Application of Stochastic Index Numbers in Inflation Measurement – the Case of Poland

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Abstract

The stochastic approach is a specific way of viewing index numbers, in which uncertainty and statistical properties play a central role. This approach, applied to the prices, treats the underlying rate of inflation as an unknown parameter that has to be estimated from the individual prices. Thus, the stochastic approach provides the whole probability distribution of inflation. In this paper we present and discuss several basic stochastic index numbers. We propose a simple stochastic model, which leads to a price index formula being a mixture of the previously presented specifications. We verify the considered indices on a real data set.

Keywords	***JEL code***
Price indices, stochastic index numbers, price index theory	*E17, E21, E30*

INTRODUCTION

The weighted price index is a function of a set of prices and quantites of the considered group of N commodities comming from the given moment t and the basic moment s. In reality, the price index formula is a quotient of some random variables and thus, it is really difficult to construct a confidence interval for that formula. The so called new stochastic approach (NSA) in the price index theory gives a solution for the above-mentioned problem. Within this approach, a price index is a regression coefficient (unknown parameter[2] θ) in a model explaining price variation. Having estimated sampling variance of the estimator ($\hat{\sigma}_{\theta}^{2}$) we can build the $(1-\alpha)$ confidence interval[3] as $\hat{\theta} \pm t_{1-\alpha/2,n-1}\hat{\sigma}_{\theta}$, where n is the sample size and $t_{1-\alpha/2,n-1}$ is the $100(1-\alpha/2)$ percentile of the t distribution with $n-1$ degrees of freedom (see von der Lippe (2007)). The individual prices are observed with error and the problem is a signal-extraction one of how to combine noisy prices so as to minimize the effects of measurement errors. Under certain assumptions, the stochastic approach leads to known price index formulas (such as Divisa, Laspees, etc.), but their foundations differ from the classical deterministic approach. Within this approach we can also

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2 In the next part of the paper we consider only the least squares and maximum likelihood estimator for θ.
3 We can build this confidence interval under the additional assumption that the residuals are normally distributed.

obtain some new price index formulas having some desired economical and statistical properties (Clements et al. (2006)).

The stochastic approach originated in the work of Jevons (1863, 1869) and Edgeworth (1887, 1888, 1889). Aldrich (1992) attributes the introduction of the term "stochastic" in this context to Frisch (1936), and it was adopted by Allen (1975), to describe Edeworth's analysis. The recent resurrection of the stochastic approach to index number theory is due to Balk (1980), Clements and Izan (1981, 1987), Bryan and Cecchetti (1993) and Selvanathan and Prasada Rao (1992). This literature is still expanding and has been the subject of a book by Selvanathan and Prasada Rao (1994), who emphasise the versatility and the usefulness of the stochastic approach. Although some papers have critical tone (see for example Diewert (1995)), some other and more recent papers extend this approach in new directions (see Diewert (2004, 2005), Prasada Rao (2004)). In this paper we present and discuss only some basic stochastic index numbers. We propose a simple stochastic model, which leads to a price index formula being a mixture of the previously presented specifications.

1 STOCHASTIC INDEX NUMBERS IN INFLATION MEASUREMENT

The main attraction of the stochastic approach over competing approaches to the index number theory is its ability to provide confidence intervals for the estimated inflation rates:

"Accordingly, we obtain a point estimate of not only the rate of inflation, but also its sampling variance. The source of the sampling error is the dispersion of relative prices from their trend rates of change -- the sampling variance will be larger when the deviations of the relative prices from their trend rates of change are larger. This attractive result provides a formal link between the measurement of inflation and changes in relative prices." (Clements and Izan (1987), p. 339)

There are many directions and stochastic models in the field of the stochastic approach. To make the exposition of stochastic index numbers as clear as possible, we concentrate on the simplest possible cases. Let $Dp_{i,t} = \ln p_{i,t} - \ln p_{i,t-1}$ be the log-change in price of commodity i ($i = 1,2,..., N$) from year $t - 1$ to t. Suppose that each price change is made up of a systematic part that is common to all prices (θ_t) and a random component $\varepsilon_{i,t}$,

$$Dp_{i,t} = \theta_t + \varepsilon_{i,t} , \tag{1}$$

where we assume that $E(\varepsilon_{i,t}) = 0$ and thus $E(Dp_{i,t}) = \theta_t$. We can see that the parameter θ_t is interpreted here as the common trend in all prices, or the underlying rate of inflation. Let all $\varepsilon_{i,t}$ have variances and covariances of the form $\hat{\sigma}^2_{ij,t}$ and let $\Sigma_t = [\hat{\sigma}^2_{ij,t}]$ be the corresponding $N \times N$ covariance matrix. Under above significations we can write (1) in vector form as:

$$Dp_t = \theta_t u + \varepsilon_t , \tag{2}$$

where $Dp_t = [Dp_{i,t}]'$, $u = [1,...,1]'$, $\varepsilon_t = [\varepsilon_{i,t}]'$ are all $N \times 1$ vectors.

Using the generalized least squares method for estimating θ_t we obtain the BLUE estimator as follows (see Clements et al. (2006)):

$$\hat{\theta}_t = (u'\Sigma_t^{-1}u)^{-1}u'\Sigma_t^{-1}Dp_t, \tag{3}$$

with variation:

$$\hat{\sigma}^2_{\theta_t} = (u'\Sigma_t^{-1}u)^{-1}. \tag{4}$$

The presented formulas (3) and (4) have a general form and in the remaining part of the paper we consider some special cases of this model. Let us notice that $\varepsilon_{i,t}$ is interpreted as the change in the i – th relative price. Let us suppose that $\varepsilon_{i,t}$ and $\varepsilon_{j,t}$ are independent (for $i \neq j$) and

$$\sigma^2_{ii,t} = \frac{\lambda^2_t}{w_{i,t}}, \tag{5}$$

where λ_t is a parameter independent of i and $w_{i,t}$ is the i – th budget share, with $q_{i,t}$ the quantity consumed of i -th good in the year t, namely:

$$w_{i,t} = \frac{p_{i,t}q_{i,t}}{\sum_{i=1}^{N} p_{i,t}q_{i,t}}. \tag{6}$$

The assumption (5) means that the variance of $\varepsilon_{i,t}$ is inversely proportional to the corresponding budget share $w_{i,t}$. There are several justifications for the specification (5). One of them (see Clements et al. (2006)) can be written as follows: since a commodity absorbs a large part of the of the overall economy (its budget share rises), there is less scope for its relative price to vary as there is simply a lesser amount of other goods against which its price can change. In other words the variance of a large good is smaller than the variances of other goods. It can be easily shown that then we get (see Clements et al. (2006)):

$$\Sigma_t = \lambda^2_t W_t^{-1}, \tag{7}$$

where $W_t = diag[w_{1,t}, w_{2,t}, ..., w_{N,t}]$.

From (3), (4) and (7) we obtain[4] (see also von der Lippe (2007)):

$$\hat{\theta}^I_t = \sum_{i=1}^{N} w_{i,t} Dp_{i,t}, \tag{8}$$

$$\hat{\sigma}^2_{\hat{\theta}^I_t} = \frac{1}{N-1}\sum_{i=1}^{N} w_{i,t}(Dp_{i,t} - \hat{\theta}^I_t)^2. \tag{9}$$

In other words, the estimator $\hat{\theta}^I_t$ of the underlying rate of inflation is a budget-share weighted average of the N price log-changes. It makes intuitive sense. Moreover, we can notice that $\exp(\hat{\theta}^I_t)$ is a logarithmic Paasche price index, and if we use as weights the arithmetic average of the observed budget shares in years $t - 1$ and t, we obtain in (8) the Divisia price index, also known as the Törnqvist (1936)-Theil (1967) index, that has many of desirable properties.

As it was already mentioned, Diewert (1995) criticizes the stochastic approach. One of his remarks is that the variance assumptions are not consistent with the facts. Diewert argues that equation (5) is not in line with observed behavior of prices.[5] Some authors reject this specification (see Clements and Izan (1987)) but let us notice, that variance specification (5) is just one of multitude of possibilities. In the paper by Clements et al. (2006) authors give three other specifications to show how the stochastic approach deals with different specifications of Σ_t – case I: prices are independent (Σ_t is a diagonal matrix with elements $\sigma^2_{11,t}, \sigma^2_{22,t}, ..., \sigma^2_{NN,t}$); case II: prices have a common variance σ^2_t and a common correlation coefficient ρ_t at time t ($\Sigma_t = \sigma^2_t[(1 - \rho_t)I + \rho_t uu']$, where I is an identity matrix); case III: $\Sigma_t = D_t(I + \lambda_t) D_t$, where D_t is a diagonal matrix with the standard deviation of N prices on the main diagonal, $\lambda_t = [\lambda_{ij,t}]$ is an $N \times N$ symmetric matrix with diagonal elements zero and (i,j) – th off-diagonal element the relevant correlation, it means $\lambda_{ij,t} = \sigma^2_{ij,t}/(\sigma_{ij,t}\sigma_{jj,t})$.

The afore-mentioned authors show that depending on the case we get:

$$\text{case I: } \hat{\theta}^I_t = \sum_{i=1}^{N} w^I_{i,t} Dp_{i,t}, \quad \hat{\sigma}^2_{\hat{\theta}^I_t} = \frac{1}{\sum_{i=1}^{N}\sigma^{-2}_{ii,t}}, \text{ where } w^I_{i,t} = \frac{\sigma^{-2}_{ii,t}}{\sum_{i=1}^{N}\sigma^{-2}_{ii,t}};$$

4 To distinguish estimators coming from different models we use the following notation: $\hat{\theta}^I_t$, $\hat{\theta}^{II}_t$,...

5 Diewert (1995) gives the following example: food has a big share while energy has a small share and the volatility of price components is simply not highly correlated with the corresponding expenditure shares.

case II: $\hat{\theta}_t^{II} = \frac{1}{N}\sum_{i=1}^{N} Dp_{i,t}$, $\hat{\sigma}^2_{\hat{\theta}_t^{II}} = \sigma_t^2[\rho_t + \frac{1-\rho_t}{N}]$;

case III: $\hat{\theta}_t^{III} = \sum_{i=1}^{N} w_{i,t}^{III} Dp_{i,t}$, $\hat{\sigma}^2_{\hat{\theta}_t^{III}} = \frac{1}{\sum_{i=1}^{N}(\sigma_{ii,t}^{-2} - \lambda_{.i}^*)}$,

where $w_{i,t}^{III} = \frac{\sigma_{ii,t}^{-2} - \lambda_{.i,t}^*}{\sum_{i=1}^{N}(\sigma_{ii,t}^{-2} - \lambda_{.i,t}^*)}$

and $\lambda_{i,t}^*$ is the sum of elements in the i – th row of the matrix $\lambda_t^* = D_t^{-1}\lambda_t D_t^{-1}$,

namely $\lambda_{.i,t}^* = \sum_{j=1}^{N} \lambda_{ij,t}^*$.

As we can see in the first case the estimated rate of inflation is still a weighted average of the price changes, but now the weights are proportional to the reciprocals of the variances of the respective relative prices. Obviously, the weights are positive and have a unit sum. In the second case, the estimated rate of inflation is an unweighted average of price changes, while its variance is increasing in the common correlation ρ_t. In this case, if prices are independent we obtain $\hat{\sigma}^2_{\hat{\theta}_t^{II}} = \sigma_t^2/N$ and if prices are perfectly and positively correlated we have $\hat{\sigma}^2_{\hat{\theta}_t^{II}} = \sigma_t^2$. In case III, which is the most realistic, the estimated rate of inflation is again a weighted average of price changes[6] but now the weights $w_{i,t}^{III}$ are related to the variances and covariances of the relative prices. The fraction $w_{i,t}^{III}$ is larger when the i – th variance is lower and the i – th relative price is less correlated with the others. In cases II and III the value of the estimator does not depend on the budget share. Other specifications of the covariance matrix are clearly possible (see Crompton (2000)) and we propose one of them in the next part of the paper. Although the form of the matrix Σ_t determines the final results, still the main idea is to think of the rate of inflation as the underlying common trend in prices. As we can notice, in the presented stochastic models this trend is estimated by a type of a mean of the considered N price changes.

2 A BASIC MODEL AND A NEW PRICE INDEX FORMULA

Let us assume the following specification[7] of the matrix Σ_t:

$$\Sigma_t = D_t(I - \lambda_t)^{-1} D_t W_t^{-1}, \tag{10}$$

where D_t is diagonal matrix with the standard deviations of N relative prices on the main diagonal, $\lambda_t = [\lambda_{ij,t}]$ is an $N \times N$ symmetric matrix with diagonal elements zero and (i,j) – th off-diagonal element the relevant correlation[8] $\lambda_{ij,t} = \sigma_{ij,t}^2/(\sigma_{ii,t}\,\sigma_{jj,t})$ and $W_t = diag[w_{1,t}, w_{2,t},..., w_{N,t}]$ is an $N \times N$ diagonal matrix. The following theorem is true.

Theorem 1

In the stochastic model described by (1) with the corresponding covariance matrix defined by (10) we obtain the following estimator of the rate of inflation[9] and its variation:

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[6] To be precise the formula describing the estimator in case III is an approximation, since it holds that $(I - \lambda_t)^{-1} \approx I + \lambda_t$.

[7] The specification (10) is similar to the specification presented previously as case III, namely $\Sigma_t = D_t(I + \lambda_t)D_t$. In fact, from the known result that $(I - \lambda_t)^{-1} = I + \lambda_t + \lambda_t^2 + ...$ for small elements of λ_t we have $(I - \lambda_t)^{-1} \approx I + \lambda_t$. The last component, the matrix W_t, corresponds to the budget share model (BSM – see von der Lippe (2007)), also presented previously. In other words, the present model is some kind of mixture of the earlier models.

[8] We assume here the realistic scenario that prices are correlated. Otherwise, we should take $\Sigma_t = D_t^2 W_t^{-1}$.

[9] We still use the generalized least squares method for estimating.

$$\hat{\theta}_t^* = \sum_{i=1}^{N} w_{i,t}^* Dp_{i,t}, \tag{11}$$

$$\hat{\sigma}_{\hat{\theta}_t^*}^2 = \frac{1}{\sum_{i=1}^{N} w_{i,t}(\sigma_{ii,t}^{-2} - \lambda_{.i,t}^*)}, \tag{12}$$

where:

$$w_{i,t}^* = \frac{w_{i,t}(\sigma_{ii,t}^{-2} - \lambda_{.i,t}^*)}{\sum_{i=1}^{N} w_{i,t}(\sigma_{ii,t}^{-2} - \lambda_{.i,t}^*)}, \tag{13}$$

and $\lambda_{i,t}^*$ described as in case III (see section 1).

Proof
Firstly, from (10) we obtain:

$$\Sigma_t^{-1} = W_t D_t^{-1}(I - \lambda_t)D_t^{-1}, \tag{14}$$

and thus, we have:

$$[u'\Sigma_t^{-1}u]^{-1} = [u'W_t D_t^{-1}(I - \lambda_t)D_t^{-1}u]^{-1} = [u'W_t(D_t^{-2}u - D_t^{-1}\lambda_t D_t^{-1}u)]^{-1} =$$

$$= [\sum_{i=1}^{N} w_{i,t}(\sigma_{ii,t}^{-2} - \sum_{j=1}^{N} \sigma_{ii,t}^{-1}\lambda_{ij,t}\sigma_{jj,t}^{-1})]^{-1} = [\sum_{i=1}^{N} w_{i,t}(\sigma_{ii,t}^{-2} - \lambda_{.i,t}^*)]^{-1}. \tag{15}$$

The second part of the right-hand side of the equation (3) is as follows:

$$u'\Sigma_t^{-1}Dp_t = u'[W_t D_t^{-1}(I - \lambda_t)D_t^{-1}]Dp_t = u'[W_t(D_t^{-2} - D_t^{-1}\lambda_t D_t^{-1})]Dp_t =$$

$$= u'[W_t(D_t^{-2} - \lambda_t^*)]Dp_t = \sum_{i=1}^{N} w_{i,t}(\sigma_{ii,t}^{-2} - \lambda_{.i,t}^*)Dp_{i,t}. \tag{16}$$

From (3), (15) and (16) we obtain:

$$\hat{\theta}_t^* = (u'\Sigma_t^{-1}u)^{-1}u'\Sigma_t^{-1}Dp_t = \sum_{i=1}^{N} \frac{w_{i,t}(\sigma_{ii,t}^{-2} - \lambda_{.i,t}^*)}{\sum_{i=1}^{N} w_{i,t}(\sigma_{ii,t}^{-2} - \lambda_{.i,t}^*)} Dp_{i,t} = \sum_{i=1}^{N} w_{i,t}^* Dp_{i,t}. \tag{17}$$

Let us notice that from (4) and (15) we get directly the variation of the estimator :

$$\hat{\sigma}_{\theta_t^*}^2 = (u'\Sigma_t^{-1}u)^{-1} = \frac{1}{\sum_{i=1}^{N} w_{i,t}(\sigma_{ii,t}^{-2} - \lambda_{.i,t}^*)}. \tag{18}$$

Remark
As we can see the estimated rate of inflation (11) with weights described by (13) is still a weighted arithmetic mean of the price log-changes, where the weights are proportional to the reciprocals of the variances of the relative prices, proportional to the budget-shares and it also takes into account correlations among prices. In the next part of the paper (see the empirical study) we compare results obtained by using estimators $\hat{\theta}_t^{III}$ and $\hat{\theta}_t^*$.

3 EMPIRICAL STUDY

In our empirical illustration of the presented measures of inflation we use monthly data[10] on price indices of consumer goods and services in Poland for the time period I 2010–XII 2012 (36 observations). The weights $w_{i,t}$ also are taken from data published by the Central Statistical Office.[11] The calculated standard deviations of considered relative prices and their correlations for each considered year are presented in (respectively) Table 1 and Table 2. The estimated rates of inflation for years: 2010–2012 with the corresponding variations and confidence intervals are presented in Table 3.

Table 1 Standard deviations of the log-change prices of the considered goods and services in Poland

Year	Standard deviations											
	X1	*X2*	*X3*	*X4*	*X5*	*X6*	*X7*	*X8*	*X9*	*X10*	*X11*	*X12*
2010	0.0131	0.0173	0.0079	0.0038	0.0043	0.0007	0.0240	0.0037	0.0091	0.0024	0.0041	0.0042
2011	0.0140	0.0051	0.0154	0.0044	0.0053	0.0123	0.0123	0.0166	0.0042	0.0105	0.0031	0.0036
2012	0.0080	0.0050	0.0122	0.0062	0.0022	0.0125	0.0243	0.0138	0.0038	0.0106	0.0024	0.0045

Source: Own calculations

Table 2 Correlations of the considered log-change prices for years 2010–2012 in Poland

Year: 2010	*X1*	*X2*	*X3*	*X4*	*X5*	*X6*	*X7*	*X8*	*X9*	*X10*	*X11*	*X12*
X1	1.000											
X2	–0.687	1.000										
X3	0.591	–0.863	1.000									
X4	0.753	-0.451	0.476	1.000								
X5	–0.511	0.870	–0.959	–0.418	1.000							
X6	–0.132	0.244	–0.483	0.037	0.357	1.000						
X7	–0.399	0.854	–0.680	–0.052	0.770	0.233	1.000					
X8	0.324	0.030	0.147	0.655	–0.028	–0.306	0.371	1.000				
X9	–0.058	0.593	–0.547	0.246	0.676	0.086	0.830	0.692	1.000			
X10	–0.070	–0.256	–0.082	–0.166	–0.009	0.227	–0.468	–0.258	–0.202	1.000		
X11	–0.409	0.888	–0.898	–0.268	0.954	0.259	0.860	0.114	0.754	–0.233	1.000	
X12	–0.604	0.920	–0.895	–0.363	0.951	0.324	0.845	0.039	0.679	–0.115	0.924	1.000

Year: 2011	*X1*	*X2*	*X3*	*X4*	*X5*	*X6*	*X7*	*X8*	*X9*	*X10*	*X11*	*X12*
X1	1.000											
X2	–0.805	1.000										
X3	0.602	–0.518	1.000									
X4	–0.634	0.324	–0.003	1.000								
X5	–0.405	–0.002	0.256	0.828	1.000							
X6	–0.240	–0.163	0.046	0.615	0.745	1.000						
X7	–0.407	0.005	–0.194	0.717	0.610	0.630	1.000					
X8	–0.494	0.084	–0.468	0.581	0.467	0.390	0.777	1.000				
X9	0.138	-0.563	0.288	0.402	0.688	0.596	0.566	0.433	1.000			
X10	–0.608	0.176	–0.265	0.795	0.767	0.603	0.761	0.713	0.555	1.000		
X11	0.129	–0.346	0.760	0.456	0.734	0.359	0.108	0.006	0.602	0.274	1.000	
X12	–0.171	–0.245	0.472	0.690	0.930	0.582	0.482	0.341	0.766	0.635	0.882	1.000

[10] We use highly-aggregated data taking into account price indices of the following group of consumer goods and services in Poland: food and non-alkoholic beverages (*X1*), alcoholic beverages, tobacco (*X2*), clothing and footwear (*X3*), housing, water, electricity, gas and other fuels (*X4*), furnishings, household equipment and routine maintenance of the house (*X5*), health (*X6*), transport (*X7*), communications (*X8*), recreation and culture (*X9*), education (*X10*), restaurants and hotels (*X11*) and miscellaneous goods and services (*X12*).

[11] Główny Urząd Statystyczny (GUS) in Poland.

Table 2 Correlations of the considered log-change prices for years 2010–2012 in Poland continuation

Year: 2012	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12
X1	1.000											
X2	–0.603	1.000										
X3	–0.260	0.396	1.000									
X4	–0.222	0.736	0.353	1.000								
X5	–0.155	0.563	0.580	0.871	1.000							
X6	0.129	0.510	0.344	0.770	0.827	1.000						
X7	0.051	0.596	0.514	0.791	0.846	0.943	1.000					
X8	0.056	0.160	–0.372	0.517	0.236	0.166	0.078	1.000				
X9	–0.003	0.607	0.269	0.431	0.322	0.575	0.515	–0.017	1.000			
X10	–0.143	0.603	0.441	0.829	0.737	0.715	0.709	0.315	0.501	1.000		
X11	0.101	0.022	0.734	0.245	0.538	0.460	0.568	–0.397	0.074	0.500	1.000	
X12	–0.133	0.498	0.771	0.667	0.782	0.713	0.845	–0.106	0.283	0.770	0.832	1.000

Source: Own calculations

Table 3 Values of the considered estimators of a rate of inflation, their variances and the corresponding 95% confidence intervals for years 2010–2012 in Poland

Measure	Year: 2010 (published[12] rate of inflation –0.031)		
$\hat{\theta}_t^{III}$	0.0334	$\hat{\theta}_t^{*}$	0.0325
$\hat{\sigma}^2_{\hat{\theta}_t^{III}}$	0.0129	$\hat{\sigma}^2_{\hat{\theta}_t^{*}}$	0.0023
95% confidence interval	(0.0049; 0.0620)	95% confidence interval	(0.0274; 0.0376)
Measure	**Year: 2011 (published rate of inflation –0.046)**		
$\hat{\theta}_t^{III}$	0.0405	$\hat{\theta}_t^{*}$	0.0474
$\hat{\sigma}^2_{\hat{\theta}_t^{III}}$	0.0083	$\hat{\sigma}^2_{\hat{\theta}_t^{*}}$	0.0011
95% confidence interval	(0.0220; 0.0588)	95% confidence interval	(0.0450; 0.0498)
Measure	**Year: 2012 (published rate of inflation –0.024)**		
$\hat{\theta}_t^{III}$	0.0183	$\hat{\theta}_t^{*}$	0.0239
$\hat{\sigma}^2_{\hat{\theta}_t^{III}}$	0.0061	$\hat{\sigma}^2_{\hat{\theta}_t^{*}}$	0.0009
95% confidence interval	(0.0049; 0.0317)	95% confidence interval	(0.0219; 0.0259)

Source: Own calculations

CONCLUSIONS

It is not unexpected that values of estimators $\hat{\theta}_t^{III}$ and $\hat{\theta}_t^{*}$ differ from each other and values of $\hat{\theta}_t^{*}$ are closer to the published rates of inflation, because only $\hat{\theta}_t^{*}$ and CPI[13] take into account budget shares. However, $\hat{\theta}_t^{III}$ and $\hat{\theta}_t^{*}$ have the same merit – they also take into account variances and correlations of the relative prices. Moreover, the general conclusion is that the variance of the $\hat{\theta}_t^{*}$ estimator (for each year of the research) is smaller then the variance of $\hat{\theta}_t^{III}$ and thus, the confidence intervals for $\hat{\theta}_t^{*}$ are more narrow then confidence

[12] This is an official yearly rate of inflation in Poland published by the Central Statistical Office in December of a given year. To be more precise it is a value of the general price index of consumer goods and services (December of the previous year is a base period) minus one. This value should be approximated by $\exp(\hat{\theta}_t) - 1$, but we use $\hat{\theta}_t$ as an approximation since $\exp(\hat{\theta}_t) - 1 \approx \hat{\theta}_t$ for small values of $\hat{\theta}_t$.

[13] CPI (Consumer Price Index) in Poland takes the Laspeyres form.

intervals calculated for $\hat{\theta}_t^{III}$. In particular, the published rate of inflation in Poland seems to be too small in 2010 (it equals 3,1%, when $\hat{\theta}_t^{III}$ = 3,34% and $\hat{\theta}_t^*$ = 3,25%) and overestimated in 2012 (it equals 2,4%, when $\hat{\theta}_t^{III}$ = 1,83% and $\hat{\theta}_t^*$ = 2,39%). Let us also notice that all confidence intervals for estimated rate of inflation include the value of this rate published by the Central Statistical Office in the corresponding year.

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