \int e^{M(T-\tau)}NY(\tau)\delta(\tau). \tag{11}$$

The last equation reflects the equality between the total volume of capital used in future year and contemporary volume of capital plus final consumption. The investments into fixed capital are endogenized in this closed model with help of discontinuous model on the basis of differences. Final consumption stays as the only unknown and exogenous variable (except constant). As already mentioned, the final consumption variable is often modeled as a residual variable or with a use of a specific utility function.

2.2 Technical Capital Matrix

The main part of the dynamic Input-Output model consists of the matrix **B**. This matrix is a capital-intensity coefficients matrix or the capital stock matrix as already mentioned. This matrix depicts a quantity of technical capital produced in time *t* and employed in time *t+1* (Leontief, 2007a, p. 295, p. 316) calls this matrix "Capital stock coefficients"; matrix which depicts states of technical capital. The matrix is then supposed to encompass all capital flows into particular industry in individual years controlled for depreciation.

It is possible to interpret the meaning and definition of this matrix differently. A classical definition assumes the elements of the matrix to illustrate the ratio of quantity of capital stock delivered from industry *i* necessary for production of one unit of output in industry *j*. Following this definition, one might express this matrix as:

$$\text{Matrix: } \mathbf{B}^F = (b_{ij})_{n\times n}, \qquad \text{with elements: } b_{ij} = \frac{f_{ij}}{x_j}. \tag{12}$$

That is ratio between all capital flows from industry *i* to industry *j* designated for one unit of production in industry *j*. For all coefficients relation $0 \leq b_{ij}$ must stay true in this definition of matrix, as industries cannot keep negative amount of capital.

One can also define the matrix **B** as investment coefficients matrix (Goga, 2011). Therefore, the construction of the matrix utilizes on the relationship about the long-run stability of technical coefficients of the matrix **B** and on the equations of the dynamic model (you can find the proof below). Keeping these relations in mind one can calculate the matrix as:

$$\text{Matrix: } \mathbf{B}^I = (b_{ij})_{n\times n}, \qquad \text{with elements: } b_{ij} = \frac{i_{ij}}{\Delta x_j}. \tag{13}$$

The elements are hence calculated as the ratio of capital flow in one year (new investment) and production change. However, this computation method does not guarantee the validity of previous limitations about their non-negativity. In this case, investments can be negative due to depreciation, the production difference might appear below zero and investments may grow. For this reason, negative coefficients are transformed to null ones. This way defined matrix does not fully correspond to the basic dynamic model in its meaning. It rather artificially sets states of capital according to the capital flow in one year (investment) and to production changes dependent on this alternation.

In respect to the equations one can find several conditions, data and model must meet while using the computation method for the matrices **B** and **F**:

1. Change in capital variable (investment) tends to long-run equilibrium even in the short-run and hence does not undergo any long-run volatility; the same stays true for investment and production.

Taking into consideration the relation, $f_{ij} = \frac{i_{ij}}{\Delta x_j} x_j$, one can deduce proportional influence of the change in investment or production on the total stock of capital accumulated in the previous periods of time.

2. Previous-year or older effect of investment changes does not affect the present change in production; otherwise the total stock of technical capital would alter.
3. The entire effect of investment adjustment is carried on this year, as the technical capital would alter otherwise.
4. Situation where a industry does not invest into a specific flow of capital (i, j) must not occur even if it satisfies the above stated conditions and thus confirms the principle of linearity of relationships between investment and production changes. The reason is the zero value of estimated stock of capital obtained by this method (under the assumption that the capital is being used) regardless the capital accumulation recorded in the previous year.
5. Situation when production of a sec equals zero must not occur since the equations would not have solution.

These conditions are rather strict for calculation of necessary matrices. Most probably the outcome is not going to be robust from the standpoint of long-run stability of coefficients. In the short-run coefficients might appear volatile, one can assume delayed effect of investments and finally, the effect of investment adjustments from previous years might affect the output within the observed period of time. It is not possible to fully eliminate such problem but it is possible to minimize it by applying calculations based on longer time series with a consideration of an existence of depreciation. This idea will be discussed in section 2.3.1 in bigger detail.

Matrix $\mathbf{B}^F$ appears from the calculation perspective as more robust than matrix $\mathbf{B}^I$. One of my arguments is a possibility of significant differences between matrix $\mathbf{B}^I$ and matrix $\mathbf{B}^F$ in a reaction to production and investment volatility within one year. Then this relation seems as more reasonable:

$$\lim_{t \to \infty} b_{ij}^I \approx b_{ij}^F . \tag{14}$$

It depicts the raise in approximation of elements of the matrix $\mathbf{B}^I$ to elements of matrix $\mathbf{B}^F$ in long-run.

Last but not least, this analysis might be accompanied with a problem of singularity of matrices. Since not all industries produce technical capital, the matrix $\mathbf{B}$ is practically always singular. This characteristic aggravates the utilization of dynamic Input-Output models. One can partially but not absolutely evade this obstacle by applying pseudo-inverse methods as already mentioned. According to many results (see Miller, Blair, 2009; Šafr, 2014), such modification of model with pseudo-inversion leads to unstable estimations, which inclines to "exponential" growth. Such outcome also results in low values of the matrix $\mathbf{F}$.

2.3 Construction of Capital Stock Matrix (B)

2.3.1 Extrapolated (model) approach

There are several methods how to construct a capital stock matrix. Common diversification understands direct methods based on primary collection of data and indirect methods. The latter types of methods derive the capital matrix from other sources of data or from direct calculation from the model. The Czech Statistical Office does not publish capital flows/stock matrix classified by industry or production. For this reason, I need to find a different way of dataset collection for the matrix, for example its calculation from other sources of data.

First and second presented method for matrix computation is based on detail knowledge of production allocation for GFCF (non-symmetric matrix). I am going to assemble a symmetric capital flow matrix within one year using symmetrizing methods for the matrix "product x industry". These methods have been originally derived for symmetrizing SIOT tables. This part is common for both methods.

Dynamic Input-Output models can serve for a calculation of matrix **B** by the approximation procedure (Goga, 2011, p. 81). The core assumption of I-O models understands production function as constant in the long run. Mathematical form of this assumption:

$$f_j^t(x_j^t) = f_j^{t+p}(x_j^t). \tag{15}$$

Application of this assumption for dynamic Input-Output models especially for the capital matrix could be written as (Eurostat, 2008, p. 520):

$$\mathbf{I}^F(t) = \mathbf{BX}(t+1) - \mathbf{BX}(t). \tag{16}$$

This relation displays the investment matrix in time *t* as the difference between production-flow matrix multiplied by capital coefficients in time *t+1* and the same matrix in time *t*. Then this equation must stay true (similarly as for the technical coefficients matrix ***A***):

$$\frac{f_{ij}(t)}{x_j(t)} = \frac{f_{ij}(t+p)}{x_j(t+p)}. \tag{17}$$

The above equality states coefficients of matrix **B** as stable and constant in the long run. One might obtain the same results by using dataset in time *t*, *t+1* or *t+p*. If this assumption holds then:

$$f_{ij}(t) = \frac{f_{ij}(t)}{x_j(t)} x_j(t), \qquad f_{ij}(t+p) = \frac{f_{ij}(t)}{x_j(t)} x_j(t+p), \tag{18}$$

$$f_{ij}(t) = \frac{f_{ij}(t+p)}{x_j(t+p)} x_j(t), \qquad f_{ij}(t+p) = \frac{f_{ij}(t+p)}{x_j(t+p)} x_j(t+p),$$

These equations can be applicable for construction of GFCF matrix or investment matrix $\mathbf{I}^F$. The above-mentioned formula (18) stays valid for this matrix (**F**). Using **previous formulas** (18) along with the formulas (15) and (17) one can obtain these relations:

$$\sum_t^{t+p} i_{ij}(t) d_{ij}(p) = f_{ij}(t+p) - f_{ij}(t), \tag{19}$$

then:

$$f_{ij}(t+p) - f_{ij}(t) = \frac{f_{ij}(t+p)}{x_j(t+p)} x_j(t+p) - \frac{f_{ij}(t)}{x_j(t)} x_j(t), \tag{20}$$

modified to:

$$\sum_t^{t+p} i_{ij}(t) d_{ij}(p) = \frac{f_{ij}(t)}{x_j(t)} \left[x_j(t+p) - x_j(t)\right], \tag{21}$$

where I substitute:

$$\Delta x_j^{p,t} = x_j(t+p) - x_j(t), \tag{22}$$

to obtain:

$$\sum_{t}^{t+p} i_{ij}(t) d_{ij}(p) = \frac{f_{ij}(t)}{x_j(t)} \Delta x_j^{p,t} . \tag{23}$$

Our aim is to express capital value (**F**) and coefficients of matrix **B**, therefore:

$$f_{ij}(t) = x_j(t) \left(\Delta x_j^{p,t}\right)^{-1} \left[\sum_{t}^{t+p} i_{ij}(t) d_{ij}(p) \right] , \tag{24}$$

and

$$b_{ij}(t) = \left(\Delta x_j^{p,t}\right)^{-1} \left[\sum_{t}^{t+p} i_{ij}(t) d_{ij}(p) \right] , \tag{25}$$

where:

$$\text{Matrix: } \mathbf{B}(t) = \left(b_{ij}(t)\right)_{n \times n} , \quad \text{where: } b_{ij}(t) \approx \frac{f_{ij}(t)}{x_j(t)} \approx \frac{f_{ij}(t+p)}{x_j(t+p)} , \tag{26}$$

$0 \leq b_{ij}$, $\quad i, j = 1,2,\ldots\ldots,n.$

For matrix of flows:

$$\text{Matrix: } \mathbf{F}(t) = \left(f_{ij}(t)\right)_{n \times n} , \qquad i, j = 1,2,\ldots\ldots,n. \tag{27}$$

and investment:

$$\text{Matrix: } \mathbf{I}^F(t) = \left(\sum_{t}^{t+p} i_{ij}(t) d_{ij}(p) \right)_{n \times n} , \qquad i, j = 1,2,\ldots\ldots,n. \tag{28}$$

Aggregate form of these two outcomes are:

$$\mathbf{F} = \mathbf{I}^F \overline{(\Delta \mathbf{x}(t))}^{-1} \overline{(\mathbf{x}(t))} , \tag{29}$$

$$\mathbf{B} = \mathbf{I}^F \overline{(\Delta \mathbf{x}(t))}^{-1} , \tag{30}$$

where:

$\overline{(\Delta \mathbf{x}(t))}^{-1}$ – Inverse matrix with diagonal elements of newly created investments.

$\overline{(\mathbf{x}(t))}$ – Matrix, which diagonal elements are vector of total production in time t.

This model procedure of computation of technical capital matrix might carry several already discussed problems. For this reason, I derive general solution for longer time period:

$$f_{ij}(t) = x_j(t) \left(x_j(t+p) - x_j(t)\right)^{-1} \left[\sum_{t}^{t+p} i_{ij}(t) d_{ij}(p) \right] . \tag{31}$$

Analogically for time t + p:

$$f_{ij}(t+p)=x_j(t+p)(x_j(t+p)-x_j(t))^{-1}\left[\sum_{t}^{t+p} i_{ij}(t)d_{ij}(p)\right], \tag{32}$$

where $d_{ij}(p)$ represents depreciation rate of investment $i_{ij}(t + p)$ in time p. This depreciation is determined for capital type (i) as well as for industries (j) employing the capital in specific time ($t + p$).

For time period p, which does not have to be necessarily continuous from the perspective of stability of technical coefficients, implementation of this way of calculation yields $N_{F/B}$ various ways of solutions for matrices **F** and **B**, where:

$$N_{F/B}=(p-1)p. \tag{33}$$

The final averaged matrix $\overline{\mathbf{B}}$ can be obtained by calculating averages for individual matrices:

$$\overline{\mathbf{B}}=\left(\overline{b}_{ij}\right)_{n\times n}, \qquad \overline{b}_{ij}=\sum_{w=1}^{(p-1)p}\frac{b_{ij}^{w}}{(p-1)p}. \tag{34}$$

Resulting matrix $\overline{\mathbf{B}}$ symbolizes extrapolated solution. It is an averaged technical coefficients matrix.

Non-solved question arises, namely if such computed matrix is in accordance with the original theory. The construction way of the matrix classifies it as rather matrix of willingness to invest (Díaz, Carvajal, 2002, p. 12), which could be viewed as positive from the standpoint of construction of a dynamic model.

I expect this matrix to significantly vary from the matrix formulated with the second method. However, this matrix should include all effects potentially affecting the matrix of fixed states not necessarily apparent in statistics such as faster capital depreciation.

2.3.2 Construction of the CPA matrix from the AN matrix (AN approach)

Second method utilized in the application part to compose the matrix **B** originates in the available capital state matrix according to the classification AN x NACE. First, transformation of the AN matrix to CZ-CPA takes place. The conversion procedure employs knowledge about the GFCF matrix (in classification CZ-CPA x NACE) along with suitable interference concerning individual flows.

The total value of capital states is based on the realization of the matrix AN x CZ-CPA. Following this classification, the resulting CZ-CPA x NACE matrix is symmetrized in accordance with the method "A" in the manual of Eurostat (2008). To be precise, the procedure is not matrix symmetrization by definition but transformation to CPA x CPA classification, as it does not have to yield equality between the sum of values of columns and rows.

2.4 Complete Final model

The outline of the complete final model enters the matrix form as:

$$\mathbf{x}(t+1)=\mathbf{B}^{-1}[(\mathbf{I}-\mathbf{A}+\mathbf{B})\mathbf{x}(t)-\mathbf{y}(t)], \tag{35}$$

$$\mathbf{y}(t+1)=(\mathbf{A}-\mathbf{I})\mathbf{B}^{-1}[(\mathbf{I}-\mathbf{A}+\mathbf{B})\mathbf{x}(t)-\mathbf{y}(t)], \tag{36}$$

$$\mathbf{F}(t+1)=[(\mathbf{I}-\mathbf{A}+\mathbf{B})\mathbf{x}(t)-\mathbf{y}(t)], \tag{37}$$

$$\mathbf{I}(t+1)=(\mathbf{I}-\mathbf{A}+\mathbf{B})(\mathbf{x}(t)-\mathbf{x}(t-1))-\mathbf{y}(t)+\mathbf{y}(t-1), \tag{38}$$

The common recommendation to avoid unstable fluctuations of resulting aggregates is an inclusion of an investment variable into a total demand (as consumption and export) and modeling it along with its outcome as a stable element. Increasing/decreasing capacities of dynamic Input-Output model then endeavors to reflect upon the varying final demand of an economy (Eurostat, p. 523).

I will replace final consumption of the model with real consumption to verify defined hypothesis about prediction capability of the model concerning production. Next, since the model does not specify calculation procedure for capital/capital coefficients in period t=0, I will apply the two above elucidated methods and compare them in the concluding ex-post/ex/ante analysis. However, regarding time period t>0 I will utilize the above described equations. In other words, the model will be calibrated with regards to real economic variables in time t=0. Production and capital estimated for subsequent time period will initiate final consumption, in which most solutions of other models (Idenburg, Wilting, 2000) vary. Math representation:

$$\mathbf{x}(t+p+1) = f\left(\mathbf{x}(t+p), \mathbf{A}(t), \mathbf{B}(t), \mathbf{I}(t+p), \mathbf{y}(t+p+1)\right). \tag{39}$$

Differences between predicted and real future total production in time t+p+1 should primarily originate in production functions. For this reason, this study abstracts from the issue of utility function. Potential inclusion of utility function could conceal and undervalue/overvalue real prediction capability of those production functions. Finally, I assume stable relationship between production growth and gross fixed capital formation such as for example Eurostat (2008, p. 523).

3 DATA

Dataset used for the computation of the model comes from the Czech Statistical Office. This institution publishes symmetric Input-Output tables in a five-year period; the most recent SIOT table is for year 2010. I also utilize ESA 95 valid for the 31st of December 2013, in regard to the structure and data classification for capital. To calculate coefficients of the matrix **A** I work with dataset for the year 2010. The technical capital matrix is based on the time span 2005–2010 (n = 5) except for the year 2009. Exclusion of the latter year stems from my aim to minimize the bias of the model due to the economic crisis.

I compose the GFCF matrix in correspondence to the matrix product x industries (CPA x NACE) within time period of one year. I treat this non-symmetric matrix for price level changes and then symmetrize both matrices for individual years according to the structure of the use of intermediate consumption matrix. This symmetrizing part of the methodology seems to be the most simplifying and problematic. In reality, the structure of capital utilization does not probably correspond to that of intermediate consumption. Nevertheless, this procedure appears to be the only contemporary way of the matrix calculation without necessity to construct new dataset.

The resulting GFCF matrix reveals to be for both methods irregular. For this reason, I apply pseudoinverse method to be able to solve the equations (for more information see: Jódar, Merello, 2010). The final dynamic Input-Output model reflects depreciation of gross fixed capital. The value of the capital depreciation is consistent with depreciation rates of the Czech Statistical Office commonly used for calculating the stock of GFCF.

I consider several different views of depreciation. First, I scrutinize the longevity of the capital employment; second the capital type and third the place of its depreciation. Therefore, not only various types of capital generate different depreciation, one type of capital in different industry shows to have a different length of its utilization/depreciation as well. Then the final depreciation parameters matrix yields three parameters – WHEN, WHICH and WHERE capital depreciates. Data for final consumption and GFCF are collected from the website of the Czech Statistical Office.

Model and prediction respect the classification to 82 products. For the purpose of analysis, acquired outputs aggregates to the top CZ-CPA categories.

4 RESULTS

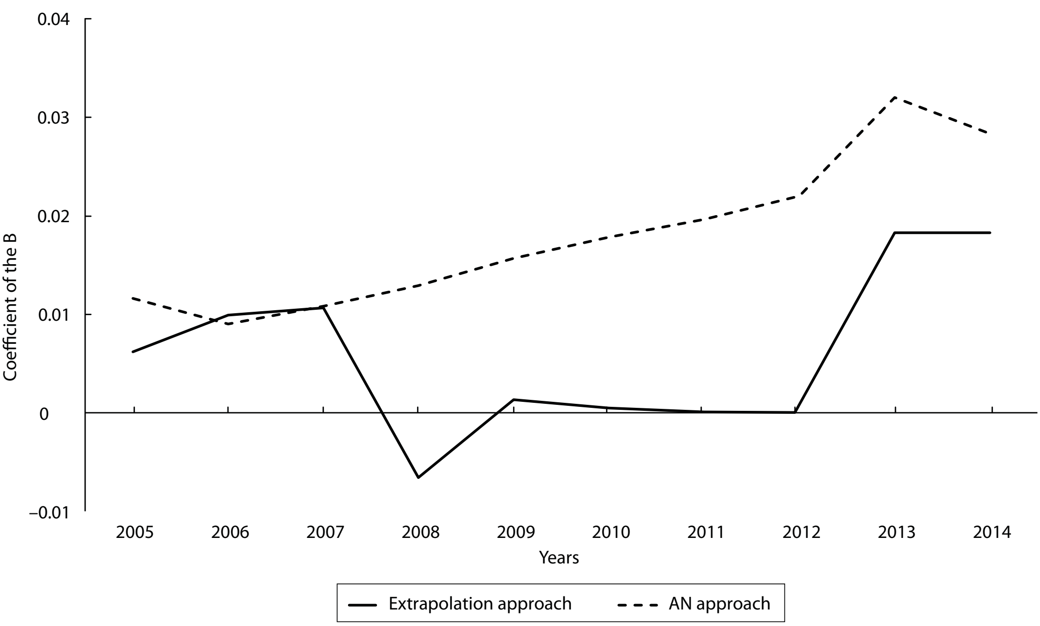
Case study of the Czech Republic for years 2005 to 2010 starts with an estimation of missing data, paying attention to the technical coefficients matrix and the capital state matrix. With respect to the available dataset I construct two solutions. The first solution (extrapolated matrices **B** and **F**) exploits the above-derived procedure for the composition of the capital matrices based on the assumption about validity of the dynamic I-O model. The second solution, the transformation of AN x NACE matrix to CPA x CPA, derives from statistically determined dataset and its manipulation to obtain the CPA x CPA matrix.

After the estimation of missing data I choose the year 2010 to serve as a base year. Next, I predict future values of production and GDP for time period 2010 to 2014 and past values for years 2005 to 2009. Regarding the base year, the minimum errors in the structure of prediction will exist in the vicinit of the year 2010.

4.1 Matrices

The coefficients of the matrix **B** as well as the matrix **A** are assumed to be stable in the long-run. The subsequent graph depicts the evolution of the median non-zero coefficients of the matrix **B**. The dashed line illustrates these coefficients obtained by extrapolation and dotted line stands for coefficients from the classification AN (see Figure 1).

Figure 1 Median non-zero coefficients of B



Source: Author's calculations

One can notice significant instability of the coefficients in the matrix **B** calculated by the method of extrapolation. The drop in the year 2008 reflects the violation of the model assumptions as a result of the financial crisis.

The next table displays the values from the extrapolated calculation divided for the two time periods before and after the critical year 2008 as well as for the entire observation period:

Table 1 Statistics of extrapolated calculation

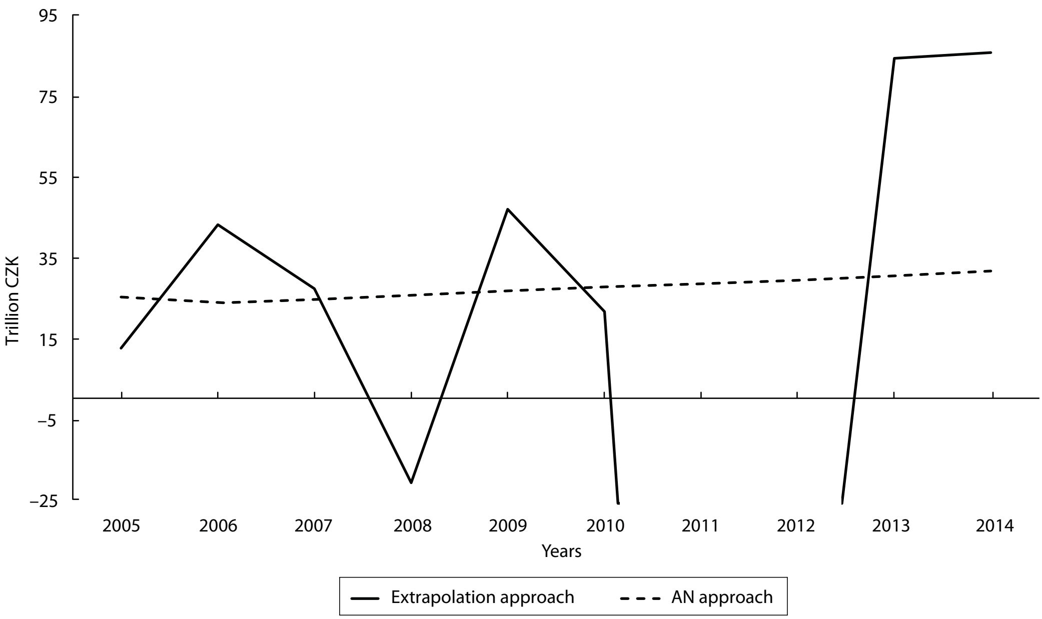
Time period	Average coefficient	% of negative values in matrix B	Average Coefficient of Variation of coefficient matrix B
2005–2013	−0.1384	5.19%	−34.56%
2005–2008	+0.0073	5.03%	+18.44%
2009–2013	−0.2551	5.31%	+836.07%

Source: Author's calculations

Negative values of the matrix **B** were transferred to null ones by applying method RAS.

Regarding the extrapolated solution, the construction of the matrix **B** precedes that of the matrix **F**. In respect to the second composition method based on the AN classification, the calculation of the matrix **B** follows that of the matrix **F** (see Figure 2).

Figure 2 Amount of fixed capital in the economy (trillion CZK)



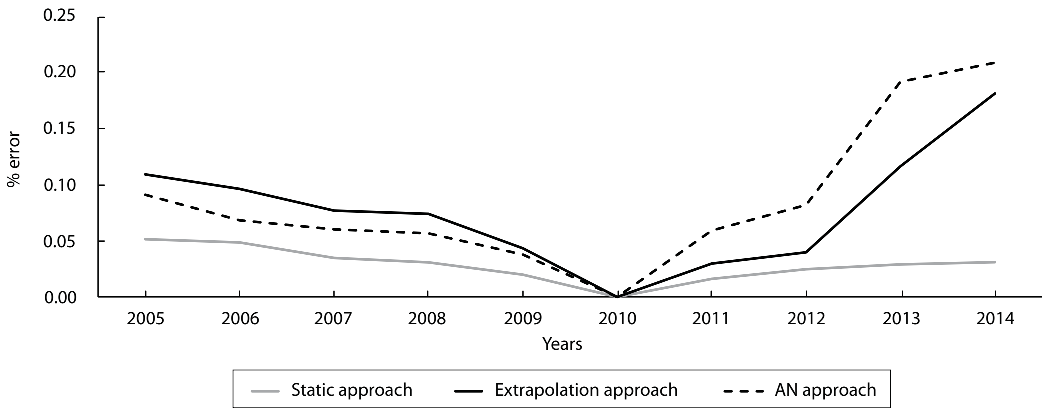
Source: Author's calculations

The volume of the stock derived from the AN classification stands for the statistically measured stock in the economy. In contrast, the stock calculated by the extrapolated method demonstrates the volatility of the model estimation caused by a violation of the assumption essential for the applied method of extrapolation. The extrapolated solution for the matrix **B** but also for the matrix **F** reflects the effect of crisis. The turmoil period is especially apparent in the growth of the negative changes in values of production. The most accurate calculation of the value of capital states occurs in the extrapolated solution between years 2005 and 2007. During this time period the capital reached on average 20.37% of the real value of measured capital.

4.2 Predictions

Dynamic Input-Output models originate in the structurally analytical models with a potential for product enhancement as an outcome of investment. Since these models are primarily structural, I start with a comparison of deviations from the predicted structure of production at the Figure 3.

Figure 3 The gradual growth of the error (%)

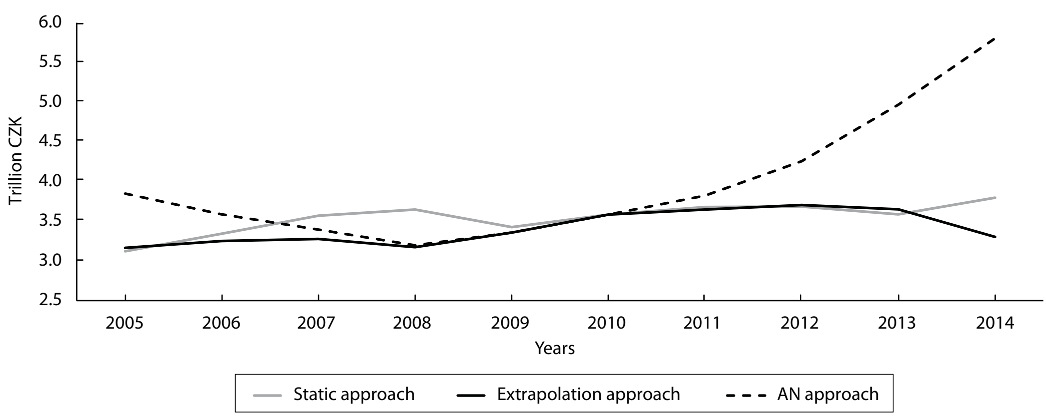


Source: Author's calculations

This graph illustrates a gradual growth of the error in the structure of prediction for the total production since 2010. The static solution exhibits a minimum deviation concerning the values before and after the year 2010. The cumulated error in the structure of prediction for the static solution reaches the value of 5.16% for the entire observation period and 3.11% if one regards only the future values. This outcome also reflects the alteration of the structure of SIOT tables with a violation of linear relationships in these tables. Extrapolated solution indicates 10.95% of error for the past values and 18.18% for the future values. On the contrary, the solution of the matrix computation from the AN classification exhibits error of 9.13% for the past and 20.92% for the future values.

The resulting prediction of gross value added (GVA) would according to dynamic and static I-O model appear between years 2005 and 2014 is at Figure 4.

Figure 4 The prediction of gross value added (trillion CZK)



Source: Author's calculations

It is worth noting the absence of real measured evolution of GVA for domestic production on this graph. The reason for such deficiency is the lack of GVA data; they are published in five-year period therefore regarding the observation period, data only for years 2005 and 2010 are present.

This graph depicts a prognosis of production drop during the period of crisis (most apparent for year 2008) for both dynamic variants. The line standing for the extrapolated solution is also decreasing. The downward trend is a consequence of the above-mentioned troublesome computational procedure concerning the capital stock, which was calculated with help of production and capital changes from one year to another. Considering the line calculated from the AN x NACE matrix, such prognosis during financial crisis reflects a reduction of specific type of investment during these years. Static model delays the reduction prognosis till 2009 as a consequence of consumption diminution. The forecast of the evolution for the extrapolated and static version assimilates development of GVA. In contrast, the method from the AN assumes a non-realistic high growth of values for the period 2010–2014.

5 DISCUSSION

The above-mentioned results signal numerous obstacles. We can categorize these problems into the question of calculation to complete the missing dataset and the issue of calculation of the respective prediction.

Both computation methods of matrices approach the matrices **B** and **F** from a different point of view. Extrapolated calculation proceeds from the matrix **B** to the matrix **F** while the calculation based on the classification AN starts with the matrix **F** to derive the matrix **B**. This seemingly trivial fact leads to the raise of cumulated errors mostly apparent in the matrix **F** when using the extrapolated calculation and in the matrix **B** in case of the calculation from the classification AN. Absolute level of capital evaluated from the extrapolated solution is then significantly lower than if one applies the calculation from AN. The capital state matrix produces significant unstable estimations of the aggregate stock of capital **F** caused mostly due to the violation of the assumption about non-negativity (or rather same direction) of the production in respect to the investment in GFCF. On the other hand, the matrix **B** indicates substantially lower variability in case of the extrapolated solution. Nevertheless, the elements of the matrix **B** are also negative for the extrapolated solution. Most authors agree upon the importance of non-negativity of the matrix **B**. Despite this concurrence, there exists an economic explanation of this phenomenon.

Next, the computation of the prediction also exhibits the inversion problem regarding the singular matrix **B**. The main difficulty occurs in the values of the matrix approximating zero and thus leading to exponential trends. Other authors indicate this problem (Miller, Blair, 2009). Some estimated predictions of product as well as GVA illustrate divergent oscillations even for such short time period.

Minimum deviations in the structure of industrial prediction occur around the year 2010. This outcome is given by a rather slower adjustment in the structure of national economy than the model assumes as well as by an absence of some effects and differences caused by a capacity enlargement, which are expected in the model for both dynamic versions. Divergence of all models from their real counterparts grows over time. The disparity between the static variant and reality rests entirely on the alteration of the structure of national economy since 2010. The static model does not reflect changes in the structure and wholly neglects the influence of capital.

The outcome of the product and GVA predictions stay consistent with the previous research especially concerning the problematic utilization of the capital matrix (Miller, Blair, 2009). Unsolved question regards the content of the matrix **B**, respectively **F**. The neglect of assets and their real effectiveness generates substantial differences between the theoretical and the extrapolated stock of capital (Diaz, Carvajal, 2002).

Furthermore, the model contains an assumption about the full impact of investment taking place within one time period, which might not necessary come true. In that case, total production becomes overestimated and the structure of respective production modifies faster. Some succeeding models eliminate this problem and exhibit better results than the dynamic I-O model alone (Mönnig, 2012). We could indisputably classify INFORUM models as such types of successful models, which combine features of a dynamic I-O model with models on the basis of CGE or other endogenous and exogenous information (Zhang, 2000; Liew, 2000).

CONCLUSION

This article aims to test the applicability of dynamic I-O models for the Czech data. Despite the attainment of construction of the respective dynamic model, range of problems and unsolved difficulties arises. Regarding absence of complete datasets, this paper shows possible construction procedures of unavailable matrices. However, both above discussed and applied ways face computation problems. Applying extrapolated solution often leads to the violation of assumptions crucial for this calculation method. The obstacle of the second approach stems from the troublesome transformation of the classification from AN to CPA. Next, utilization of the latter method necessitates a difficult decision about the content of such matrix. The particular calculation of the prediction faces problem of irregularity of capital matrices as well as display of oscillatory and exponential evolution. In the end, the resulting prognosis distinctly varies from the statistically measured real values especially for the structure of production in national economy. Namely, there appear to be eleven to twenty-one per cent erroneously allocated values during five periods. This inaccuracy might originate in the expectations integrated in the model about the impact of all effects on the economy within one year, which might not necessarily correspond to reality. Next, the model assumes the same effect to result from different types of capital. Finally, utilization of statistically computed depreciation in the model might not be truly consistent with reality.

Most of the outlined problems could be overcome by an incorporation of the production equations into an enhanced model framework. Examples of those models are based on the models such as INFORUM or DSGE. These models constitute a complex infrastructure possibly defining restricted production functions along with possible stickiness.

The subsequent research could focus on an inclusion of these production functions into a more complex economic model based on the contemporary economic theory. Next, following analysis could more thoroughly explain the phenomenon surrounding capital matrices. Difficulties concerning diverging structure over time could be explained by the steadiness and different effectiveness of capital. Finally, other endogenous variables such as employment could be used to enhance the presented model along with an inclusion of individual utility functions for various economic factors such as households or government.

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