An Estimated DSGE Model with a Housing Sector for the Czech Economy – Technical Appendix

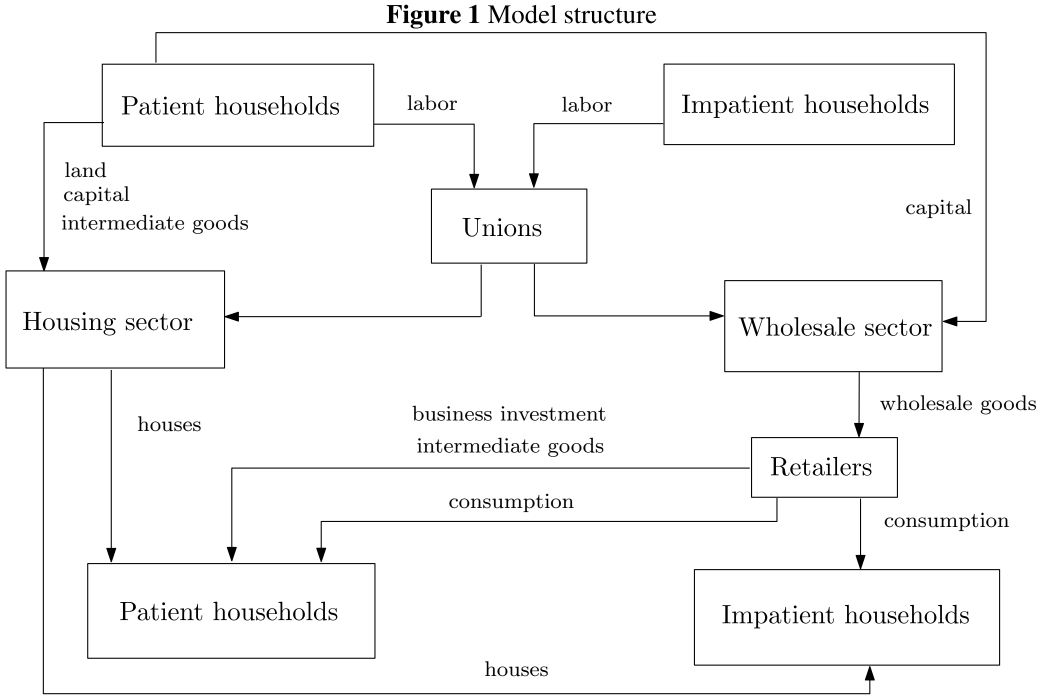
1 Model

The model is taken from Iaccoviello and Neri (2010). This part describes optimization problems of agents in the model. Denoting of the variables is the same as in original paper. Figure 1 helps to provide basic orientation in the model structure.

Agents in the model

- Patient households (work, consume, accumulate housing, provide funds)
- Impatient households (work, consume, accumulate housing, borrow)
- Wholesale firms (wholesale goods/new houses)
- Retailers (consumption, investment goods)
- Unions (differentiated labour)
- Aggregators/labour packers
- Monetary authority

Figure 1 Model structure



1.1 Households

Patient households

Patient households work, consume, accumulate housing and save – provide funds to firms and impatient households. They also own final goods firms. Their utility function is

$$E_0 \sum_{t=0}^{\infty} (\beta G_C)^t \mathrm{z}_t \left(\Gamma_c \ln(c_t - \varepsilon c_{t-1}) + \mathrm{j}_t \ln h_t - \frac{\tau_t}{1+\eta} \left(n_{c,t}^{1+\xi} + n_{h,t}^{1+\xi} \right)^{\frac{1+\eta}{1+\xi}} \right) \tag{1}$$

where c, h, n_c, n_h are consumption, housing, hours in the consumption sector and hours in the housing sector. β is discount factor, ε is habit in consumption, $\xi, \eta \geq 0$ elasticity of substitution of hours in two sectors. $\mathrm{z}_t, \mathrm{j}_t$ and τ_t are shock to intertemporal preferences, housing preference shock and shock to labour supply which are specified as

$$\ln \mathrm{z}_t = \rho_z \ln \mathrm{z}_{t-1} + u_{z,t}$$

$$\ln \mathrm{j}_t = (1-\rho_j) \ln j + \rho_j \ln \mathrm{j}_{t-1} + u_{j,t}$$

$$\ln \tau_t = \rho_\tau \ln \tau_{t-1} + u_{\tau,t}$$

where $u_{z,t}, , u_{j,t}, u_{\tau,t}$ are i.i.d. processes with variances $\sigma_z^2, \sigma_\tau^2, \sigma_j^2$. G_C is the growth rate of consumption along the balanced growth path. The scaling factors $\Gamma_c = (G_C - \varepsilon)/(G_C - \beta \varepsilon G_C)$ and $\Gamma_c = (G_C - \varepsilon')/(G_C - \beta' \varepsilon' G_C)$ ensure that the marginal utility of consumption is $1/c$ in the steady state.

Budged constraint (in real terms) for patient households is

$$\begin{aligned} c_t + \frac{k_{c,t}}{\mathsf{A}_{k,t}} + k_{h,t} + k_{b,t} + q_t h_t + p_{l,t} l_t - b_t &= \frac{w_{c,t} n_{c,t}}{X_{wc,t}} + \frac{w_{h,t} n_{h,t}}{X_{wh,t}} \\ + \left(R_{c,t} z_{c,t} + \frac{1-\delta_{kc}}{\mathsf{A}_{k,t}} \right) k_{c,t-1} + \left(R_{h,t} z_{h,t} + 1 - \delta_{kh} \right) k_{h,t} &+ p_{b,t} k_{b,t-1} - \frac{R_{t-1} b_{t-1}}{\pi_t} \\ + (p_{l,t} + R_{l,t}) l_{t-1} + q_t (1-\delta_h) h_{t-1} + Div_t - \phi_t &- \frac{a(z_{c,t}) k_{c,t-1}}{\mathsf{A}_{k,t}} - a(z_{h,t}) k_{h,t-1} \end{aligned} \tag{2}$$

Patient households choose consumption c_t, capital in the consumption sector $k_{c,t}$, capital $k_{h,t}$ and intermediate goods $k_{b,t}$ (priced at $p_{b,t}$) in the housing sector, housing h_t (priced at q_t), land l_t, (priced at $p_{l,t}$), hours in consumption and housing sector $n_{c,t}$ and $n_{h,t}$, capital utilization rates $z_{c,t}$ and $z_{h,t}$ and borrowing b_t (loans if b_t is negative) to maximize utility function 1 subject to budget constraint 2. $\mathsf{A}_{k,t}$ is investment-specific technology shock which represents the marginal cost (in terms of consumption goods) of producing capital used in the non-housing sector. Loans are set in nominal terms and yields a riskless nominal return R_t. Real wages are denoted by $w_{c,t}$ and $w_{h,t}$, real rental rates by $R_{c,t}$ and $R_{h,t}$ and depreciation rates by δ_{kc} and δ_{kh}. The terms $X_{wc,t}$ and $X_{wh,t}$ denote mark-up between the wage paid by the wholesale firm and wage paid to the households that goes to labour unions (there is a monopolistic competition in labour market with nominal rigidity of wages). $\pi_t = P_t/P_{t-1}$ is inflation rate in the consumption sector, Div_t are lump-sum profits from final good firms and from labour unions, ϕ_t denotes convex adjustment costs for capital. $a(.)$ is the convex cost of setting capital utilization rate z (the capital utilization rate z transforms physical capital k into effective capital zk). The equations for ϕ_t, $a(.)$ and Div_t are specified in part 1.4.

Impatient households

Impatient households consume, work and accumulate housing. Their utility function is

$$E_0 \sum_{t=0}^{\infty} (\beta' G_C)^t \mathrm{z}_t \left(\Gamma_c' \ln(c_t' - \varepsilon' c_{t-1}') + \mathrm{j}_t \ln h_t' - \frac{\tau_t}{1+\eta} \left((n_{c,t}')^{1+\xi'} + (n_{h,t}')^{1+\xi'} \right)^{\frac{1+\eta'}{1+\xi'}} \right) \tag{3}$$

They do not accumulate capital and do not own finished goods firms or land (their dividends come only from labour unions). Their maximum borrowing b_t' is given by the expected present value of their home times the loan-to-value (LTV) ratio m:

$$c_t' + q_t h_t' - b_t' = \frac{w_{c,t}' n_{c,t}'}{X_{wc,t}'} + \frac{w_{h,t}' n_{h,t}'}{X_{wh,t}'} + q_t (1-\delta_h) h_{t-1}' - \frac{R_{t-1} b_{t-1}'}{\pi_t} + Div_t' \tag{4}$$

$$b'_t \leq mE_t\left(\frac{q_{t+1}h'_t\pi_{t+1}}{R_t}\right) \tag{5}$$

The discount factor or impatient households is lower than that of patient households $\beta' < \beta$. This assumption implies that for small shocks the constraint 5 hold with equality near the steady state. Impatient agents decumulate wealth quickly to some lower bound. When they accumulate housing they need to borrow the maximum possible amount against it. Along the BGP, fluctuations in housing values affect borrowing and spending capacity of constrained households through 5. This effect is larger, the larger is m.

1.2 Firms

Wholesale firms

Wholesale firms hire labour and capital services and buy intermediate goods from households to produce wholesale goods Y_t and new houses IH_t. Their optimization problem is

$$\max \frac{Y_t}{X_t} + q_t IH_t - \left(\sum_{i=c,h} w_{i,t}n_{i,t} + \sum_{i=c,h} w'_{i,t}n'_{i,t} + \sum_{i=c,h} R_{i,t}z_{i,t}k_{i,t-1} + R_{l,t}l_{t-1} + p_{b,t}k_{b,t}\right) \tag{6}$$

subject to the production functions

$$Y_t = \left(\mathsf{A}_{c,t}\left(n_{c,t}^{\alpha}n'^{\,1-\alpha}_{c,t}\right)\right)^{1-\mu_c}(z_{c,t}k_{c,t-1})^{\mu_c} \tag{7}$$

$$IH_t = \left(\mathsf{A}_{h,t}\left(n_{h,t}^{\alpha}n'^{\,1-\alpha}_{h,t}\right)\right)^{1-\mu_h-\mu_b-\mu_l}(z_{h,t}k_{h,t-1})^{\mu_h}k_{b,t}^{\mu_b}l_{t-1}^{\mu_l} \tag{8}$$

The wholesale good Y_t is produced using technology 7 with labour and capital inputs only. New houses IH_t are produced using technology 8 with labour, capital, land and the intermediate input k_b. The terms $\mathsf{A}_{c,t}$and $\mathsf{A}_{h,t}$ denotes productivity in the non-housing and housing sector. Parameter α measures labour income share of patient households.

Retailers and labour unions

There are nominal wage rigidities in both production sectors and price rigidity in the consumption goods sector. It is allowed by existence of monopolistic competition at those markets. In the retail sector, the firms have market power in price setting but the prices could be changed at some costs. This mechanism is modeled in Calvo (1983) style. Retailers buy wholesale good Y_t from wholesale firms at the price P_t^w at competitive market, differentiate them at no cost and sell the goods at a markup $X_t = P_t^w/P_t$ over marginal cost. The aggregators pack these goods into homogenous consumption and investment goods of households. Only fraction $1-\theta_\pi$ of retail firms can set price optimally, while the fraction θ_π cannot do so. They only index prices to the previous period inflation with elasticity ι_π. The optimization problem of these firms implies hybrid New Keynesian Phillips curve (in consumption-sector)

$$\ln\pi_t - \iota_\pi\ln\pi_{t-1} = \beta G_C(E_t\ln\pi_{t+1} - \iota_\pi\ln\pi_t) - \varepsilon_\pi\ln(X_t/X) + u_{p,t} \tag{9}$$

where $\varepsilon_\pi = \frac{(1-\theta_\pi)(1-\beta G_C\theta_\pi)}{\theta_\pi}$ and $u_{p,t}$ are cost shocks, which are assumed i.i.d. with zero mean and variance σ_p^2. These shocks can affect inflation independently from changes in the markup.

Wage setting is modelled in analogous way to price setting. Patient and impatient households supply homogenous labour services to labour unions. The unions differentiate labour at no cost and sell with markup to labour packers. They reassemble these services into homogenous labour composites used in the production of wholesale goods and houses. Again, the mechanism of wage setting follows Calvo scheme with partial indexation to past (price) inflation. The optimization problem of labour unions results in four wage Phillips curves for each sector/type of household where $\omega_{i,t}$ is nominal wage inflation defined as $\omega_{i,t} = \frac{w_{i,t}\pi_t}{w_{i,t-1}}$.

$$\ln\omega_{c,t} - \iota_{wc}\ln\pi_{t-1} = \beta G_C(E_t\ln\omega_{c,t+1} - \iota_{wc}\ln\pi_t) - \varepsilon_{wc}\ln(X_{wc,t}/X_{wc}) \tag{10}$$

$$\ln\omega'_{c,t} - \iota_{wc}\ln\pi_{t-1} = \beta' G_C(E_t\ln\omega'_{c,t+1} - \iota_{wc}\ln\pi_t) - \varepsilon'_{wc}\ln(X'_{wc,t}/X_{wc}) \tag{11}$$

$$\ln\omega_{h,t} - \iota_{wh}\ln\pi_{t-1} = \beta G_C(E_t\ln\omega_{h,t+1} - \iota_{wh}\ln\pi_t) - \varepsilon_{wh}\ln(X_{wh,t}/X_{wh}) \tag{12}$$

$$\ln\omega'_{h,t} - \iota_{wh}\ln\pi_{t-1} = \beta' G_C(E_t\ln\omega_{h,t+1} - \iota_{wh}\ln\pi_t) - \varepsilon'_{wh}\ln(X'_{wh,t}/X_{wh}) \tag{13}$$

where $\varepsilon_{wc} = \frac{(1-\theta_{wc})(1-\beta G_C\theta_{wc})}{\theta_{wc}}$, $\varepsilon'_{wc} = \frac{(1-\theta_{wc})(1-\beta' G_C\theta_{wc})}{\theta_{wc}}$, $\varepsilon_{wh} = \frac{(1-\theta_{wh})(1-\beta G_C\theta_{wh})}{\theta_{wh}}$, $\varepsilon'_{wh} = \frac{(1-\theta_{wh})(1-\beta' G_C\theta_{wh})}{\theta_{wh}}$.

1.3 Monetary authority

Monetary authority sets the (gross) interest rate R_t according to the monetary rule with response to past interest rate, inflation and output growth

$$R_t = R_{t-1}^{r_R} \pi_t^{1-r_R} r_\pi \left(\frac{Y_t}{G_C Y_{t-1}} \right)^{1-r_R} r_Y \overline{rr}^{1-r_R} \frac{u_{r,t}}{\mathsf{s}_t} \tag{14}$$

where $\overline{rr}$ is the steady-state real interest rate, $u_{R,t}$ is an i.i.d. monetary policy shock with variance σ_R^2, s_t is shock to inflation target which is assumed highly persistent and follows AR(1) process $\ln \mathsf{s}_t = \rho_s \ln \mathsf{s}_{t-1} + u_{s,t}$ where, $\rho_s > 0$ and $u_{s,t} \sim N(0, \sigma_s)$.

1.4 Market clearing and equilibrium condition

The goods market produces consumption and investment goods and intermediate inputs and the housing market produces new homes IH_t. The equilibrium conditions are:

$$C_t + IK_{c,t}/\mathsf{A}_{k,t} + IK_{h,t} + k_{b,t} = Y_t - \phi_t \tag{15}$$

$$H_t - (1-\delta_h) H_{t-1} = IH_t \tag{16}$$

where $C_t = c_t + c_t'$ is aggregate consumption, $H_t = h_t + h_t'$ is aggregate stock of housing, and $IK_{c,t} = k_{c,t} - (1 - \delta_{kc}) k_{c,t-1}$ and $IK_{h,t} = k_{h,t} - (1-\delta_{kh}) k_{h,t-1}$ are two components of business investment. Total land is fixed and normalized to one.

The dividents paid to households are in equlibrium equal to:

$$Div_t = \frac{X_t - 1}{X_t} Y_t + \frac{X_{wc,t} - 1}{X_{wc,t}} w_{c,t} n_{c,t} + \frac{X_{wh,t} - 1}{X_{wh,t}} w_{h,t} n_{h,t}$$

$$Div_t' = \frac{X_{wc,t}' - 1}{X_{wc,t}'} w_{c,t}' n_{c,t}' + \frac{X_{wh,t}' - 1}{X_{wh,t}'} w_{h,t}' n_{h,t}'$$

The capital adjustment cost and utilization rates are given as:

$$\phi_t = \frac{\phi_{k,c}}{2G_{IK_c}} \left(\frac{k_{c,t}}{k_{c,t-1}} - G_{IK_c} \right)^2 \frac{k_{c,t-1}}{(1+\gamma_{AK})^t} + \frac{\phi_{k,h}}{2G_{IK_h}} \left(\frac{k_{h,t}}{k_{h,t-1}} - G_{IK_h} \right)^2 k_{h,t-1}$$

$$a(z_{c,t}) = R_c \left(\varpi z_{c,t}^2 + (1-\varpi) z_{c,t} + (\varpi/2 - 1) \right)$$

$$a(z_{h,t}) = R_h \left(\varpi z_{h,t}^2 + (1-\varpi) z_{h,t} + (\varpi/2 - 1) \right)$$

where R_c an R_h are steady state values of the real rental rates of the two types of capital.

1.5 Trends and balanced growth paths

The are heterogenous productivity trends in consumption, non-housing and housing sector. These processes are:

$$\ln \mathsf{A}_{c,t} = t \ln(1+\gamma_{AC}) + \ln Z_{c,t} \qquad \ln Z_{c,t} = \rho_{AC} \ln Z_{c,t-1} + u_{C,t}$$

$$\ln \mathsf{A}_{k,t} = t \ln(1+\gamma_{AK}) + \ln Z_{k,t} \qquad \ln Z_{k,t} = \rho_{AK} \ln Z_{k,t-1} + u_{K,t}$$

$$\ln \mathsf{A}_{h,t} = t \ln(1+\gamma_{AH}) + \ln Z_{h,t} \qquad \ln Z_{h,t} = \rho_{AH} \ln Z_{h,t-1} + u_{H,t}$$

where $u_{C,t}$, $u_{H,t}$ and $u_{K,t}$ are serially uncorrelated with zero mean and standard deviations σ_{AC}, σ_{AH} and σ_{AK}. The terms γ_{AC}, γ_{AH} and γ_{AK} denote the net growth rates of technology in each sector.

1.6 Growth rates

Growth rates of the real variables along balanced growth path

$$G_C = G_{IK_h} = G_{q\times IH} = 1 + \gamma_{AC} + \frac{\mu_c}{1-\mu_c}\gamma_{AK} \tag{17}$$

$$G_{IK_c} = 1 + \gamma_{AC} + \frac{1}{1-\mu_c}\gamma_{AK} \tag{18}$$

$$G_{IH} = 1 + (\mu_h + \mu_b)\gamma_{AC} + \frac{\mu_c(\mu_h + \mu_b)}{1-\mu_c}\gamma_{AK} + (1-\mu_h-\mu_l-\mu_b)\gamma_{AH} \tag{19}$$

$$G_q = 1 + (1-\mu_h-\mu_b)\gamma_{AC} + \frac{\mu_c(1-\mu_h-\mu_b)}{1-\mu_c}\gamma_{AK} + (1-\mu_h-\mu_l-\mu_b)\gamma_{AH} \tag{20}$$

The trend growth rates of $IK_{h,t}$, $IK_{c,t}/\mathsf{A}_{k,t}$ and $q_t IH_t$ are all equal to the trend growth rate of real consumption G_C. Business investment G_{IK_c} grows faster than consumption, as long as $\gamma_{AK} > 0$. Next, the trend growth rate in real house prices G_q offsets differences in the productivity growth between the consumption, G_C, and the housing sector G_{IH}. These differences are due to the heterogenous rates of technological progress in the two sectors and to the presence of land in the production function for new homes.

The equations describing equilibrium of the model are linearized around balanced growth path before the model is taken to data.

2 Calibration

The calibrated parameter are quoted in Table 2. They were mostly set according to data in national accounts and with reference to Iacoviello and Neri (2010).

Table 1 Calibrated parameters

Parameter	Interpretation	Value
β	discount factor, parient HH	0.9957
β'	discount factor, imparient HH	0.97
j	weight on housing in U	0.12
μ_c	capital share in PF (non-housing sec.)	0.35
μ_h	capital share in PF (housing sec.)	0.1
μ_l	land share in PF (housing sec.)	0.1
μ_b	intermediate goods share in PF (housing sec.)	0.1
δ_h	depreciation rate of housing	0.01
δ_{kc}	depreciation rate of capital (non-housing sec.)	0.034
δ_{kh}	depreciation rate of capital (housing sec.)	0.05
X,X_{wc},X_{wh}	markups (C goods, wages in both sec.)	1.15
ι_p, ι_{wh},ι_{wk}	indexation parameters (C goods, wages in both sec.)	0
m	loan-to-value ratio	0.75
ρ_s	inflation target shock (persistence)	0.97

3 Data description

Empirical counterpart for following model variables were used for estimation: consumption (C_t), residential investment (IH_t), non-residential investment (IK_t), real house prices (q_t), inflation (π_t) nominal interest rate (R_t), worked hours and wage inflation in housing (NH_t, WH_t) and wholesale sector (NC_t, WC_t). Time series are quarterly, they are obtained from the Czech Statistical Office and the Czech National Bank databases and cover time period 1998:Q1 – 2013:Q2. Number of observations: 62. The data are shown in Figure 2.

- **Consumption** Final consumption expenditure of households (from Gross Domestic Product by type of expenditure), mil. CZK, constant prices 2005, seasonally adjusted from source; divided by labour force 15+ (source: labour Force Survey, Czech Statistical Office). Source: CZSO, `tab_vs_2q13.xls`.
- **Residental investment** Gross Fixed Capital Formation by Type of Capital, Dwellings, mil. CZK, constant prices 2005, seasonally adjusted by Tramo-seats (Demetra+); divided by labour force 15+. Source: CZSO, `tab_k_2q13.xls`
- **Business investment** Gross Fixed Capital Formation by Type of Capital, sum of Cultivated assets, Other machinery and equipment, Transport equipment, Other buildings and structures and Intangible fixed assets, mil. CZK, constant prices 2005ì, seasonally adjusted by Tramo-seats (Demetra+); divided by labour force 15+. Source: CZSO, `tab_k_2q13.xls`.
- **Inflation** Quarter on quarter change of Consumer Price Index, Average (2005 = 100), demeaned. Source: CZSO.
- **Nominal short-term interest rate** PRIBOR 3M, p.a., HP filtered, Source: Czech National Bank.
- **Real house prices** Price indices of apartments, average indices (2005 = 100) of realized prices of flats (offering prices of flats); deflated with Consumer Price Index. Source: CZSO `ceby11q401.xls` and tables `7009xxxx.xls`.
- **Hours in housing sector** Total Employment, hours worked in Construction sector, thousand hours, seasonally adjusted from source; divided by labour force 15+, demeaned. Source: CZSO

- **Hours in consumption goods sector** Total Employment, hours worked in all sectors exluding construction, thousand hours, seasonally adjusted from source; divided by labour force 15+, demeaned. Source: CZSO, `tab_h_2q13.xls`.
- **Wage inflation in housing sector** Quarterly change of Wages and salaries in Construction sector, current prices, mil. CZK, seasonally adjusted by Tramo-seats (Demetra+), demeaned. Source: CZSO `tab_m_2q13.xls`.
- **Wage inflation in consumption goods sector** Quarterly change of Wages and salaries in all sectors excluding construction, current prices, mil. CZK, seasonally adjusted by Tramo-seats (Demetra+), demeaned. Source:CZSO `tab_m_2q13.xls`.

For data of consumption, both type of investments and the real house prices the logarithm was taken and index with 0 in 1998:Q1 was calculated. Data for interest rate were detrended using Hodrick-Prescott filter (with $\lambda = 1600$), data for price and wage inflation and hours worked were demeaned.

4 Results of estimation

Parameters were estimated using Bayesian techniques. Posterior distribution of the parameters was obtained by Random Walk Chain Metropolis-Hastings algorithm. It was generated 1,000,000 draws in two chains with 500,000 replications each, 80 % of replications were discarded so as to avoid influence of initial conditions and calculate posterior moments from converged draws. MCMC diagnostics were used for verification of the convergence. The results of estimation is quoted in Tables 2 and 3 and in Figures 3 to 6. The estimated shocks are shown in Figure 7, Figure 8 shows multivariate diagnostics of the algorithm convergence.

Table 2 Prior and posterior distribution of structural parameters

	Prior distribution			Posterior distribution		
Parameter	Density	Mean	S.D.	Mean	2.5 %	97.5 %
Habit formation						
ε	beta	0.50	0.08	0.42	0.33	0.52
ε'	beta	0.50	0.08	0.52	0.38	0.65
Inverse labour elast.						
η	gamma	0.50	0.10	0.51	0.35	0.68
η'	gamma	0.50	0.10	0.49	0.33	0.65
Elast. of substitution of labour between sectors						
ξ	normal	1.00	0.10	1.00	1.17	0.83
ξ'	normal	1.00	0.10	1.04	1.20	0.87
Capital adjustment cost						
$\phi_{k,c}$	gamma	5.00	2.50	9.10	7.09	10.97
$\phi_{k,h}$	gamma	5.00	2.50	4.58	1.76	7.34
labour income share						
α	beta	0.65	0.05	0.72	0.65	0.80
Taylor rule						
r_R	beta	0.75	0.10	0.91	0.89	0.93
r_π	normal	1.50	0.10	1.34	1.17	1.50
r_Y	normal	0.00	0.10	0.23	0.09	0.37
Calvo parameters						
θ_π	beta	0.67	0.05	0.73	0.67	0.79
$\theta_{w,c}$	beta	0.67	0.05	0.76	0.72	0.80
$\theta_{w,h}$	beta	0.67	0.05	0.69	0.62	0.75
Capital utilization						
ζ	beta	0.50	0.20	0.88	0.78	0.98
Technology growth rates						
$100\gamma_{AC}$	normal	0.50	1	0.41	0.37	0.46
$100\gamma_{AH}$	normal	0.50	1	-0.53	-0.95	-0.09
$100\gamma_{AK}$	normal	0.50	1	0.10	0.05	0.14

Figure 2 Data for estimation

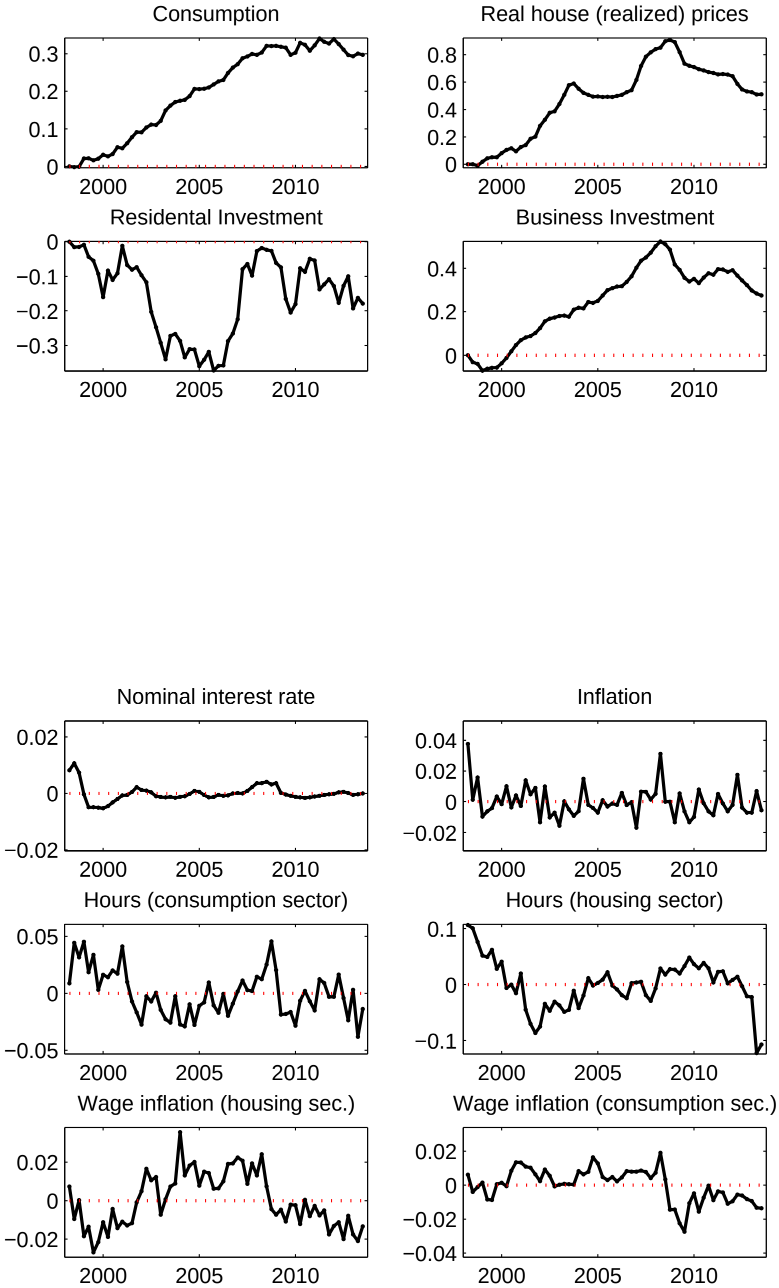


Table 3 Prior and posterior distribution of shock processes

Parameter	Prior distribution Density	Mean	S.D.	Posterior distribution Mean	2.5 %	97.5 %
Persistence of shocks						
ρ_{AC}	beta	0.80	0.10	0.9584	0.9390	0.9794
ρ_{AH}	beta	0.80	0.10	0.9942	0.9865	0.9998
ρ_{j}	beta	0.80	0.10	0.9714	0.9449	0.9968
ρ_{AK}	beta	0.80	0.10	0.8135	0.6749	0.9537
ρ_{τ}	beta	0.80	0.10	0.7826	0.6854	0.8814
ρ_{z}	beta	0.80	0.10	0.6757	0.5402	0.8141
Volatility						
σ_{AC}	inv. gamma	0.001	0.01	0.0225	0.0190	0.0260
σ_{R}	inv. gamma	0.001	0.01	0.0014	0.0011	0.0016
σ_{AH}	inv. gamma	0.001	0.01	0.0310	0.0263	0.0357
σ_{j}	inv. gamma	0.001	0.01	0.1513	0.0457	0.2617
σ_{AK}	inv. gamma	0.001	0.01	0.0010	0.0002	0.0019
σ_{p}	inv. gamma	0.001	0.01	0.0115	0.0091	0.0138
σ_{s}	inv. gamma	0.001	0.01	0.0074	0.0056	0.0093
σ_{τ}	inv. gamma	0.001	0.01	0.0444	0.0314	0.0573
σ_{z}	inv. gamma	0.001	0.01	0.0219	0.0156	0.0278
$\sigma_{n,h}$	inv. gamma	0.001	0.01	0.1289	0.1062	0.1509
$\sigma_{w,h}$	inv. gamma	0.001	0.01	0.0181	0.0141	0.0219

Figure 3 Priors and Posteriors

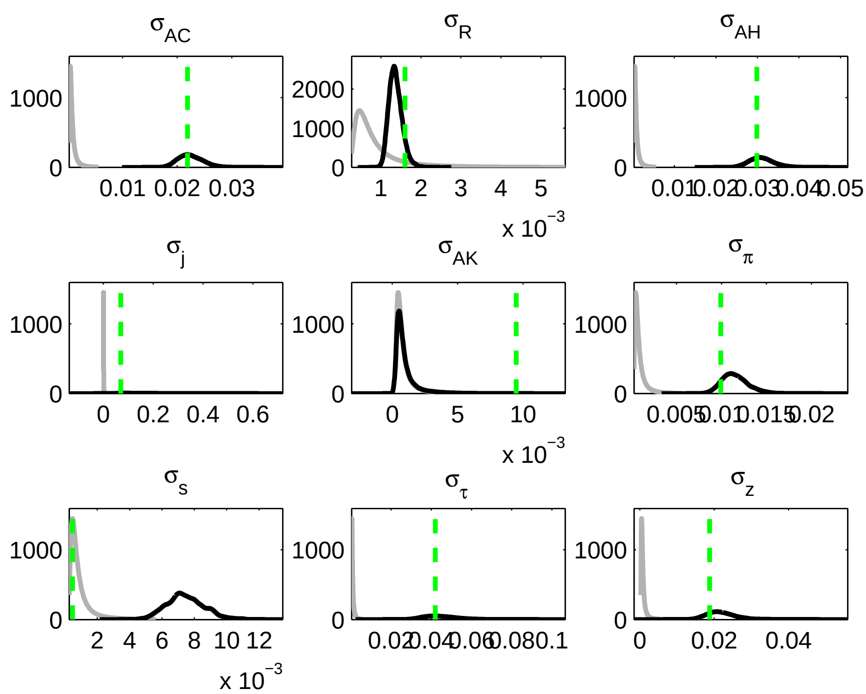


Figure 4 Priors and Posteriors

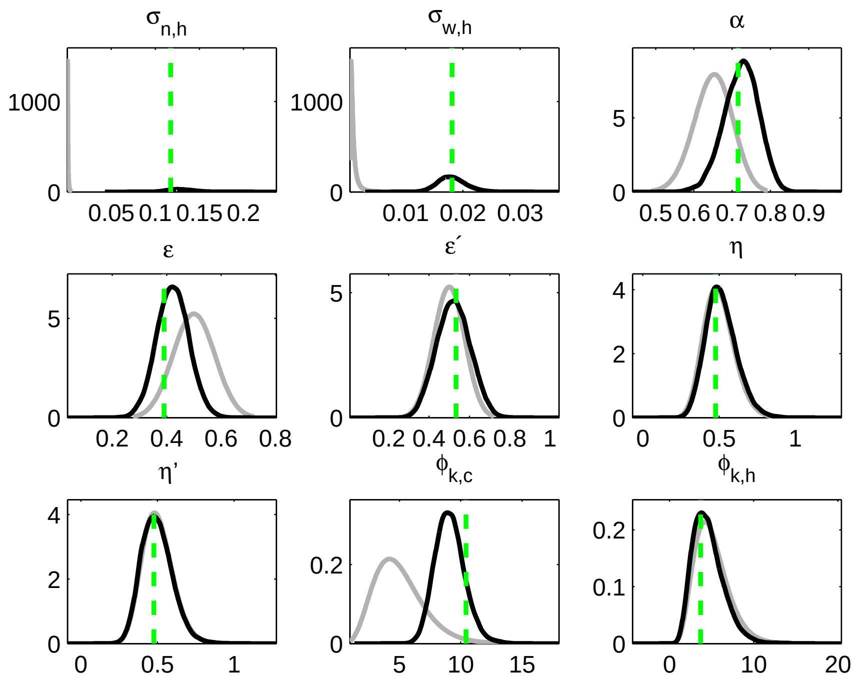


Figure 5 Priors and Posteriors

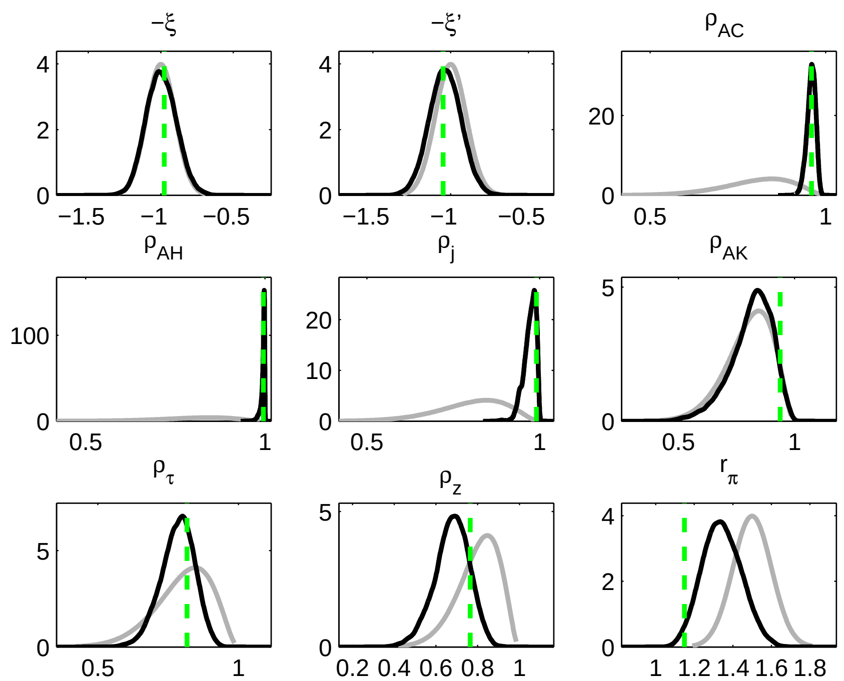


Figure 6 Priors and Posteriors

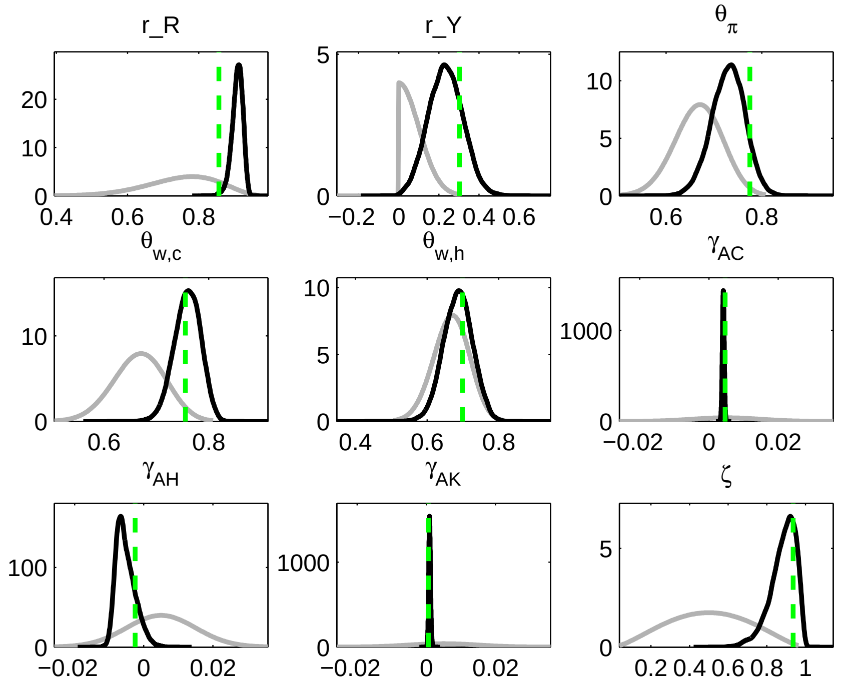


Figure 7 Smoothed shocks

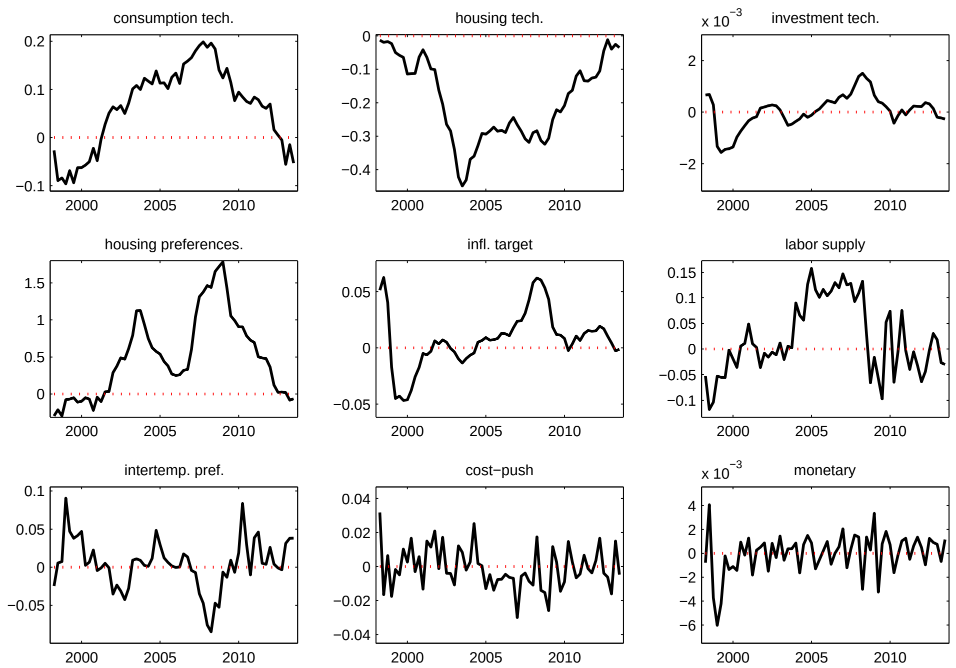


Figure 8 Multivariate convergence diagnostics

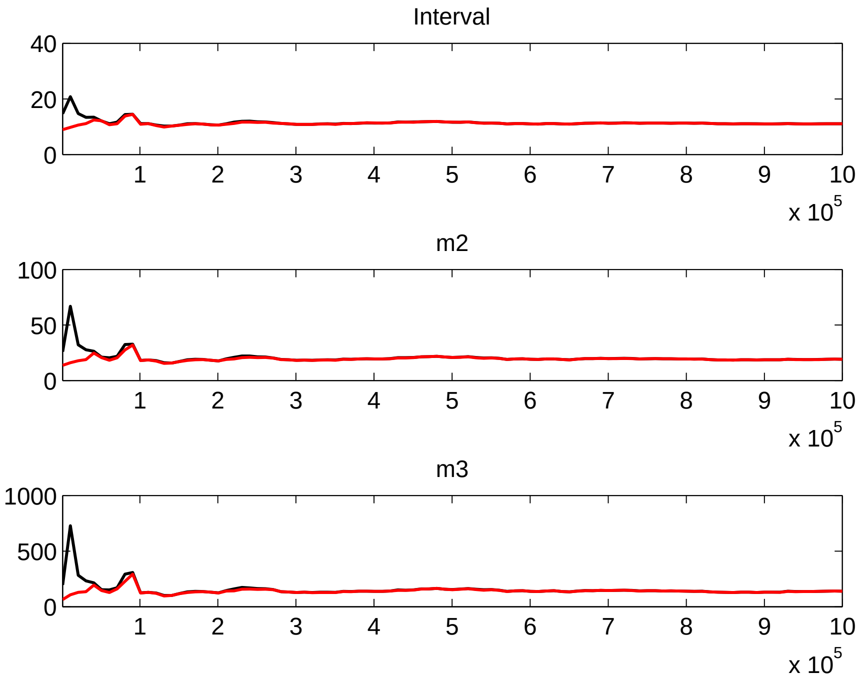
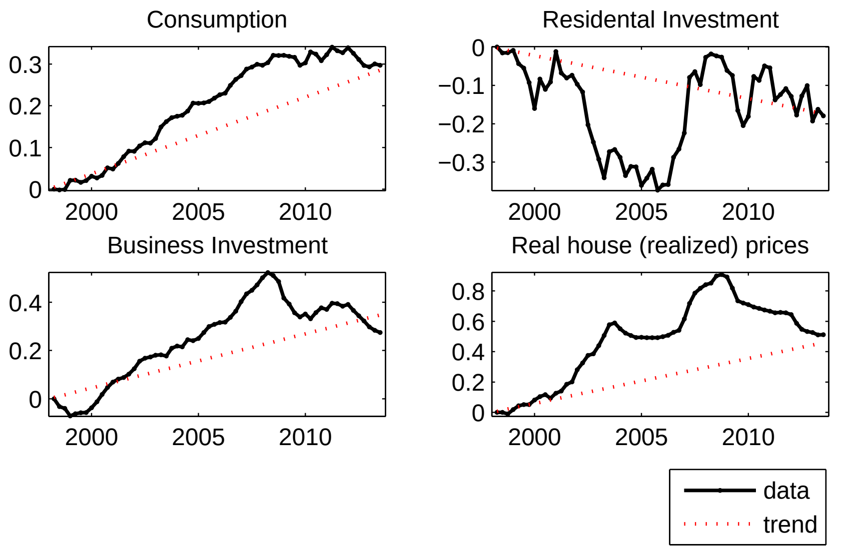


Figure 9 Data and estimated trends



5 Data fit

Figure 9 compares estimated trends with data of consumption, both types of investment and real house prices. Table 4 shows moments calculated from the data, and moments obtained from model simulations (with 90% probability intervals). The simulation is performed for the vector of the parameters at their posterior mean. Confidence bands were obtained by performing one thousand simulations of an artificial time series of the same length as the data. The model variable Y denotes private domestic demand (also called output), which consists of consumption, business and residential investment. The same definition is used for the data. Time series for the real variables were linearly de-trended so that they correspond to the treatment of the variables in the model.

The empirical performance of the model appears acceptable. The volatility of the variables is matched quite precisely; volatilities of all variables, with the exception of house prices, fall within the probability bands. The model under-predicts the volatility of real house prices q, which is understandable because booms and busts in house prices were unusual during recent years.

As far as autocorrelations are concerned,[1] the model has difficulty matching the autocorrelation of business investment IK and inflation π, although other autocorrelation values are within probability bands. When it comes to the correlations, the data fit is quite satisfactory. The model correctly predicts positive correlations between house price and output, and consumption (q,Y and q,C) but the correlation coefficient is lower than that in the data. The model also produces a positive correlation between residential investment and output (IH,Y) and residential investment and house prices (IH,q) but the correlation in the data is virtually zero or negative. However, given the parameter uncertainty, these moments come within the probability intervals.

Table 4 Moments from data and model

	Model			Data
	Mean	5 %	95 %	
Volatiliy (std)				
C	4.48	2.77	7.15	3.49
IH	14.45	10.07	20.65	11.24
IK	8.77	6.62	12.75	9.06
q	8.31	4.76	12.45	15.67
π	1.22	1.00	1.50	1.00
R	0.36	0.20	0.58	0.28
Y	5.54	3.49	7.97	4.73
Autocorrelations				
C	0.90	0.83	0.96	0.90
IH	0.71	0.43	0.86	0.90
IK	0.80	0.72	0.90	0.93
q	0.86	0.69	0.94	0.93
π	0.28	0.02	0.58	-0.03
R	0.87	0.73	0.96	0.79
Y	0.85	0.76	0.92	0.92
Correlations				
C,Y	0.92	0.82	0.97	0.96
IH,Y	0.41	-0.22	0.78	-0.01
IK,Y	0.95	0.90	0.98	0.97
q,Y	0.43	-0.41	0.87	0.90
q,C	0.33	-0.59	0.85	0.90
q,IH	0.37	-0.30	0.87	-0.16
q,π	-0.01	-0.38	0.38	-0.05

[1] Autocorrelations of data are obtained from estimation of the VAR(1) model.

6 Variance decomposition

Table 5 shows conditional variance of the model variables explained by each shock for a one-, eight- and twenty-quarter forecast horizon, and an unconditional forecast error variance decomposition. This table provides numerical base to Figure 6 in the paper.

Table 5 Conditional variance decomposition (1, 8 and 20 quarters) and unconditional variance decomposition

	cons. tech.	monet.	housing tech.	housing pref.	invest. tech.	cost-push	infl. targ.	labor supply	inter-temp.
1 q									
C	21.5	17.5	0.4	0.2	0.0	9.4	11.8	2.2	37.0
IH	1.3	7.1	31.6	50.8	0.0	0.5	4.5	3.7	0.5
IK	13.3	34.0	0.1	0.3	0.3	28.2	22.6	1.2	0.1
q	6.8	7.3	6.9	68.0	0.0	5.9	4.6	0.4	0.2
π	10.5	9.7	0.1	0.1	0.0	62.2	13.5	2.1	1.9
R	2.1	35.8	0.1	1.6	0.0	33.4	22.2	0.5	4.1
Y	16.4	30.6	2.2	5.9	0.1	18.4	20.3	2.8	3.3
8 q									
C	59.3	9.9	0.2	0.1	0.0	4.0	7.2	7.5	11.7
IH	0.9	2.9	46.3	39.0	0.0	0.2	1.8	8.1	0.7
IK	50.9	15.5	0.5	0.2	0.2	11.5	10.6	8.4	2.3
q	13.5	2.0	8.6	71.0	0.0	1.5	1.3	1.5	0.6
π	10.1	13.1	0.1	0.1	0.0	45.7	26.2	2.8	2.0
R	11.8	8.3	0.1	1.1	0.0	7.1	62.2	4.4	5.1
Y	53.8	12.9	2.4	2.9	0.1	6.7	9.0	10.9	1.4
20 q									
C	74.9	5.7	0.1	0.1	0.0	2.3	4.3	6.2	6.5
IH	0.8	1.7	56.2	34.5	0.0	0.1	1.1	5.1	0.4
IK	63.1	11.2	0.6	0.2	0.2	8.2	7.6	7.2	1.8
q	15.3	1.1	12.4	68.4	0.0	0.8	0.7	1.0	0.3
π	9.8	12.4	0.1	0.1	0.0	43.2	29.7	2.7	1.9
R	15.5	4.3	0.0	0.7	0.0	3.7	69.6	3.3	2.8
Y	66.8	8.0	3.4	2.6	0.0	4.1	5.6	8.6	1.0
∞									
C	82.9	3.8	0.1	0.1	0.0	1.5	2.9	4.4	4.3
IH	0.6	0.7	76.7	19.2	0.0	0.1	0.5	2.2	0.2
IK	68.0	9.6	0.6	0.3	0.1	7.1	6.6	6.2	1.5
q	8.5	0.4	58.4	31.7	0.0	0.3	0.2	0.4	0.1
π	9.9	11.8	0.1	0.1	0.0	41.2	32.5	2.6	1.8
R	21.5	2.9	0.1	0.5	0.0	2.5	68.1	2.4	1.9
Y	70.5	5.7	7.3	2.5	0.0	2.9	4.1	6.2	0.7