Progressivity of Taxes, Skeweness of Income Distribution and Violations of the Progressive Principle in Income Tax Systems

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Abstract

Kakwani and Lambert state the three axioms, which should be respected by an equitable tax system. They also proposed a measurement system to evaluate the violations of the axioms. One of the axioms, axiom 2, formulates the progression principle in income tax systems. Vernizzi and Pellegrino improved the alternative index to evaluate violations concerning the progressive command in a tax system. The main aim of this paper is to compare the two indexes in order to evaluate violations of progressive principle in income tax system using the real data. We also check how the progressivity of taxes and skewness of income distribution affect the measurement of the progressive principle violation.

Keywords	***JEL code***
Personal income tax, progressive principle, redistributive effect, progressive of taxes, equitable tax system	*C81, H23, H24*

INTRODUCTION

Many authors define equity in income taxation by horizontal and vertical equity [Urban, Lambert 2008]. In this paper the equity in income taxation is defined by means of three axioms, introduced by Kakwani and Lambert in 1998. Tax system is equitable if all axioms are satisfied. Violation of them – by a personal income tax system – produces negative influence on the redistributive effect of the tax. This negative influence provides the means to characterize the type of inequity present in a tax system.

The three general rules requirement for the personal income tax system are named axioms by Kakwani and Lambert. As an axiom is defined as a mathematical statement that is accepted as being true without a mathematical proof (it is a logical statement that is assumed to be true), we propose to name

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these postulates as rules. Despite their arbitrary character, tax systems that violate them intentionally are very rare. Practical solutions in personal income tax systems are not, however, so clear. Tax deduction and exemptions – commonly used tax instruments – often cause violation of these rules.

Let $x_1, x_2, ..., x_n$ mean pre-tax income of n income units, who are paying $t_1, t_2, ..., t_n$ in tax. We can write X as a vector of $x_1, x_2, ..., x_n$ and T as a vector of $t_1, t_2, ..., t_n$. In our analysis, household is set as an income unit, so:

x_1 will denote pre-tax income of household i and
t_1 tax payment of household *i*.

In this notation $y_i = x_1 - t_1$ denotes post-tax income of household *i* and $a_i = \frac{t_i}{x_i}$ - tax rate for household *i*.

The first rule – **Rule 1** – says that tax duty should increase monotonically with respect to taxpayers' ability to pay. This rule they written as:

$$x_i \geq x_j \Rightarrow t_i \geq t_j. \tag{1}$$

Because the inequalities are weak, postulate of "equal treatment of equals" could be treated as a special case of this rule. It also enables government to exempt taxpayers with the lowest incomes from having to pay tax. This rule is named *minimal progression principle*.

According to **Rule 2**, the richer people must pay taxes at higher rates. Of course, a violation of minimal progression automatically entails a violation of this principle. The weak inequalities in rule 2 mean that proportional taxation is permitted.

This second rule – *progression principle* – is defined in the following way:

$$x_i \geq x_j \text{ and } t_i \geq t_j \Rightarrow a_i \geq a_j. \tag{2}$$

If tax system is ruled out by principles 1 and 2 taken together then it means existence of regression in the tax system.

The last rule – **Rule 3** – says that a tax, which satisfies the other two rules, should cause no reranking in taxpayers' post-tax income. This rule is called no-reranking criterion and can be written as:

$$x_i \geq x_j \text{ and } t_i \geq t_j \textit{ and } a_i \geq a_j \Rightarrow x_i - t_i \geq x_j - t_j. \tag{3}$$

The Rule 3 can be seen as a vertical restriction, ruling out "too much" progression.

1 VIOLATION OF THE PROGRESSIVE PRINCIPLE

The most important for this paper is the second rule, the *progression principle* [Lambert 2001]. Violations of progression principle (also the others of rules) produces negative influence on the redistributive effect of the tax. In this context we should to be able to assess when the progression principle is not upheld and how much lost the redistributive effect produces.

The redistributive effect is defined as difference between the Gini index for pre-tax income and the Gini index for post-tax income [Lambert 2001] could be decomposed into following way [Kakwani and Lambert 1998]:

$$RE = G_X - G_{X-T} = V - S_1 - S_2 - S_3 \tag{4}$$

where: S_1 – measures loss in redistributive effect, caused by a violation of rule 1,
S_2 – loss in redistributive effect, caused by a violation of rule 2,
S_3 – loss in redistributive effect, caused by a violation of rule 3,
V – value of redistributive effect that might be achieved if all rules are upheld.

The measures in decomposition (4) are defined by Gini and concentration index.

Let $C_{Z,X}$ denotes concentration index for attribute Z. This measure is calculated in the same way as Gini index, but vector values of Z is ordered by incomes before taxation (X). If both orderings are identical (attribute Z causes no reranking of income), Gini and concentration indexes calculated for the same vector of incomes take the same value.

Whereas:

$$V = \tau \cdot \left(\left(C_{T,X} - G_X \right) + \left(G_{\frac{T}{X}} - C_{\frac{T}{X},X} \right) \right),$$

$$S_1 = \tau \cdot \left(G_T - C_{T,X} \right),$$

$$S_2 = \tau \cdot \left(\left(G_{\frac{T}{X}} - C_{\frac{T}{X},X} \right) - \left(G_T - C_{T,X} \right) \right)$$

and

$$S_3 = G_{X-T} - C_{X-T,X},$$

$$\tau = \frac{\sum_{i=1}^{n} t_i}{\sum_{i=1}^{n} (x_i - t_i)}.$$

S_2 always takes non-negative values and according Kakwani-Lambert methodology:

The progression principle is violated

$$\Updownarrow$$

$$S_2 > 0$$

If S_2 is zero, the progression principle is upheld.

Violation of rule 1 about minimal progression automatically entails a violation of the progressive principle (rule 2). It means that income unit pairs (i,j) for which rule 1 fails cannot provide violations of the progressive principle.

In next section we check how S_2 measures violation of progression principle for real data.

2 EMPIRICAL ANALYSIS

The values of measure S_2 as a measure of violation of progression principle was analyzed on the basis of Polish data from Wrocław-Fabryczna tax office for fiscal year 2007. This set of data contains information on income and tax paid for taxpayers that file their tax return in the Municipality of Wrocław, tax office (district identification) Fabryczna. In this analysis households are equated with couples of taxpayers who take advantage of joint taxation and filled up the formulate PIT 37. The analyses were performed by author'own programmes, written in the "*R*" language.

Population of 19 487 households was divided into subpopulations with respect to the number of dependent children. We created 4 sets: family without children, family with one child, family with two children, family with three or more children.

Table 1 presents measures S_2 for each type of family and for pairs of units income which satisfied or not rule 1. When rule 1 is violated and rule 2 is upheld, the measure of loss in RE due to violation rule to should be equal 0. It is not true for S_2.

Table 1 Values of the measure S_2 for fourth type of family

Parameter	Rule 1 violated	Rule 1 upheld	Total
family without children			
S_2	0.001379	0.001717	0.003096
family with one child			
S_2	0.001213	0.000729	0.001942
family with two children			
S_2	0.001174	0.000443	0.001617
family with three or more children			
S_2	0.001582	0.00021	0.001792

Source: Own calculations

For each group of taxpayers we observe that the measures S_2 are greater than 0 for pairs of units (i,j) for which rule 1 is violated. According to Kakwani and Lambert methodology it means that, progression principle is violated. If we look at the mathematical record of the rule 2 (see formula (2)) it could be observed that for pairs of units (i,j) for which rule 1 is violated the rule 2 is not violated and the measure S_2 could be zero. If we want to use this measure S_2 we should firstly eliminate from set of data the pairs of units (i,j) for which rule 1 is violated and next calculate measure S_2. Elimination from data the pairs of units (i,j) for which rule 1 is violated is not a simple the task.

Pellegrino and Vernizzi (2013) introduced the correction of the measure of loss in redistributive effect, caused by a violation of rule 2 – S_2^* – which can be used for full set of data. The measures is defined as follows:

$$S_2^* = \frac{\tau}{2n^2} \cdot \sum_{i=1}^{k} \sum_{j=1}^{k} \frac{a_i - a_j}{\mu_A} \cdot \left[\left(I_{i-j}^{A} - I_{i-j}^{A/X} \right) - \left(I_{i-j}^{T} - I_{i-j}^{T/X} \right) \right] \cdot p_i p_j , \tag{5}$$

where:
n is a sample size, $a_i = \frac{t_i}{x_i}$, p_i, p_j are weights associated to a_i and a_j, $\sum_{i=1}^{k} p_i = n$, μ_A is the average of a_i, $i=1,\ldots k$. I_{i-j}^{Z}, $I_{i-j}^{Z/X}$ are indicator function for attribute Z:

$$I_{i-j}^{Z} = \begin{cases} 1 & z_i \geq z_j \\ -1 & z_i < z_j \end{cases} \qquad I_{i-j}^{Z/X} = \begin{cases} 1 & \text{if} \quad x_i > x_j \\ -1 & \text{if} \quad x_i < x_j \\ I_{i-j}^{Z} & \text{if} \quad x_i = x_j \end{cases} .$$

Table 2 presents values of the measure S_2^* for analyzed sets of data.

Table 2 The measure S_2 for each type of family

Parameter	Rule 1 violated	Rule 1 upheld	Total
family without children			
S_2^*	0	0.001717	0.001717
family with one child			
S_2^*	0	0.000729	0.000729
family with two children			
S_2^*	0	0.000443	0.000443
family with three or more children			
S_2^*	0	0.00021	0.00021

Source: Own calculations

We can observe that if $S_2^* = 0$, it not necessarily true that S_2. The measure S_2^* is demonstrating appropriate behaviors for each of analysed data sets. In every case of pairs of units (i,j) for which rule 1 is violated the value measure S_2^* is zero correctly. It proves that S_2^* could be better measure for lost of redistributive effect due to violation of progressive principle. Table 3 presents the results of decomposition of RE according formula (4) for four Polish data sets. For families without children, the personal income tax system reduces the inequality of income by 1.6 percentage points. Losses in this redistributive effect due to violation of rule 1, 2 and 3 are 0.4 percentage points according to KL methodology or 0.3 percentage points according to VP methodology. The difference appears as a result of difference between estimation of the loss of redistributive effect due to violation of Rule 2 according to KL and VP. The inequity, resulting from violation of Rule 2, reduces overall redistributive effect by 0.31 (according to KL) percentage points which is 19.13 % of *RE* or 0.17 (according to VP) percentage points which is only 10.80 % of RE.

Table 3 RE decomposition for taxpayers divided into subpopulations with respect to the number of dependent children

family without children

	Gini for pre-tax income	Gini for post-tax income	RE	Potential equity	Rule 1	Rule 2	Rule 3	Total Rules
Kakwani and Lambert	0.371782	0.355399	0.016382	0.020589	0.000951	0.003134	0.000121	0.004206
	percentage of RE (%):		100.00	125.68	5.80	19.13	0.74	25.68
Vernizzi and Pellegrino	0.371782	0.355399	0.016382	0.019195	0.000951	0.001740	0.000121	0.002812
	percentage of RE (%):		100.00	119.19	5.92	10.80	0.75	17.46

family with 1 child

	Gini for pre-tax income	Gini for post-tax income	RE	Potential equity	Rule 1	Rule 2	Rule 3	Total Rules
Kakwani and Lambert	0.346474	0.323702	0.022772	0.025682	0.000854	0.001942	0.000115	0.00291
	percentage of RE (%):		100.00	112.78	3.75	8.53	0.50	12.78
Vernizzi and Pellegrino	0.346474	0.323702	0.022772	0.02447	0.000854	0.000729	0.000115	0.001698
	percentage of RE (%):		100.00	107.45	3.75	3.20	0.50	7.45

family with two children

	Gini for pre-tax income	Gini for post-tax income	RE	Potential equity	Rule 1	Rule 2	Rule 3	Total Rules
Kakwani and Lambert	0.346507	0.318711	0.027796	0.030329	0.000797	0.001617	0.000119	0.002533
	percentage of RE (%):		100.00	109.11	2.87	5.82	0.43	9.11
Vernizzi and Pellegrino	0.346507	0.318711	0.027796	0.029155	0.000797	0.000443	0.000119	0.001359
	percentage of RE (%):		100.00	104.89	2.87	1.59	0.43	4.89

family with three or more children

	Gini for pre-tax income	Gini for post-tax income	RE	Potential equity	Rule 1	Rule 2	Rule 3	Total Rules
Kakwani and Lambert	0.387007	0.353281	0.033726	0.036399	0.000782	0.001792	9.95E-05	0.002673
	percentage of RE (%):		100.00	107.93	2.32	5.31	0.29	7.93
Vernizzi and Pellegrino	0.387007	0.353281	0.033726	0.034817	0.000782	0.00021	9.95E-05	0.001091
	percentage of RE (%):		100.00	103.23	2.32	0.62	0.29	3.23

Source: Own calculations

It is almost twofold increase for KL methodology in comparison with VP methodology. The difference is so big that it is worth doing an investigation of why $S_2 - S_2^* > 0$and conditions when S_2 can be a reasonable approximation of S_2^*. The second aspect of this problem is the fact that value of S_2 influences on the potential redistributive effect. It is important because potential redistributive effect informs us, what is worth mentioning, how removal of inequities due to violation of rules could potentially improve the redistributive effect of taxation without increasing the marginal tax rates for the taxpayers groups.

The total inequity in the Polish tax system reduces the redistributive effect of taxation for group of family without children by 0.42 percentage points (according to KL) or by 0.28 percentage points (according to VP). These results suggest that the absence of all mentioned inequities could reduce the inequality of income by 2.06 percentage points or by 1.91 percentage points (instead of 1.64 percentage points).

For the group of taxpayers with one, two or three or children we observe similar connection between S_2.and S_2^*. Table 4 presents differences between this two values for each type of family, which are always greater than 0. The differences are from 4.22 to 8.51 points of percentage of *RE* for different type of families.

Table 4 The differences between S_2 and S_2^* for each type of family

Type of family	S_2	S_2^*	$S_2 - S_2^*$	$S_2 - S_2^*$ as percentage of RE (%)
0 children	0.003134	0.00174	0.001394	8.51
1 child	0.001942	0.000729	0.001213	5.33
2 children	0.001617	0.000443	0.001174	4.22
3 or more children	0.001792	0.00021	0.001582	4.69

Source: Own calculations

Where do the differences come from? We are looking for conditions when we can use S_2 as good approximation of S_2^*. We can give some thought if difference $S_2 - S_2^*$ depends on tax progressivity or on the skweness of income distribution. Below table presents $S_2 - S_2^*$ and the measure of tax progressivity defined by Kakwani (1977) as a difference between the concentration index of taxes and the Gini index of the pre-tax income.

$$\Pi^K = D_T - G_X. \tag{6}$$

Values of this measure are included in a range: $\Pi^K \in [-1 - G_X, 1 - G_X]$. Positive values, $\Pi^K > 0$, mean the progressive tax system. For the proportional system we receive: $\Pi^K = 0$. The negative values $\Pi^K < 0$ are describing the regressive tax system. The measure Π^K could be interpreted as the percent of total fiscal charges which remained changed from worse earning to the better earning for the effect of the progressions of tax system.

Table 5 The name and way of create data sets for different types of skwenesses

name of set	description
0 children 80%	contains 80% taxpayers with the lowest income from set 0 children
0 children 90%	contains 90% taxpayers with the lowest income from set 0 children
0 children 95%	contains 95% taxpayers with the lowest income from set 0 children
0 children 97%	contains 97% taxpayers with the lowest income from set 0 children
0 children 99%	contains 99% taxpayers with the lowest income from set 0 children
0 children 100%	taxpayers with 0 dependent children

Source: Own presentation

We can give also some thought if the difference $S_2 - S_2^*$ depends on skweness of income distribution. In order to do that we created data sets by cutting down the origin sets. In this way we created the following sets presented in Table 5.

In the same way we created the sets of taxpayers with 1, 2 or 3 dependent children. Table 6 presents results of analysis from each data sets. We calculated apart from differences $S_2 - S_2^*$, influence the differences on redistributive effect – $\frac{S_2 - S_2^*}{RE}$ as well as the skewness of income distribution and progressivity index.

Table 6 Results of analysis for created data sets

Data set	$S_2 - S_2^*$	RE	$\frac{S_2 - S_2^*}{RE}$	Skweness	Π^K
0 children 80%	0.00112	0.00693	0.1618	0.14	0.089936
0 children 90%	0.00109	0.00700	0.1562	0.44	0.086222
0 children 95%	0.00111	0.00842	0.1321	0.74	0.097939
0 children 97%	0.00114	0.00966	0.1183	0.96	0.108245
0 children 99%	0.00122	0.01202	0.1016	1.48	0.126605
0 children 100%	0.00139	0.01638	0.0851	3.82	0.157335
1 child 80%	0.000853	0.009299	0.0918	0.06	0.163209
1 child 90%	0.000871	0.011029	0.0790	0.35	0.171039
1 child 95%	0.000910	0.013396	0.0679	0.66	0.188018
1 child 97%	0.000950	0.014993	0.0634	0.90	0.198983
1 child 99%	0.001041	0.018037	0.0577	1.44	0.219137
1 child 100%	0.001212	0.022772	0.0532	3.77	0.247409
2 children 80%	0.000713	0.010824	0.0659	0.03	0.240979
2 children 90%	0.000763	0.013927	0.0548	0.38	0.252356
2 children 95%	0.000815	0.016688	0.0488	0.69	0.264156
2 children 97%	0.000856	0.018205	0.0470	0.89	0.270102
2 children 99%	0.000960	0.021927	0.0438	1.46	0.289748
2 children 100%	0.001174	0.027796	0.0422	8.23	0.317156
3 children 80%	0.000580	0.008092	0.0716	0.12	0.362515
3 children 90%	0.000738	0.012809	0.0576	0.61	0.377318
3 children 95%	0.000839	0.016909	0.0496	0.9	0.383747
3 children 97%	0.000924	0.019023	0.0486	1.23	0.384923
3 children 99%	0.001122	0.024289	0.0462	1.89	0.399589
3 children 100%	0.001582	0.033726	0.0469	6.64	0.419609

Source: Own calculations

We can observe that for the lowest tax progressivity we have the biggest value of difference $S_2 - S_2^*$ for each group. Consistently for the highest tax progressivity we have the smallest value of difference $S_2 - S_2^*$. Generally, the higher the skewness is – the higher the difference between S_2 and S_2^*. Only for *0 children 90%* set we have lowest difference $S_2 - S_2^*$ and highest skweness in compare with 0 children 80%. On the other side if we are looking for conditions when we can use S_2 as good approximation of S_2^* we should analyze influence the $S_2 - S_2^*$ on redistribution effect. 4th column in Table 6 presents this influence. We observe that the higher the skewness is, the lower the influence of the $S_2 - S_2^*$ difference on redistribution effect. This relation we observe also for *0 children 90%* set.

CONCLUSIONS

We presented and compared two measures of violations of progressivity principle: S_2 and S_2^*. We carried out an investigation for different income distribution and one tax system. We tried to understand the difference between these indexes and conditions when S_2 could be a reasonable approximation of S_2^*.

If we want to only check if progression principle is upheld or not we can use both methods: original Kakwani and Lambert or modified by Vernizzi and Pellegrino. If we want to assess the loss in the redistributive effect, caused by a violation of progression principle we should use recast index S_2^*.

We observe that the higher the skewness of income distribution is, the lower influence difference $S_2 - S_2^*$ on the redistributive effect. We observe similar simple correlation between the tax progressivity index and influence difference on the redistribution effect.

References

KAKWANI, N. C. Measurement of tax progressivity: an international comparison. *Economic Journal of Political Economy*, 1977, 87, pp. 71–80.

KAKWANI, N. C., LAMBERT, P. J. On measuring inequity in taxation: A new approach. *European Journal of Political Economy*, 1998, Vol. 143, pp. 369–380.

LAMBERT, P. J. *The Distribution and Redistribution of Income.* Manchester and New York: Manchester University Press, 2001.

LAMBERT, P. J., ARONSON, J. R. Decomposition Analysis and the Gini Coefficient Revisited. *The Economic Journal*, 1993, Vol. 103, No. 420, pp. 1221–1227.

PELLEGRINO, S., VERNIZZI, A. On Measuring Violations of the Progressive Principle in Income Tax Systems. *Empirical Economics*, 2013, 45, pp. 239–245.

URBAN, I., LAMBERT, P. J. Redistribution, horizontal inequity and reranking: how to measure them properly. *Public Finance Review,* 2008, 36(5), pp. 563–587.