Application of Data Envelopment Analysis to Measure Cost, Revenue and Profit Efficiency

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Abstract

The literature analysing efficiency of financial institutions has enveloped rapidly over the last years. Most studies have focused on the input side analysing input technical and cost efficiency. Only few studies have examined the output side evaluating output technical and revenue efficiency. We know that both sides are relevant when evaluating efficiency of financial institutions. Therefore the primary purpose of this paper is to review a number of approaches for efficiency measurement. In particular, the concepts of cost, revenue and profit functions are discussed. We apply Data Envelopment Analysis (DEA) to a sample of Slovak and Czech commercial banks during years 2009–2013 comparing the efficiencies by either minimizing cost or maximizing revenue and profit. The results showed that the level of average revenue efficiency was the highest and the average profit efficiency was the lowest one. As can be seen the Czech banks were more cost, revenue and profit efficient than Slovak ones during the whole analysed period.

Keywords	*JEL code*
Commercial banks, Cost efficiency, Revenue efficiency, Profit efficiency, DEA models	*C14, C61, G21*

INTRODUCTION

Efficiency of banks and other financial institutions is very frequently discussed topic. Efficiency of banking system is one of the most important issues in the financial market as efficiency of banks can affect the stability of banking industry and thus the effectiveness of whole monetary system (Yilmaz, 2013).

In modern society there exists number of approaches how to define efficiency. Our definition is based on the study of Farrell (1957), who proposed that the efficiency of a firm consists of two components: technical efficiency and allocative efficiency. Technical efficiency reflects the ability of a firm to obtain maximal output from a given set of inputs. On the other hand, allocative efficiency reflects the ability of a firm to use the inputs in optimal proportions, given their prices and the production technology. These two types of efficiency are then combined into an overall economic efficiency, which can be examined

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from the perspective of input or output based models. Then, we can talk about overall cost efficiency (input perspective) or overall revenue efficiency (output perspective).

Farrell paper led to development of many approaches to measuring the input and output efficiency. Greatest importance was assigned to a Stochastic Frontier Approach (SFA), created by Aigner, Lovell and Schmidt (1977); and Data Envelopment Analysis (DEA) developed by Charnes, Cooper and Rhodes (1978).

The conventional banking theories assume that banks earn profits by purchasing transactions deposits from the depositors at a low interest rate, then reselling those funds to the borrowers at higher interest rate, based on its comparative advantage at gathering information and underwriting risk (Santos, 2000). In other words, commercial banks make profits from spread between the interest rate received from borrowers and interest rate paid to depositors (Bader et all, 2008).

Using DEA we can assess the banks´ profitability from a different perspective. According Bader (2008) profit efficiency indicates how well a bank is predicted to perform in term of profit relative to other banks in the same period for producing the same set of outputs. We can also define cost efficiency and revenue efficiency. Cost efficiency gives a measure of how close a bank´s cost is to what a best-practice bank´s cost would be for producing the same bundle of output under the same conditions. Revenue efficiency indicates how well a bank is predicted to perform in terms of revenue relative to other banks in the same period for producing the same set of outputs.

Most studies have focused on the input side, estimating cost efficiency (Berger, Hunter and Timme (1993); Resti (1997)). Only few studies have examined the output side evaluating revenue and profit efficiency (Maudos and all (2002); Bader and all (2008)). We know that both sides are relevant when evaluating efficiency of financial institutions. Therefore this paper deals with DEA method and describes its application in measuring cost, revenue and profit efficiency.

The structure of the paper is as follows. The review of relevant literature is described in section 1. Used methodology is discussed in detail in the section 2. Section 3 contains the practical application of methods for measuring cost, revenue and profit efficiency of Slovak and Czech commercial banks during years 2009–2013 using the R software. Finally, the paper ends with some concluding remarks.

1 LITERATURE REVIEW

Data envelopment analysis (DEA) is a non-parametric mathematical (linear) programming approach to frontier estimation. The basic DEA model developed by Charnes, Cooper and Rhodes (1978) was based on the assumption of constant return to scale. This basic model has been modified by Banker, Charnes and Cooper (1984) and based on the assumption of variable return to scale. Both these DEA models have been created in both forms – the input and output oriented. Basic models are discussed in works of many authors and applied in many areas.

Sherman and Gorld (1985) applied DEA to banking as the first. They used DEA analysis to evaluate operating efficiency of 14 saving bank branches. As the result of analysis they not only measured the level of efficiency, but also defined how to eliminate inefficiency by adjusting input and output of inefficient bank branches. Motivated by the DEA results, management indicated that the service outputs and the resources used to provide these would be further evaluated as distinct from the liquidity issues.

For example, Pastor and col. (1997) analysed efficiency of banks in US and in selected countries of Europe. For comparison of different European and US banking systems they used the value added approach. They found out, that France, Spain and Belgium appeared as the countries with the most efficient banking systems, whereas the UK, Austria and Germany show the lowest efficiency levels.

On the other hand, Casu and Molyneux (2003) in their study used the intermediation approach to evaluate efficiency of 750 selected European banks. Overall, the results showed relatively low average efficiency scores, nevertheless, it was possible to detect a slight improvement in the efficiency levels through time.

In the case of using DEA in evaluating banks within a banking sector was DEA analysis used in evaluating the efficiency in Turkish banking system. Yilmaz and col. (2013) measured efficiency of 30 Turkish commercial banks between 2007 and 2010, while the intermediation approach was used. In their study they compared the efficiency of foreign and domestic banks and they found out, that the domestic banks were more efficient in all evaluated years.

In case of Indian banking sector was DEA used by Karimzadeh (2012). In this study was examined the efficiency of 8 major commercial banks during 2000–2010. The results suggest that the mean overall efficiency was 100% in 2000, decreasing to 98% in 2002, and remained unstable from 2003 to 2009 with fluctuating in percentage till 2010–2011, which reached 100% again. The increase of efficiency contributed positively to many economic and financial reforms, which were implemented during the evaluated period, IT innovation, competition, better supervision, and enlarged investment in new information technology.

In the Slovak Republic as well as in the Czech Republic DEA models have been used to measure efficiency of financial institutions for example in works of Stavárek (2006), Jablonský and Grmanová (2009); Stavárek and Řepková (2012). Moreover, DEA was used to measure efficiency in other areas. For example Dlouhý, Jablonský and Novosádová used DEA to measure efficiency of hospitals. They analysed 22 Czech acute-care hospitals with constant return to scale model and variable return to scale model. Also Koróny and Gavurová (2013) used DEA analysis as indicator of eight Slovak regions with one day healthcare during 2009–2011. They separately evaluated efficiency from the viewpoint of junior patients and from the viewpoint of patients over 18 years. DEA models can be also used to measure efficiency of education. Lima (2013) employed DEA to briefly describe some evidence on functioning and dynamics of labour markets and to evaluate efficiency of use of knowledge as strategy to increase the growth in PIGS economies. Jeck and Sudzina (2009) applied DEA models directly to evaluate relative efficiency of faculties of Slovak universities. Another application area is evaluating of efficiency of public road transport. Kráľ and Roháčová (2013) used an input oriented slack-based model under variable return to scale to measure efficiency of transport companies in the Slovak Republic.

2 METHODOLOGY

In this paper we discuss some popular extensions of basic DEA models. If price data are available then it is possible to measure allocative, technical efficiency as well as overall cost, revenue and profit efficiency. To calculate these main types of efficiency, a set of linear programs should be solved.

The input-oriented DEA model under the assumption of variable return to scale can be used for calculation of input-oriented technical efficiency and cost efficiency. Input-oriented model under the assumption of variable return to scale is often termed as BCC model, which can be written in the following form (Dlouhý et all, 2007):

$$\begin{aligned} \min \quad & \theta_q & & (1) \\ \text{s.t.} \quad & \sum_{j=1}^{n} x_{ij} \lambda \le \theta_q x_{iq} & & i = 1, 2,\ldots,m, \\ & \sum_{j=1}^{n} y_{rj} \lambda_j \ge y_{rq} & & r = 1, 2,\ldots,s, \\ & \sum_{j=1}^{n} \lambda_j = 1 & & \\ & \lambda_j \ge 0 & & j = 1, 2,\ldots,n. \end{aligned}$$

Where θ_q is input-oriented technical efficiency (TE_q) of Decision Making Unit (DMU_q) in the input-oriented DEA model, y_{rq} is produced amounts of r^{th} output ($r = 1,2,\ldots,s$) for DMU_q, x_{iq} is consumed amounts of i^{th} input (i = 1,2,...,m) for DMU_q, y_{rj} is produced amounts of r^{th} output (r = 1,2,...,s) for DMU_j (j = 1, 2,...,n), x_{ij} is consumed amounts of i^{th} input (i = 1,2,...,m) for DMU_j (j = 1, 2,...,n), λ_j is weight assigned to the DMU_j (j = 1,2,…,n).

To calculate cost efficiency is necessary to solve the following cost minimisation DEA model (Coelli and all, 2005):

$$\begin{aligned} \min \quad & \sum_{i=1}^{m} w_{iq} x^{*}_{iq} && (2) \\ \text{s.t.} \quad & \sum_{j=1}^{n} x_{ij} \lambda_j \le x^{*}_{iq} && i = 1, 2,...,m, \\ & \sum_{j=1}^{n} y_{rj} \lambda_j \ge y_{rq} && r = 1, 2,...,s, \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \ge 0 && j = 1, 2,...,n. \end{aligned}$$

Where wiq is a vector of input prices of DMU_q and x^{*}_{iq} is the cost minimising vector of input quantities for DMU_q, given the input prices w_{iq} and the output levels y_{rq}.

The overall cost efficiency (CE_q) is defined as the ratio of minimum cost of producing the outputs to observed cost of producing the outputs for the DMU_q (Coelli and all, 2005):

$$CE_q = \frac{\sum_{i=1}^{m} w_{iq} x^{*}_{iq}}{\sum_{i=1}^{m} w_{iq} x_{iq}}. \qquad (3)$$

The overall cost efficiency can be expressed as a product of technical and allocative efficiency measures. Therefore, the allocative efficiency of the DMU_q can be calculated as ratio of overall cost efficiency (CE_q) to input-oriented technical efficiency (TE_q). These three measures (technical, allocative and overall cost efficiency) can take values ranging from zero to one, where a value of one in case of TE, AE and CE indicates full efficiency. If production unit is fully technically efficient (TE_q = 1) and displays allocative efficiency (AE_q = 1); it is also overall cost efficient (CE_q = 1). This production unit uses minimum amount of inputs for producing of given outputs, while the proportion of inputs will guarantee the minimum possible costs. The production unit which is technically efficient (TE_q = 1) but does not demonstrate allocative efficiency (AE_q<1) is not also overall cost efficient (CE_q<1). This production unit uses minimum amount of inputs for producing the given outputs, but the proportion of inputs will not guarantee the minimum possible costs. The production unit which reaches allocative efficiency (AE_q = 1), but does not reach technical efficiency (TE_q<1) cannot be marked as overall cost efficient (CE_q<1). The proportion of inputs will guarantee the minimum possible costs, but this combination of inputs is not minimal for a producing of given outputs. If the production unit fails to demonstrate any of these three types of efficiency (TE_q<1; AE_q<1; CE_q<1), then the value of overall cost efficiency can be interpreted as a potential costs saving that can be achieved if the production unit uses the inputs in optimal combination. Potential costs saving can be calculated by subtracting the value of overall cost efficiency from the number one.

The output-oriented DEA model under the assumption of variable return to scale can be used for calculation of output-oriented technical efficiency and revenue efficiency. Output-oriented model under the assumption of variable return to scale can be written in the following form (Dlouhý et all, 2007):

$$\begin{aligned} \max \quad & \phi_q \\ \text{s.t.} \quad & \sum_{j=1}^{n} x_{ij}\lambda_j \le x_{iq} && i = 1, 2,...,m, \\ & \sum_{j=1}^{n} x_{rj}\lambda_j \ge \phi_q y_{rq} && r = 1, 2,...,s, \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \ge 0 && j = 1, 2,...,n. \end{aligned} \tag{4}$$

Where ϕ_q is output-oriented technical efficiency (TE_q) of DMU_q in the output-oriented DEA model.

To calculate revenue efficiency the following revenue maximisation DEA problem is necessary to solve (Coelli and all, 2005):

$$\begin{aligned} \max \quad & \sum_{r=1}^{s} p_{rq} y^{*}_{rq} \\ \text{s.t.} \quad & \sum_{j=1}^{n} x_{ij}\lambda_j \le x_{iq} && i = 1, 2,...,m, \\ & \sum_{j=1}^{n} y_{rj}\lambda_j \ge y^{*}_{rq} && r = 1, 2,...,s, \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \ge 0 && j = 1, 2,...,n. \end{aligned} \tag{5}$$

Where p_{rq} is a vector of output prices of DMU_q and x^{*}_{rq} is the revenue maximising vector of output quantities for DMU_q, given the output prices p_{rq} and the input levels x_{iq}.

The overall revenue efficiency (RE_q) is defined as the ratio of observed revenue to maximum revenue for the DMU_q (Coelli et all, 2005):

$$RE_q = \frac{\sum_{r=1}^{s} p_{rq} y_{rq}}{\sum_{r=1}^{s} p_{rq} y^{*}_{rq}} . \tag{6}$$

The overall revenue efficiency can be expressed as a product of technical and allocative efficiency measures. Therefore, the allocative efficiency of the DMU_q can be calculated as the ratio of revenue efficiency (RE_q) to output-oriented technical efficiency (TE_q) of the DMU_q. These three measures (technical, allocative and overall revenue efficiency) can take values ranging from zero to one, where a value of one in case of TE, AE and RE indicates full efficiency. If the production unit is fully technically efficient ($TE_q = 1$) and displays allocative efficiency ($AE_q = 1$); it is also overall revenue efficient ($RE_q = 1$). This production unit achieve the maximum possible outputs at given inputs, while the proportion of outputs will guarantee the maximum possible revenues. If the production unit is technically efficient ($TE_q = 1$) but doesn't demonstrate allocative efficiency ($AE_q<1$), it isn't also overall revenue efficient ($RE_q<1$). This production unit achieves the maximum possible outputs using a given inputs, but the proportion of inputs will not guarantee the maximum possible revenues. If the production unit fails to demonstrate any of these three types of efficiency ($TE_q<1$; $AE_q<1$; $RE_q<1$), then the value of overall revenue efficiency can be interpreted as potential revenue increasing that can be achieved if the production unit uses the outputs in optimal combination. Potential revenues increasing can be calculated by subtracting the value of overall revenue efficiency from the number one.

If we have access to price data on both inputs and outputs, then the profit efficiency can also be calculated. The profit maximisation DEA problem is specified as follows (Coelli and all, 2005):

$$\max \quad \sum_{r=1}^{s} p_{rq} y^{*}_{rq} - \sum_{i=1}^{m} w_{iq} x^{*}_{iq} \tag{7}$$

$$\begin{aligned} \text{s.t.} \quad & \sum_{j=1}^{n} y_{rj} \lambda_j \geq y^{*}_{rq} && r = 1, 2, \ldots, s, \\ & \sum_{j=1}^{n} x_{ij} \lambda_j \leq x^{*}_{iq} && i = 1, 2, \ldots, m, \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \geq 0 && j = 1, 2, \ldots, n. \end{aligned}$$

Where all notations used comply with previous definition.

The overall profit efficiency (PE_q) can be defined as the ratio of observed profit to maximum profit for the DMU_q (Coelli and all, 2005):

$$PE_q = \frac{\sum_{r=1}^{s} p_{rq} y_{rq} - \sum_{i=1}^{m} w_{iq} x_{iq}}{\sum_{r=1}^{s} p_{rq} y^{*}_{rq} - \sum_{i=1}^{m} w_{iq} x^{*}_{iq}} . \tag{8}$$

However, this measure need not be bounded by zero and one. It could be negative if a profit is negative, or it could be undefined if maximum profit is zero. (Coelli et all, 2005). The value of overall profit efficiency can be interpreted as potential profit increasing that can be achieved if the production unit uses the inputs and outputs in optimal combination.

3 EMPIRICAL ANALYSIS AND RESULTS

This section describes practical application of methods for measuring cost, revenue and profit efficiency of Slovak and Czech commercial banks during years 2009–2013 using the R software. R is a free software environment for statistical computing and graphics. It compiles and runs on a wide variety of UNIX platforms, Windows and MacOS and can be downloaded on the web page: <http://www.r-project.org>. R is very much a vehicle for newly developing methods of interactive data analysis. It is developing fast, and has been extended by a large collection of packages. However, most programs written in R are essentially ephemeral, written for a single piece of data analysis. One of packages is a package "Benchmarking", prepared by Bogetoft and Otto (2013). Bogetoft and Otto (2011) in their work used software R for calculating efficiency not only for DEA models, but also for SFA models. The package "Benchmarking" contains methods to estimate technologies and measure efficiency using DEA while supporting different technology assumptions (Free disposability hull, Variable return to scale, Constant return to scale, Decreasing return to scale, Increasing return to scale), and using different efficiency measures (input based, output based, hyperbolic graph, additive, super, directional).

The study evaluates a cross-country level data of 18 large commercial banks (8 from Slovakia and 10 banks from Czech Republic) for the period 2009–2013 compiled from the database BankScope. To evaluate banks the intermediation approach which was suggested by Sealey and Lindley (1977) was used. This approach views bank as an intermediary of financial services and assumes that banks collect funds (deposits and purchases funds) with the assistance of labour and capital and transform them into loans and other assets. For each bank in each country in the sample it was necessary to select inputs, outputs, input prices and output prices. All input and output variables, selected types of costs and revenues are measured in thousands of EUR.

We consider three inputs, namely, deposits (x_1), number of employees (x_2) and fixed assets (x_3). Each of these inputs generates costs, referred to total interest expenses, personnel expenses and other operating expenses. Therefore, we can easily calculate prices for each input as a ratio of relevant cost

Table 1 Descriptive statistics on variables used for efficiency measurement

Variable		Year	Minimum	Maximum	Average	Standard deviation
Total deposits (in EUR th)	x_1	2013	806369	25857283	7388564	8171782
		2012	717820	26058434	7208081	8215899
		2011	755524	26752814	7077829	8315440
		2010	663341	25250387	6746347	7974709
		2009	686719	23991794	6444349	7569328
Number of employees	x_2	2013	182	10760	3024	3055
		2012	170	10661	3012	3050
		2011	155	10711	2984	3092
		2010	114	10722	2982	3101
		2009	119	10865	2993	3190
Fixed assets (in EUR th)	x_3	2013	2279	580467	107031	145827
		2012	2260	597311	114601	153821
		2011	1752	639199	114962	155821
		2010	1338	581616	115710	146707
		2009	553	572025	114949	147297
Total loans (in EUR th)	y_1	2013	921392	18728106	5940969	5933283
		2012	779792	19072432	5858226	6045909
		2011	764764	17591450	5548703	5694316
		2010	566647	17175586	5185611	5321655
		2009	328280	16806925	4938354	5054287
Other earning assets (in EUR th)	y_2	2013	224204	17834056	3962460	5300869
		2012	223040	16847694	3700826	4903676
		2011	183478	17520681	3712726	5158410
		2010	195900	17926955	3696638	4942729
		2009	138570	15962431	3525830	4708079
Price of deposits	w_1	2013	0.00593	0.02915	0.01223	0.00579
		2012	0.00642	0.02498	0.01265	0.00533
		2011	0.00486	0.02669	0.01217	0.00575
		2010	0.00035	0.02762	0.01291	0.00588
		2009	0.00412	0.03948	0.01735	0.00752
Price of labour	w_2	2013	25.64516	59.93252	33.85554	8.43658
		2012	24.22452	40.19036	31.77743	5.35486
		2011	23.39632	43.22487	31.30455	5.65688
		2010	18.34522	37.97383	28.96051	5.20046
		2009	19.07984	38.85767	28.41732	5.49254
Price of physical capital	w_3	2013	0.45587	10.77854	2.48491	2.97297
		2012	0.46007	11.66216	2.42749	3.08905
		2011	0.46091	13.21868	2.01414	2.77587
		2010	0.43249	13.74011	1.85727	2.91354
		2009	0.57547	22.89933	2.47004	4.99914
Price of loans	p_1	2013	0.03702	0.10545	0.05658	0.02085
		2012	0.03860	0.14682	0.06084	0.02742
		2011	0.03893	0.14352	0.06195	0.02627
		2010	0.03293	0.11864	0.06090	0.01962
		2009	0.03983	0.09434	0.06366	0.01422
Price of other earning assets	p_2	2013	0.01325	0.04651	0.02856	0.00969
		2012	0.00774	0.06139	0.03128	0.01397
		2011	0.00850	0.05497	0.03170	0.01204
		2010	0.00748	0.06217	0.03081	0.01369
		2009	0.01374	0.29068	0.06023	0.06017

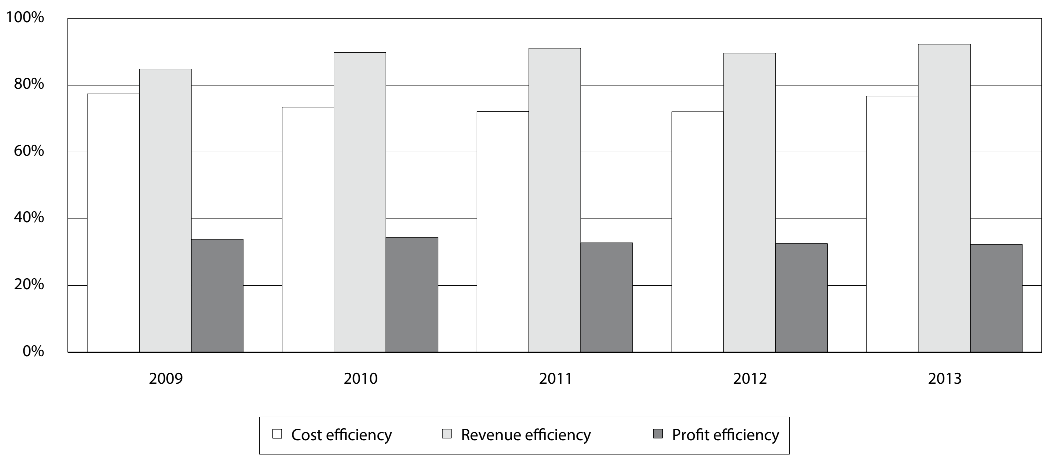
Source: Author´s calculations

to selected input. The price of deposits (w_1) can be calculated as the ratio of total interest expenses to total deposits; price of labour (w_2) as the ratio of personnel expenses to number of employees; and price of physical capital (w_3) as the ratio of other operating expenses to value of fixed assets. On the output side we consider two types of outputs: total loans (y_1) and other earning assets (y_2), which refer to non-lending activities. The income generated by first output is interest income, so the output price (p_1) is defined as the ratio of interest income to value of loans. The second output generates other interest income, therefore consider price for this output (p_2) as the ratio of other interest income to other earning assets. We report descriptive statistics of these variables in Table 1.

To solve the cost minimization problem using R, we first load the data from MS Excel file that must be saved in CSV (Comma-separated values) format. The solution of the cost minimisation DEA model requires using the procedure *cost.op*t from the Benchmarking package. This command estimates the optimal input vector that minimizes cost in the context of the DEA technology. The part of the command is to define which variables will act as inputs (the matrix of inputs, *x*), outputs (the matrix of outputs, *y*), input prices (as a matrix, *w*) and used technology (variable return to scale "*vrs*") of applied model. To calculate overall cost efficiency, we have to find the actual costs and the optimal costs. By dividing these values we obtain overall cost efficiency of evaluated production units. Analogically, we can use R to calculate the revenue or profit maximisation problem and to calculate overall revenue efficiency, or overall profit efficiency.

Figure 1 shows the development of average cost, revenue and profit efficiency of Slovak and Czech commercial banks during years 2009–2013. It is obvious that commercial banks showed most revenue efficiency, while the average revenue efficiency increased gradually in the analysed period. On the other hand commercial banks recorded the lowest profit efficiency, while the average profit efficiency decreased gradually in the analysed period.

Figure 1 Average cost, revenue and profit efficiency of commercial banks in Slovakia and the Czech Republic



Source: Author´s calculations

Table 2 reports minimum, maximum and average values on all three kinds of estimated scores (cost, revenue and profit efficiency) in whole sample and particularly in case of Slovak and Czech banks.

Table 2 Cost, revenue and profit efficiency

			2009	2010	2011	2012	2013
Cost efficiency	SR	Min	35.96%	31.21%	37.56%	44.90%	40.20%
		Max	80.77%	75.30%	61.57%	67.45%	73.74%
		Average	55.85%	54.49%	52.36%	54.87%	58.60%
	CR	Min	71.72%	36.14%	52.07%	47.16%	52.96%
		Max	100.00%	100.00%	100.00%	100.00%	100.00%
		Average	94.64%	88.51%	87.90%	85.80%	91.20%
	SR+CR	Min	35.96%	31.21%	37.56%	44.90%	40.20%
		Max	100.00%	100.00%	100.00%	100.00%	100.00%
		Average	77.40%	73.39%	72.10%	72.05%	76.71%
Revenue efficiency	SR	Min	40.28%	47.22%	61.71%	59.29%	68.22%
		Max	95.06%	100.00%	100.00%	94.25%	95.03%
		Average	72.12%	80.50%	85.83%	80.79%	84.83%
	CR	Min	77.80%	88.64%	78.53%	87.29%	90.27%
		Max	100.00%	100.00%	100.00%	100.00%	100.00%
		Average	95.01%	97.16%	95.23%	96.61%	98.15%
	SR+CR	Min	40.28%	47.22%	61.71%	59.29%	68.22%
		Max	100.00%	100.00%	100.00%	100.00%	100.00%
		Average	84.84%	89.75%	91.05%	89.58%	92.23%
Profit efficiency	SR	Min	0.89%	0.06%	1.21%	0.87%	0.48%
		Max	31.69%	28.42%	26.08%	23.94%	26.88%
		Average	11.56%	10.79%	10.35%	9.21%	9.64%
	CR	Min	1.42%	1.46%	-0.63%	0.86%	0.80%
		Max	100.00%	100.00%	100.00%	100.00%	100.00%
		Average	51.59%	53.35%	50.79%	51.19%	50.48%
	SR+CR	Min	0.89%	0.06%	-0.63%	0.86%	0.48%
		Max	100.00%	100.00%	100.00%	100.00%	100.00%
		Average	33.80%	34.44%	32.82%	32.53%	32.33%

Source: Author´s calculations

Results show that, on average, commercial banks were the most revenue efficient. In the whole analysing sample, the average revenue efficiency increased from value 84.84% in 2009 to value 92.23% in 2013. It indicates that on average banks could have increased their revenues by 15.16% at the beginning of analysed period and only by 7.77% at the end of analysed period. The minimum average value was reached in 2009, the maximum average value in 2013. When we look on the average revenue efficiency according countries then can be seen that the average revenue efficiency moved from value 72.12% to 84.83% in case of Slovakia; and from value 95.01% to 98.15% in case of Czech Republic. It indicates that Czech banks were more revenue efficient as the Slovak ones, which means that the level of potential revenues increasing was in case of Slovak banks higher.

The results of revenue efficiency can be illustrated on the example of an individual bank. Consider Bank 1 in 2013. This bank reached value of output-oriented technical efficiency equal to one. It indicates that Bank 1 was technical efficient, which means this bank produced the maximum possible outputs using a given inputs. When we look at the level of revenue efficiency of Bank 1 we found out, that the level of revenue efficiency was not equal to one (RE = 0.9503). This means, that Bank 1 achieved the maximum possible outputs using a given inputs, but the proportion of outputs did not guarantee the maximum possible revenues. So there existed the space for potential revenue increasing in case of Bank 1. When we looked at revenue of Bank 1 we can see, that the observed value of revenue was 528 500 th EUR and optimal value of revenue was 556 140.72 th EUR. It indicates that Bank 1 should increase its revenues by 4.97%. This potential revenue increasing in case of Bank 1 could be achieved by decreasing total loans to optimal value equal to 7 467 055 th EUR and by increasing of other earning assets to 4 912 368.4 th EUR, while maintaining a given output prices. This optimal combination of outputs allowed Bank 1 to achieve maximum revenue and shift on efficiency frontier.

The different development can be seen in case of cost efficiency. The minimum average value was reached in 2012 (72.05%), the maximum average value in 2009 (77.40%). Results show that the average cost efficiency decreased from value 77.40% in 2009 to value 72.05% in 2012 which can be a result of financial crisis. In last year the average cost efficiency increased to value 76.71%. It indicates that on average banks could save 22.60% of their costs at the beginning and 23.29% at the end of analysed period. When we look at the average cost efficiency according countries then it can be seen that the average cost efficiency moved from value 55.85% to 58.60% in case of Slovakia; and from value 94.64% to 91.20% in case of Czech Republic. It indicates that Czech banks were more cost efficient as the Slovak ones, which means that the level of potential costs savings was in case of Czech banks lower.

The results of cost efficiency can be also illustrated on the example of Bank 1 in 2013. This bank reached value of input-oriented technical efficiency equal to one, so Bank 1 was technical efficient, which means that the bank used minimum amount of inputs for producing of given outputs. When we look at the level of cost efficiency of Bank 1 we found out, that the level of cost efficiency was not equal to one (CE = 0.5652). Bank 1 used minimum amount of inputs for producing of given outputs, but the proportion of inputs did not guarantee the minimum possible costs. It means that there existed the space for potential cost saving in case of Bank 1. When we looked at cost of Bank 1 we can see, that the observed value of cost was 332 100 th EUR and optimal value of cost was 187 687.48 th EUR. It indicates that Bank 1 should use only 56.52% of its cost, so Bank 1 could decrease its cost by 43.48%. This potential cost saving achieved Bank 1 by increasing total deposits to optimal value equal to 8 012 383 th EUR, by reducing number of employees to 2121 and by reducing fixed assets to 48 034.852 th EUR, while maintaining a given input prices. This optimal combination of inputs allowed Bank 1 to achieve minimum cost and shift on efficiency frontier.

The lowest values were reached in case of profit efficiency, when the values moved from 33.80% in 2009 to 32.33% in 2013. The minimum average value was reached in 2013, the maximum average value in 2010 (34.44%). When we look at the average profit efficiency according countries then can be seen that the average profit efficiency moved from value 11.56% to 9.64% in case of Slovakia; and from value 51.59% to 50.48% in case of Czech Republic. It indicates that Czech banks were more profit efficient as the Slovak ones. The value of overall profit efficiency can be interpreted as potential profit increasing that can be achieved if the production unit uses the inputs and outputs in optimal combination. So we can conclude that the level of potential profit increasing was in case of Slovak banks higher. In case of profit efficiency the level could not be bounded by zero and one. It could be negative if profit is negative. This fact can be seen in case of profit efficiency of Czech banks in 2011, when minimum value in the sample was –0.63%.

CONCLUSION

Efficiency of banks and other financial institutions is very frequently discussed topic in literature. Most studies have focused on the input side, estimating cost efficiency, with fewer contributions dealing with revenue and profit efficiency. As we know that both sides are relevant when evaluating efficiency of financial institutions, we were dealing with DEA method and described its application in measuring cost, revenue and profit efficiency.

Results indicate that, on average, commercial banks in Slovakia and Czech Republic were the most revenue efficient, when the average revenue efficiency increased from value 84.84% in 2009 to value 92.23% in 2013. The different development can be seen in case of cost efficiency, when the average cost efficiency decreased from value 77.40% in 2009 to value 72.05% in 2012, and in last year increased to value 76.71%. The lowest values were reached in profit efficiency, when the values moved from 33.80% in 2009 to 32.33% in 2013. When we look at the average efficiency according to countries then we can conclude that the Czech banks were more cost, revenue and profit efficient than the Slovak ones during the whole analysed period. It indicates that the level of potential costs savings, potential revenue increasing and potential revenue increasing was in case of Slovak banks higher.

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