Developing Improved Predictive Estimator for Finite Population Mean Using Auxiliary Information

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Abstract

The present paper deals with the estimation of population mean using predictive method of estimation utilizing auxiliary information. In this paper, we have proposed improved ratio cum product type predictive estimators to estimate the population mean. The large sample properties of the proposed estimator have been studied. The expressions for the bias and mean square error (MSE) have been obtained up to the first order of approximation. The minimum value of MSE of proposed estimator is also obtained for the optimum value of the characterizing scalar. A comparison is made with the ratio and product type estimators and the conditions under which the proposed estimator is more efficient than the other mentioned estimators. An empirical study is carried out to justify the theoretical findings.

Keywords	***JEL code***
Predictive estimators, auxiliary information, finite population mean, MSE, efficiency	*C13, C83*

INTRODUCTION

Sampling is a method or technique of drawing sample from the population. It is used whenever the population is large and the complete enumeration is very time consuming and costly. Parameters of the population are estimated through their appropriate estimators using the information supplied by the sample and their large sample properties are studied up to a certain order of approximation.

In this paper we have studied the large sample properties of the proposed estimator up to the first order of approximation. The use of auxiliary information prevails since the use of sampling itself. It is a unanimous agreement stating that the proper use of auxiliary information enhances the efficiency of the parameter estimator of the main variable under study. The auxiliary variable which supplies the auxiliary information is highly positively or negatively correlated with the main variable under study.

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When the auxiliary variable is highly and positively correlated with the main variable then the ratio method of estimation is used in which the ratio type estimators are used for the estimation of parameters. On the other hand, product type estimators are used when the auxiliary variable is highly and negatively correlated with the main variable under study.

In certain situations, the estimation of the population mean of the study variable has received a considerable attention from experts engaged in survey-statistics. For example, in agriculture, the average production of crop is required for further planning or in manufacturing industries and pharmaceutical laboratories, the average life of their products is a necessity for their quality control. Although, in literature, a great variety of techniques have been used mentioning the use of auxiliary information by means of ratio, product and regression methods for estimating population mean and other parameters. However, some efforts in this direction are reported by many authors. Agrawal and Roy (1999) proposed the efficient estimators of population variance using ratio and regression type predictive estimators, Upadhyaya and Singh (1999) used transformed auxiliary variable and proposed the estimator of population mean, Singh (2003), suggested the improved product type estimator of population mean for negative correlated auxiliary variable, Singh and Tailor (2003), utilized the correlation coefficient of auxiliary and main variable and proposed the improved estimator of population mean, Singh *et al.* (2004, 2014), proposed improved estimators using power transformation and the predictive exponential estimators of population mean, respectively, Kadilar and Cingi (2004, 2006), utilized the different parameters of auxiliary variable and proposed improved ration type estimators of population mean, Yan and Tian (2010), suggested the estimators of population mean using coefficient of skewness of auxiliary variable, Yadav (2011), proposed an efficient ratio estimator of population variance using auxiliary variable, Subramani and Kumarapandiyan (2012), suggested the efficient estimator of population mean using coefficient of variation and the median of auxiliary variable, Solanki et al. (2012), proposed an alternative estimator of population mean using variable for improved estimation of population mean, Onyeka (2012), proposed improved estimator of population mean in poststratified random sampling scheme, Jeelani et al. (2013), suggested modified ratio estimators of population mean using linear combination of coefficient of skewness and the quartile deviation of auxiliary variable, Saini (2013), proposed a predictive class of estimators on two stage random sampling, Yadav and Kadilar (2013) proposed the improved class of ratio and product estimators of population mean and Yadav et al. (2014) suggested the improved ratio estimators for population mean based on median using linear combination of population mean and median of an auxiliary variable. They have thoroughly used auxiliary information for improved estimation of population mean through ratio and product type estimators for the main characteristic under study.

In this paper, we attempt to develop an improved estimator for population mean using predictive method of estimation with the help of auxiliary information. The proposed estimator falls under the category of ratio cum product type predictive estimator. The expressions for the bias and mean square error (MSE) have been obtained up to the first order of approximation. Efficiency condition has been attained by using numerical illustration and proved by comparison showing that present estimator is more efficient than previous ones.

1 MATERIAL AND METHODS

In surveys sampling the information on supplementary population is often used at the estimation stage to enhance the precision of estimates of a population mean and other parameters. The use of auxiliary information is common in practice for estimation of the finite population mean of a study character under consideration. A vast variety of approaches is available in literature to construct more and more efficient estimators for the population mean including design based and model based methods. Many authors have discussed the estimation of population mean using auxiliary information on design based methods. Herein, we have considered the use of auxiliary information on model based method also known as predictive method of estimation of population mean of study variable. The model-based approach or the predictive method of estimation in sampling theory is based on super population models. This approach assumed that the population under consideration was a realization of super-population random variables containing a super population model. Under this super population model the prior information about the population is formalized and used to predict the non-sampled values of the population that is the finite population quantities, mean and other parameters of the study variable. Some of the advantages of super population model approach lie in the fact as: the statistical inferences about the population parameters of the study variable may be drawn using the predictive estimation theory of survey sampling or model based theory. The very well known and popular estimators of population mean in the classical theory are the ratio, regression and other estimators. These estimators can be used as predictors in a general prediction theory under a special model. Many authors have used ratio, product and regression type estimators of population parameters for predictive estimation. It is well established in predictive estimation theory that the use of aforementioned estimators as predictors for population parameters of the unobserved units of the population results in the corresponding estimators of the population parameters for the whole population.

In this paper we propose a predictive estimator of population mean for simple random sampling design using auxiliary variable to construct ratio cum product exponential type estimator by utilizing the prediction criterion.

Let the finite population $U = (U_1, U_2, ..., U_N)$ consists of N distinct and identifiable units. Let the main variable under study be denoted by Y and the auxiliary variable by X. Thus (y_i, x_i), $(i = 1, 2, ..., N)$ denote the ith observations for the main and auxiliary variables respectively. Thus we have $\overline{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$, the population mean of the main variable to be estimated.

Further, let S denote the set of all possible samples of fixed size from the population U. Let s be a member of S and let $\vartheta(s)$ denote the effective sample size that is the number of distinct elements in s and $\overline{s}$ denote the collection of all those units of U which are not in s. We now denote:

$$\overline{Y}_s = \frac{1}{\vartheta(s)}\sum_{i\in s} y_i,$$

$$\overline{Y}_{\overline{s}} = \frac{1}{[N - \vartheta(s)]}\sum_{i\in \overline{s}} y_i.$$

For a given sample $s \in S$, , can be written as:

$$\overline{Y} = \frac{\vartheta(s)}{N}\overline{Y}_s + \frac{[N - \vartheta(s)]}{N}\overline{Y}_{\overline{s}}. \tag{1}$$

In simple random sampling procedure, the sample mean for the sample of size n (i.e. $\vartheta(s) = n$) is:

$\overline{y} = \frac{1}{n}\sum_{i\in s} y_i$ that is $\overline{Y}_s = \overline{y}$.

Thus $\overline{Y}$ in equation (1) can be written as:

$$\overline{Y} = \frac{n}{N}\overline{Y}_s + \frac{N-n}{N}\overline{Y}_{\overline{s}}. \tag{2}$$

In the light of equation (2), an appropriate estimator of population mean $\overline{Y}$ is obtained as:

$$t = \frac{n}{N}\overline{y} + \frac{N-n}{N}T, \tag{3}$$

where T is taken as the predictor of $\overline{Y}_{\overline{s}}$.

Srivastava (1983) proposed the following estimator as the predictor of the population mean $\overline{Y}_{\bar{s}}$ of unobserved units of the population as:

$T = \bar{y} = \frac{1}{n}\sum_{i\in\bar{s}} y_i$, the mean per unit estimator that is the sample mean,

$T = \bar{y}_{\bar{R}} = \overline{X}_{\bar{s}}\left(\frac{\bar{y}}{\bar{x}}\right)$, the classical ratio estimator,

where:

$$\bar{x} = \frac{1}{n}\sum_{i\in s} x_i,\ \overline{X} = \frac{1}{N}\sum_{i=1}^{N} x_i \text{ and } \overline{X}_{\bar{s}} = \frac{1}{N-n}\sum_{i\in\bar{s}} x_i = \frac{N\overline{X} - n\bar{x}}{N-n}.$$

Srivastava (1983) has shown that when above estimators are used as predictive estimators of $\overline{Y}_{\bar{s}}$, then the estimator t in (3) results in respective classical estimators:

$\bar{y} = \frac{1}{n}\sum_{i\in s} y_i$, the mean per unit estimator that is the sample mean,

$\bar{y}_R = \bar{y}\left(\frac{\overline{X}}{\bar{x}}\right)$, the classical ratio estimator.

However, for product predictive estimator $T = \bar{y}_{\bar{P}} = \bar{y}\left(\frac{\bar{x}}{\overline{X}_s}\right)$ of the population mean $\overline{Y}_{\bar{s}}$ of unobserved units of the population, he has shown that t does not result in the classical product estimator $\bar{y}_P = \bar{y}\left(\frac{\bar{x}}{\overline{X}}\right)$ of population mean $\overline{Y}$.

Thus whenever:

$T = \bar{y}_P = \bar{y}\left(\frac{\bar{x}}{\overline{X}_s}\right)$,

$$t = \frac{n\overline{X} + (N-2n)\bar{x}}{N\overline{X} - n\bar{x}} = t_p. \tag{4}$$

The biases and the mean square errors of the estimators $\bar{y}_R$, $\bar{y}_P$ and t_P, up to the first order of approximations respectively are found as:

$Bias(\bar{y}_R) = \theta\overline{Y}C_x^2(1-C)$,

$Bias(\bar{y}_P) = \theta\overline{Y}CC_x^2$,

$Bias(t_p) = \theta\overline{Y}C_x^2[C + f(1-f)^{-1}]$,

$$MSE(\bar{y}_R) = \theta\overline{Y}^2[C_y^2 + C_x^2(1-2C)], \tag{5}$$

$$MSE(\bar{y}_P) = MSE(t_p) = \theta\overline{Y}^2[C_y^2 + C_x^2(1+2C)], \tag{6}$$

where:

$\theta = (1-f)^{-1}$, $f = (n/N)$, $C_y^2 = (S_y^2/\overline{Y}^2)$, $C_x^2 = (S_x^2/\overline{X}^2)$, $C = \rho(C_y/C_x)$, $\rho = (S_{yx}/S_yS_x)$,

$S_y^2 = (N-1)^{-1}\sum_{i=1}^{N}(y_i - \overline{Y})^2$, $S_x^2 = (N-1)^{-1}\sum_{i=1}^{N}(x_i - \overline{X})^2$ and $S_{yx} = (N-1)^{-1}\sum_{i=1}^{N}(y_i - \overline{Y})(x_i - \overline{X})$.

Singh et al. (2014) proposed the following ratio and product type exponential estimators of population mean $\bar{Y}$ using Bahl and Tuteja (1991) ratio and product types exponential estimators of population mean as the predictive estimators of $\bar{Y}_{\bar{s}}$ respectively as:

$$t = t_{\mathrm{Re}} = \left[\frac{n}{N}\bar{y} + \left(\frac{N-n}{N}\right)\bar{y}\exp\left(\frac{\bar{X}_{\bar{s}} - \bar{x}}{\bar{X}_{\bar{s}} + \bar{x}}\right)\right] = \left[\frac{n}{N}\bar{y} + \left(\frac{N-n}{N}\right)\bar{y}\exp\left(\frac{N(\bar{X}-\bar{x})}{N(\bar{X}-\bar{x}) - 2n\bar{x}}\right)\right] \quad (7)$$

$$t = t_{Pe} = \left[\frac{n}{N}\bar{y} + \left(\frac{N-n}{N}\right)\bar{y}\exp\left(\frac{\bar{x} - \bar{X}_{\bar{s}}}{\bar{x} + \bar{X}_{\bar{s}}}\right)\right] = \left[\frac{n}{N}\bar{y} + \left(\frac{N-n}{N}\right)\bar{y}\exp\left(\frac{N(\bar{x}-\bar{X})}{N\bar{X} + (N-2n)\bar{x}}\right)\right] \quad (8)$$

The biases and the mean square errors of above estimators up to the first order of approximations respectively are given as:

$$Bias(t_{\mathrm{Re}}) = \frac{\theta}{8}\bar{Y}C_x^2[3 - 4(C+f)],$$

$$Bias(t_{Pe}) = \frac{\theta}{8}\bar{Y}C_x^2\left[4C - \frac{1}{(1-f)}\right],$$

$$MSE(t_{\mathrm{Re}}) = \theta\bar{Y}^2[C_y^2 + \frac{C_x^2}{4}(1-4C)], \quad (9)$$

$$MSE(t_{Pe}) = \theta\bar{Y}^2[C_y^2 + \frac{C_x^2}{4}(1+4C)]. \quad (10)$$

Note: The mean square errors of above Singh et.al (2014) estimators are equals to the mean square errors of Bahl and Tuteja (1991) estimators.

2 PROPOSED ESTIMATORS

Motivated by Singh et. al (2014), we have proposed the following improved ratio cum product type exponential estimator in predictive estimation approach as:

$$\tau = \alpha t_{\mathrm{Re}} + (1-\alpha)t_{Pe}, \quad (11)$$

where, t_{Re} and t_{Pe} are the estimators in (7) and (8) respectively and is the characterizing scalar to be determined such that the mean square error of the proposed estimator τ is minimum.

To study the large sample properties of the proposed estimator τ, we define, $\bar{y} = \bar{Y}(1+e_0)$ and $\bar{x} = \bar{X}(1+e_1)$ such that $E(e_0) = E(e_1) = 0$ and up to the first order of approximation:

$$E(e_0^2) = \theta C_y^2,\ E(e_1^2) = \theta C_x^2 \text{ and } E(e_0 e_1) = \theta C C_x^2.$$

Expressing t_{Re} in terms of e_i's, we have:

$$t_{\mathrm{Re}} = \bar{Y}(1+e_0)\left[\frac{n}{N} + \left(\frac{N-n}{N}\right)\exp\left(\frac{Ne_1}{2(N-n)+(N-2n)e_1}\right)\right],$$

$$= \bar{Y}(1+e_0)\left[f + (1-f)\exp\left(-\frac{e_1}{2(1-f)+(1-2f)e_1}\right)\right],$$

$$= \bar{Y}(1+e_0)\left[f + (1-f)\exp\left\{ -\frac{e_1}{2(1-f)}\left(1+\frac{(1-2f)}{2(1-f)}e_1\right)^{-1}\right\}\right].$$

Expanding the right hand side of above equation, simplifying after multiplication and retaining the terms up to the first order of approximation, we have:

$$t_{\text{Re}} = \bar{Y}\left\{1+e_0-\frac{e_1}{2}-\frac{e_0e_1}{2}+\frac{e_1^2}{8}(3-4f)\right\}. \quad (12)$$

Similarly expressing t_{Pe} in terms of e_i's, we have:

$$t_{Pe} = \bar{Y}(1+e_0)\left[\frac{n}{N}+\left(\frac{N-n}{N}\right)\exp\left(\frac{Ne_1}{2(N-n)+Ne_1}\right)\right],$$

$$= \bar{Y}(1+e_0)\left[f + (1-f)\exp\left(\frac{e_1}{2(1-f)+e_1}\right)\right],$$

$$= \bar{Y}(1+e_0)\left[f + (1-f)\exp\left\{\frac{e_1}{2(1-f)}\left(1+\frac{e_1}{2(1-f)}\right)^{-1}\right\}\right].$$

Expanding the right hand side of above equation, simplifying after multiplication and retaining the terms up to the first order of approximation, we have:

$$t_{Pe} = \bar{Y}\left\{1+e_0+\frac{e_1}{2}+\frac{e_0e_1}{2}-\frac{e_1^2}{8(1-f)}\right\}. \quad (13)$$

Now expressing τ from (11) in terms of e_i's, using (12) and (13), we have:

$$\tau = \bar{Y}\left[\alpha\left\{1+e_0-\frac{e_1}{2}-\frac{e_0e_1}{2}+\frac{e_1^2}{8}(3-4f)\right\}+(1-\alpha)\left\{1+e_0+\frac{e_1}{2}+\frac{e_0e_1}{2}-\frac{e_1^2}{8(1-f)}\right\}\right].$$

More simplifying, we get:

$$\tau = \bar{Y}\left[1+e_0-(2\alpha-1)\frac{e_1}{2}-(2\alpha-1)\frac{e_0e_1}{2}+\frac{e_1^2}{8}\left\{\frac{4\alpha-1-7f+4f^2}{(1-f)}\right\}\right]. \quad (14)$$

Subtracting $\bar{Y}$ on both sides of (14) and taking expectation on both sides, we get the bias of τ as:

$$Bias(\tau) = \bar{Y}\left[E(e_0)-(2\alpha-1)\frac{E(e_1)}{2}-(2\alpha-1)\frac{E(e_0e_1)}{2}+\frac{E(e_1^2)}{8}\left\{\frac{4\alpha-1-7f+4f^2}{(1-f)}\right\}\right].$$

Putting the values of different expectations, we get:

$$Bias(\tau) = \bar{Y}\left[\frac{1}{8}\left\{\frac{4\alpha-1-7f+4f^2}{(1-f)}\right\}\theta C_x^2-(2\alpha-1)\frac{1}{2}\theta CC_x^2\right]. \quad (15)$$

From (14), we have the mean square error of the estimator τ, up to the first order of approximation as:

$$MSE(\tau) \cong E(\tau - \bar{Y})^2$$

$$= E\left[\bar{Y}\left\{e_0 - (2\alpha - 1)\frac{e_1}{2}\right\}\right]^2,$$

$$= \bar{Y}^2 E\left[e_0 - \alpha_1 \frac{e_1}{2}\right]^2, \text{ where } \alpha_1 = (2\alpha - 1),$$

$$= \bar{Y}^2 E\left[e_0^2 + \alpha_1^2 \frac{e_1^4}{4} - \alpha_1 e_0 e_1\right],$$

$$= \bar{Y}^2\left[E(e_0^2) + \alpha_1^2 \frac{E(e_1^4)}{4} - \alpha_1 E(e_0 e_1)\right].$$

Putting the values of different expectations, we get:

$$MSE(\tau) = \bar{Y}^2\left[\theta C_y^2 + \alpha_1^2 \frac{1}{4}\theta C_x^2 - \alpha_1 \theta C C_x^2\right], \tag{16}$$

which is minimal for:

$$\alpha_1 = \frac{2\theta C C_x^2}{\theta C_x^2} = 2C \text{ or } \alpha = \frac{1}{2}(1 + 2C). \tag{17}$$

And the minimum mean square error of τ is:

$$MSE_{\min}(\tau) = \theta \bar{Y}^2 (C_y^2 - C C_x^2). \tag{18}$$

3 EFFICIENCY COMPARISON

Using (5) and (18), we fairly get that:

$$MSE(\bar{y}_R) - MSE(\tau) = \theta \bar{Y}^2 C_x^2 (1 - C) > 0. \tag{19}$$

In view of (6) and (18), we obtain that:

$$MSE(\bar{y}_P) - MSE(\tau) = \theta \bar{Y}^2 C_x^2 (1 + 3C) > 0. \tag{20}$$

Equations (9) and (18) are used to get the following,

$$MSE(t_{\text{Re}}) - MSE(\tau) = \theta \bar{Y}^2 \frac{C_x^2}{4} > 0. \tag{21}$$

And finally equations (10) and (18) produce that:

$$MSE(t_{Pe}) - MSE(\tau) = \theta \bar{Y}^2 C_x^2 \left(\frac{1}{4} + 2C\right) > 0. \tag{22}$$

4 EMPIRICAL STUDY

To justify the theoretical improvements of the proposed estimator τ over the estimators $\bar{y}$, $\bar{y}_R$, $\bar{y}_P$, t_p, t_{Re}, and t_{Pe} of population mean in predictive estimation theory, we have considered the following three natural populations given in the Table 1.

Table 1 The data used in the study

Population	N	n	C_y	C_x	ρ	C
I. Steel and Torrie (1960) y: Log of leaf burn in sec. x: Chlorine percentages	30	6	0.7493	0.7000	0.4996	0.4667
II. Das (1988) y: The number of agricultural laborers for 1961 x: The number of agricultural laborers for 1971	278	60	1.6198	1.4451	0.7213	0.6435
III. Cochran (1977) y: The number of persons per block x: The number of rooms per block	20	8	0.1281	0.1445	0.6500	0.7332

Source: Own construction

We wish to elaborate the tabulated values in various columns of the Table 1. In the first column of the table, various populations namely Steel and Torrie (1960), Das (1988) and Cochran (1977) are given along with their characteristics denoted as x and y respectively. Second and third columns present respective sizes of populations and samples. Coefficients of variations of two characteristics of each population are arranged in columns fourth and fifth, respectively, which evince the variation propensity of x and y characteristics. Correlation coefficients between two variables are put up in sixth column which are evident as positive. Last column is devised as a ratio of coefficient of variations of x, y and product with correlation coefficient to easily simplify the MSE involved in the discussion of result.

Table 2 The PREs of different estimators with respect to $\bar{y}$

Population	PRE($\bar{y}_R$, $\bar{y}$)	PRE($\bar{y}_P$, or t_p, $\bar{y}$)	PRE(t_{Re}, $\bar{y}$)	PRE(t_{Pe}, $\bar{y}$)	PRE(τ, $\bar{y}$)
I	92.9156	31.1004	133.0374	54.9076	**214.94**
II	156.3967	25.8171	197.7846	47.1121	**520.27**
III	157.8695	34.0327	161.2267	56.4111	**235.93**

Source: Own construction

where:

$$PRE(.,\bar{y}) = \frac{MSE(\bar{y})}{MSE(.)}\,100\,.$$

Further, Table 2 is subject to description such that first column of the table presents percentage relative efficiency of ratio estimators of all three populations. Similarly, percentage relative efficiencies of product estimator corresponding to three populations are shown in second column of the Table 2. Furthermore, relative percentage efficiencies of ratio and product exponential corresponding to the populations are fairly accommodated in third and fourth columns of the table. Last column of the table provides relative percentage efficiencies of proposed estimator corresponding to the populations under study which enabled final insight to identify them as the most efficient estimators.

RESULTS AND CONCLUSION

Sample surveys are legitimately considered as cost effective apparatus for estimation of the population parameter. The main aim of statistician is to minimize the mean square error in estimation to ideally infer the parameter of the given population. Before conducting the survey an auxiliary information is used to minimize the error in the estimation so that efficiency of the estimator goes up. To this end, we have made comparisons of desired results with previous researchers.

From the theoretical discussion of efficiency comparisons and the results in Table 2, we infer that the proposed estimator τ is better than the estimators $\overline{y}_R$, $\overline{y}_P$, t_p of Srivastava (1983) and the estimators t_{Re}, t_{Pe}, of Singh et al. (2014) as it shows smaller mean square error than all these estimators. We observe from Table 2 that the percentage relative efficiency (PRE) of the proposed estimator τ with respect to the sample mean is larger than the PREs of estimators $\overline{y}_R$, $\overline{y}_P$, t_p, t_{Re}, and t_{Pe} of population mean in predictive estimation approach. If we numerically compute the percentage increase of efficiencies it comes out to very high corresponding to the populations. For first population, percentage increase of proposed estimator with respect to ratio estimator is about 131, with respect to product estimator is about 594, with respect to ratio exponential and product exponential are 62 and 291, respectively, which are significantly higher for positive decision in favour of proposed estimator. Likewise, we can repeat the same process to get the increasing nature of proposed estimator for remaining populations. Therefore, the proposed estimator τ should be preferred for the estimation of population mean as most efficient estimator in predictive estimation approach.

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