Benford's Law and Possibilities for Its Use in Governmental Statistics[1]

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Abstract

Benford's Law (sometimes also called Benford's Distribution or Benford's Test) is one of the possible tools for verification of a data structure in a given file regarding the relative frequencies of occurrence of the first (or second, etc.) digit from the left. If it is used as a goodness-of-fit test on sample data, there are usually no problems with its interpretation. However, certain factual questions arise in connection with validity of Benford's Law in large data sets in governmental statistics; such questions should be resolved before the law is used. In this paper we discuss the application potential of Benford's Law when working with extensive data sets in the areas of economic and social statistics.

Keywords	***JEL code***
Benford's Law, goodness-of-fit test, Z-test, national accounts	*E22, C43*

INTRODUCTION

Correctness and indisputability of macroeconomic data is one of the basic principles in governmental statistics. These attributes are achieved by the use of verified methods to collect and process data, attested procedures, and balance computations with the aid of all available sources of information. The national accounts system is one of the "tools" we use for verifying the meaningfulness and cohesion of the governmental statistics. National accounts is a system of inter-related macroeconomic statistical data, arranged in the form of integrated economic accounts. We can compare this system with a crossword puzzle in which indices stand for letters. In other words, each entry is added to the total index value in the row, and one of different indices in the column, similar to letters in a crossword puzzle being parts to "down" and "across" words. This arrangement of data ensures that all items are inter-related and balanced – nothing is lost and nothing is used to excess. Without disputing the national accounts of any country, it is clear that a balanced inter-related system of data can be created from fictitious or even incorrect data items. Other

[1] This article was written thanks to support from the Institutional Support to Long-Term Conceptual Development of Research Organisation, the Faculty of Informatics and Statistics of the University of Economics, Prague.
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tools are suitable for verifying that the items of national accounts are indeed correct. In addition to the usual factual and logical checks on the data sources and procedures, such verification can be supported by certain formal tools. Benford's Distribution is one of them.

1 WHAT IS BENFORD'S LAW?

The substance of Benford's Law can easily be expressed in words: in a given set of data, the probability of occurrence as the first digit from the left is different for each of the digits 1, 2, ... 9. Numbers starting with one occur more often than those starting with two, which are in turn more frequent than those starting with three, etc., and numbers starting with nine are the least frequent ones. This observation is hard to believe at first sight. However, its validity has been empirically confirmed (first in 1881, and then again in 1938). Thanks to a new mathematical approach developed at the end of the 20^{th} century, this law found its way to be included into the theory of probability. Many a time, successful applications, including testing mathematical models and computer designs, as well as error detection in accounting, have indicated its validity.

1.1 Historical Note

By the irony of fate, it was not Frank Benford who assisted at the birth of the distribution that is now called Benford's. Neither was he the first who tried to prove it mathematically. As a matter of fact, Simon Newcomb in the late 19^{th} century first defined a distribution governing the occurrence of numbers with a given digit as the first one from the left. R. A. Raimi and T. P. Hill tried to put forth a mathematical proof of this specific law in the 1990s.

Curiosity and imagination, besides knowledge and experience, undoubtedly play an important role in scientific discoveries. This was also the case of the distribution (law) later called Benford's. American mathematician and astronomer Simon Newcomb noticed in a library that the beginning pages in logarithm table books are much more worn out than the rest. On the basis of this observation he realised that students much more often look up logarithms of numbers beginning with one than those beginning with two, the latter more often than those beginning with three, etc., and from that he deduced: the probability of occurrence for numbers beginning with one is largest, and larger than that for numbers beginning with two, etc. Empirically he derived[4] the following formula for the probability of occurrence for numbers in which digit *d* stands the first from the left:

$$P(d) = \log_{10}\left(1+\frac{1}{d}\right), \qquad \text{for } d = 1, 2, \ldots, 9. \tag{1}$$

This rule means that the probability of occurrence of a number beginning with one is 0.3010, beginning with two 0.1761, etc., to the probability of a number beginning with nine, which is 0.0458. He also derived probabilities corresponding to the digit second from the left (now, of course, zero has to be included); mutual differences are significantly lower for digits 0, 1, …, 9 at the second position: the probability of zero is 0.1197, and that of nine is 0.0850).[5]

Nowadays Newcomb's paper has hundreds of citations, but in its time it passed practically without notice and more or less fell into oblivion. Many years later American physicist Frank Benford also noticed the irregular wear of logarithmic table books' pages, and derived the same logarithmic formula for the first and second digits from the left. In 1938 he published his conclusions based on studying a large number of data sets for different areas (hydrology, chemistry, but also baseball or daily press – Benford, 1938).

[4] Cf. Newcomb (1881).
[5] Cf. Table 1.

Unlike Newcomb's paper, Frank Benford's met certain attention, perhaps thanks to recognition of his name in physics. Newcomb had been forgotten by then and the logarithmic relationship for occurrence of the first (and second) digit from the left was "christened" Benford's.

The wider use of Benford's Law in the second half of the 20th century brought about a number of questions concerning its validity. There were data sets (from natural sciences, economics, but also everyday life) in which Benford's Law was valid, but it was always possible to find situations for its rejection (phone numbers from a certain area, shoe or cloth sizes, etc.). Naturally, a question arose whether Benford's Law can or cannot be proved mathematically. In particular, T. P. Hill (Hill, 1995a; Hill, 1995b; and Hill, 1998), and R. A. Raimi (Raimi, 1969a; Raimi, 1969b; and Raimi, 1976) tried to find such a proof, but no strict mathematical proof was found.[6] If nothing else, their theoretical efforts led to an approximate formulation of Benford's Law validity: if we take random samples from arbitrary distributions, the collection of these random samples approximately obey the Benford's Law.[7]

1.2 Theoretical basis

Formula (1), first derived by Newcomb and later again by Benford, has a more general validity; or rather, it can be adapted into a form which defines occurrence of any digit at the second, third, etc. positions. In this connection, however, we have to ask whether such occurrence does or does not depend on occurrence of preceding digit(s) from the left, or is conditional with respect to such occurrence. In other words, in the former case we deal with probabilities of independent events, while in the latter conditional probabilities are due to be used.

Occurrence of a digit from 1, 2, …, 9 at the first position from the left is governed by Formula (1), but occurrence of a digit from 0, 1, …, 9 at the second position from the left (on assumption that it is independent of occurrence of a particular digit at the first position from the left) is given as

$$P(d) = \sum_{k=1}^{9} \log_{10}\left(1 + \frac{1}{10k + d}\right), \qquad \text{for } d = 0, 1, \dots, 9. \tag{2}$$

Regarding independent occurrences of digits from 0, 1, …, 9 at the third and following positions, the last formula can be generalised:

$$P(d_k) = \sum_{d_1=1}^{9} \sum_{d_2=0}^{9} \dots \sum_{d_{k-1}=0}^{9} \log_{10}\left(1 + \frac{1}{\sum_{i=1}^{k} d_i . 10^{k-i}}\right), \qquad \text{for } d_k = 0, 1, \dots, 9. \tag{3}$$

and the mutual differences between probabilities of occurrence of a particular digit get smaller already at the second position from the left; and starting at the fifth position (independent of the preceding ones) Benford's Law approaches the uniform multinomial distribution. Table 1 shows the changes in the probability values for independent occurrence of digits 0, 1, …, 9 at the first to fifth positions from the left.

The results presented above imply that, starting from the third position from the left, differences in probability values are very small and only occurrence of digits at the first and second positions from the left are interesting from the viewpoint of practical applications.

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6 Perhaps the best characterisation is that by R. A. Raimi in the conclusion of his paper (Raimi, 1969b, p. 347). Referring to the validity of Benford's Law for addresses of 5 000 people from a "Who is Who" publication, he says: "Why should the street addresses of a thousand famous men obey the logarithm law? I know no answer to this question".

7 Cf. Hill (1998) and Raimi (1969b).

Table 1 Probability of occurrence for digit *d* at the *j*th position from the left

d \ *j*	1	2	3	4	5
0	x	0.1197	0.1018	0.1002	0.1000
1	0.3010	0.1139	0.1014	0.1001	0.1000
2	0.1761	0.1088	0.1010	0.1001	0.1000
3	0.1249	0.1043	0.1006	0.1001	0.1000
4	0.0969	0.1003	0.1002	0.1000	0.1000
5	0.0792	0.0967	0.0998	0.1000	0.1000
6	0.0669	0.0934	0.0994	0.0999	0.1000
7	0.0580	0.0904	0.0990	0.0999	0.1000
8	0.0512	0.0876	0.0986	0.0999	0.1000
9	0.0458	0.0850	0.0983	0.0998	0.1000

Source: Authors' own calculations

Another situation arises when probability of occurrence of a digit from 0, 1, …, 9 at the second position from the left is conditional on occurrence of a particular digit from 1, 2, …, 9 at the first position from the left. Conditional probability of occurrence for d_2 at the second position from the left on the condition that the first digit from the left is d_1 equals

$$P(d_2 / d_1) = \frac{\log_{10}\left(1+\frac{1}{10d_1+d_2}\right)}{\log_{10}\left(1+\frac{1}{d_1}\right)}, \qquad \text{for } d_1 = 1, 2, \ldots, 9, \text{ and for } d_2 = 0, 1, \ldots, 9. \tag{4}$$

For example, probability of "2" occurring at the second position on condition of "3" being the first digit from the left is

$$P(D_2 = 2 / D_1 = 3) = \frac{\log_{10}\left(1+\frac{1}{32}\right)}{\log_{10}\left(1+\frac{1}{3}\right)} = \frac{0.0134}{0.1249} = 0.1070.$$

Values of conditional probability for pairs of digits calculated with the aid of Formula (4) are shown in Table 2.

Table 2 Conditional probability values of occurrence for d_2 on condition d_1

d_1 (first digit from the left)	d_2 (second digit from the left)									
	0	1	2	3	4	5	6	7	8	9
1	0.1375	0.1255	0.1155	0.1069	0.0995	0.0931	0.0875	0.0825	0.0780	0.0740
2	0.1203	0.1147	0.1096	0.1050	0.1007	0.0967	0.0931	0.0897	0.0865	0.0836
3	0.1140	0.1104	0.1070	0.1038	0.1008	0.0979	0.0952	0.0927	0.0903	0.0880
4	0.1107	0.1080	0.1055	0.1030	0.1007	0.0985	0.0964	0.0943	0.0924	0.0905
5	0.1086	0.1065	0.1045	0.1025	0.1006	0.0988	0.0971	0.0954	0.0938	0.0922
6	0.1072	0.1055	0.1038	0.1022	0.1006	0.0990	0.0976	0.0961	0.0947	0.0933
7	0.1062	0.1047	0.1033	0.1019	0.1005	0.0992	0.0979	0.0966	0.0954	0.0942
8	0.1055	0.1042	0.1029	0.1017	0.1005	0.0993	0.0982	0.0970	0.0959	0.0949
9	0.1049	0.1037	0.1026	0.1015	0.1004	0.0994	0.0984	0.0973	0.0964	0.0954

Source: Authors' own calculations

The relationships considered above for Benford's Law are valid for arbitrary data sets and are invariant with respect to the change of radix base or units of measurement. Equivalently expressed, data sets governed by Benford's Law will remain governed even if expressed in a base other than decimal, or in other units of measurement (physical, currency, etc.) or if the original data items are all multiplied by an arbitrary constant. This fact implies that any arithmetical operations carried out on data governed by Benford's Law will again be governed by the same law.[8]

The fact that we have at our disposal Benford's Distribution of the first (and second) digit from the left[9] provides us with an option to check any data set for a fit to the data structure governed by Benford's Law. The best choice for such a procedure is the χ^2 goodness-of-fit test, which can be used as a standard hypothesis test if the respective data set comes from a random sample. The tested hypothesis, denoted by H_0, asserts the fit of the empirical distribution with Benford's Law, and the alternative hypothesis H_1 claims the contrary. The test criterion is the statistics

$$G = n\sum_{d=1}^{9}\frac{(p_d - \pi_d)^2}{\pi_d}, \tag{5}$$

which has, under validity of H_0, approximate distribution χ^2 [8], and

where π_d – theoretical relative frequencies under Benford's "Law;
p_d – empirical relative frequencies; and
n – sample size.

The critical values are the respective quantiles of χ^2 [8]; on a 5% significance level, the 95% quantile will be used, that is, $\chi^2_{0.95}$ [8] = 15.5. For a test of the fit at the second position the procedure would be similar, but there are ten groups and nine degrees of freedom. If the underlying sample is small, we also have to respect the condition of a sufficient frequency count in each "cell" ($n\pi_d \geq 5$).

Another option for testing the fit of sample data to Benford's Law is the use of *Z*-statistics; this procedure again verifies the fit between empirical and theoretical frequencies, but separately for each digit, not as a whole. Under hypothesis H_0, the following *Z*-statistics has approximate normal distribution

$$Z_d = \frac{\sqrt{n}\left(\left|p_d - \pi_d\right| - \frac{1}{2n}\right)}{\sqrt{\pi_d(1-\pi_d)}}, \tag{6}$$

where π_d – theoretical relative frequencies under Benford's "Law;
p_d – empirical relative frequencies; and
n – sample size.

The critical value (in this case, separate for each digit) is the respective quantile $u_{1-\alpha/2}$ of the normed normal distribution. On a 5% significance level, we get $u_{0.975}$ = 1.96. Kossovsky (2015) recommends that the two-tailed test should always be used, i.e., the critical value given by quantile $u_{1-\alpha/2}$, because absolute value stands in the numerator in Formula (6), and therefore it is not necessary to distinguish between directions of the deviation from Benford's Law (it means that both lower and higher relative frequencies than the theoretical value under Benford's Law admit the same interpretation).

Although both tests lead to conclusions that are intuitively similar, there is a difference between them. Namely, the former (*G*-statistics) comprehensively assesses the validity of Benford's Law for a given set

8 Cf., e.g., Watrin et al. (2008).
9 For the above-mentioned reasons we are not going to consider more positions from the left.
10 Cf. Kossovsky (2015).

of first digits (possibly second ones as well). The particular digit for which the deviation from Benford's Law is the highest must be looked up among values

$$\frac{(p_d - \pi_d)^2}{\pi_d}, \qquad \text{for } d = 1, 2, \ldots, 9, \text{ or } d = 0, 1, \ldots, 9.$$

The second approach (Z_d-statistics) evaluates the deviation for each individual first digit independently, and it is immediately obvious which first digits do or do not comply with Benford's Law. The same considerations of course apply to testing the fit of empirical data to Benford's Law for the second digit from the left.

Mean Absolute Difference (*MAD*) is also often used to test the fit to Benford's Law. This approach, however, goes beyond standard hypothesis testing because the distribution of the *MAD* statistics is unknown. The mean absolute difference value (for the case of the first digit from the left)[11] is

$$MAD = \frac{\sum_{i=1}^{9} |p_d - \pi_d|}{9}, \tag{7}$$

where π_d – theoretical relative frequencies under Benford's "Law;
p_d – empirical relative frequencies.

Since we do not know the distribution of the *MAD* statistics, empirical threshold values[12] are used for evaluation the outcome for *MAD* – cf. Table 3.

Table 3 Degrees of fit for MAD statistics

MAD value	Degree of fit between empirical and theoretical (Benford's) distributions
0.000 – 0.006	Close fit
0.006 – 0.012	Acceptable fit
0.012 – 0.015	Loose fit
0.015 plus	No fit

Source: Nigrini (2011)

Unlike the previous approaches, which are classical statistical inference instances, the *MAD* statistics is more suitable for verifying the fit in a data set not considered a random sample because all data items in the given area are included. This is often the case when checking extensive sets in corporate accounting and macroeconomic data.

2 PRACTICAL APPLICATIONS

The simplicity and, undoubtedly, a certain degree of mystery of Benford's Law[13] have led to a large volume of literature on this subject.[14] Most often, discussions appear about the use of Benford's Law in checking accounting and macroeconomic data.

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11 For testing the second digit from the left, the calculation is similar but there are ten groups.
12 Cf. Nigrini (2011).
13 The fact that validity of Benford's Law has not been proved mathematically is also a frequent topic.
14 From among the most recent ones, we refer to Miller (2015) – it is a very good presentation of applications and experience with them, especially in the areas of economy, accounting, and also natural sciences.

Using Benford's Law for verification of accounting data correctness is one of the approaches that have recently been often used in financial auditing and (tax) inspections. However, we have to realise that this approach never will and never can substitute for professional, comprehensive and extensive effort carried out by auditors and inspectors – it can only help them find the "weak points". If an accounting data set deviates from Benford's Law, this mere fact is not evidence of data falsification or improper manipulations. It is just an indicator of where attention of auditors/inspectors should be focused. If there is such a deviation, the total fit according to (5) is usually not assessed, but deviations of individual digits are evaluated to show where the attention should be focused. In other words, tests of fit to Benford's Law should only be employed in auditing and inspections as an auxiliary tool in addition to standard procedures, or as the first step in searching for possible instances of data falsification. All authors who deal with the use of Benford's Law in auditing, taxes and inspections agree on the statement cited in the preceding sentence.[15]

Benford's Law has a similar application potential in the area of macroeconomic data. Literature in this area is substantially less extensive than in the previous case, but interesting approaches and results can even be found here. Undoubtedly the best-known contribution to the discussion on Benford's Law is that of Rauch et al. (2011). The authors of that paper focus on verification of Benford's Law validity for selected data of national accounts in 27 member states of the European Union in the period from 1999 to 2009 (data in the ESA 1995 methodology). Aware of the problem implied by the large power of a goodness-of-fit test applied to extensive data sets, they decided for a "descriptive" approach based on ordering the member states according to their values of the total deviation from Benford's Law (5). The position of each state on this scale may, in their opinion, be of assistance to Eurostat – to what extent and in what direction Eurostat's verification procedures should be used. Their analysis (based on relative frequencies of occurrence for the first digit from the left) showed that the least trustworthy, from the Benford's Law viewpoint (more exactly, the average value of the *G*-statistics) were the national accounts data of not only Greece, but also of Belgium, Romania, and Latvia. On the other hand, the best fit to Benford's Law was identified for national accounts data of Luxembourg, Portugal, the Netherlands, Hungary, Poland, and the Czech Republic.

Those excellent results of the Czech Republic inspired us to verify the validity of Benford's Law on new data of national accounts processed and published by the Czech Statistical Office according to the ESA 2010. Our ambition is not to prove the validity of Benford's Law in a wider context of national accounts time series, in which even more favourable results would certainly be achieved, but to illustrate the possibilities of this tool in checking data quality. The data set we tested for fit to Benford's Law for the first and second digits from the left was that of national accounts data of the Czech Republic in 2013 (the preliminary report for 2013). Altogether there were 2 817 digits at the first position from the left, and 2 729 digits at the second position. Statistics (5), (6), and (7) are used for testing the fit. The results for the first digit from the left are shown in Table 4.

[15] Cf., e.g., Carslaw (1988), Nigrini (2005), Nigrini (1996), Guan et al. (2006), Niskanen and Keloharju (2000) or Watrin et al. (2008).

[16] Cf., e.g., Nye and Moul (2007) or Gonzales-Garcia and Pastor (2009).

[17] Generally, data sets connected with the Stability and Growth Pact were considered. Altogether there were 36 691 numerals in 297 sets.

[18] Nonetheless, the problem with Greece's national accounts had been known before. As early as in 2002, Eurostat twice rejected data of the general government in Greece due to untrustworthiness, and again in 2004 (cf. Report by Eurostat on the Revision of the Greek Government Deficit and Debt Figures – <http://ec.europa.eu/eurostat/documents/4187653/5765001/GREECE-EN.PDF>).

[19] Data of the Czech Republic only showed a significant deviation from Benford's Law in 2002, when the value of the test criterion (5) exceeded the critical value of 15.5.

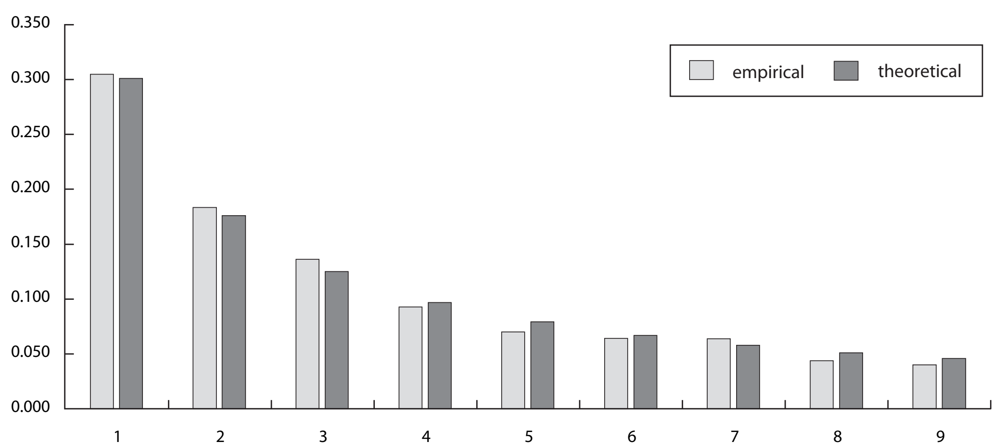
Table 4 Fit to Benford's Law – first digit from the left

First digit from the left	Absolute frequency n_d	Relative frequency p_d	Probability π_d	*G*	Z_d	*MAD*
1	858	0.305	0.301	0.000042	0.390146	0.004
2	517	0.184	0.176	0.000314	1.011605	0.007
3	384	0.136	0.125	0.001036	1.797649	0.011
4	262	0.093	0.097	0.000157	0.668436	0.004
5	198	0.070	0.079	0.000999	1.713259	0.009
6	181	0.064	0.067	0.000108	0.534417	0.003
7	180	0.064	0.058	0.000601	1.300798	0.006
8	124	0.044	0.051	0.000995	1.675934	0.007
9	113	0.040	0.046	0.000696	1.388463	0.006
Total	2 817	1.000	1.000	13.004949	x	0.006

Source: <www.czso.cz>, authors' own calculations

The entries in Table 4 clearly show that, regarding the first digit from the left, the data of the national accounts of the Czech Republic in 2013 comply with Benford's Law for all three characteristics. In the goodness-of-fit test we obtain statistics $G = 13.00$, which is smaller than the critical value of $\chi^2_{0.95}$ [8] = 15.5; hence the hypothesis is accepted that the empirical and theoretical (Benford's) distributions are identical. The values of the Z_d-statistics for each of the digits are all smaller than the critical values of the normed normal distribution ($u_{0.975} = 1.96$). We can therefore observe that, for none of the digits, the differences between the empirical and theoretical frequencies are deemed statistically significant. The *MAD* characteristic also indicates a good fit (cf. Table 3) of the data structure of the national accounts of the Czech Republic in 2013 to Benford's Law. Figure 1 illustrates the fir between the empirical frequencies and theoretical probabilities for the first digit from the left.

Figure 1 Fit to Benford's Law – first digit from the left



Source: <www.czso.cz>, authors' own calculations

The results of the comparison between the data structure of the national accounts of the Czech Republic in 2013 and Benford's Law are shown in Table 5.

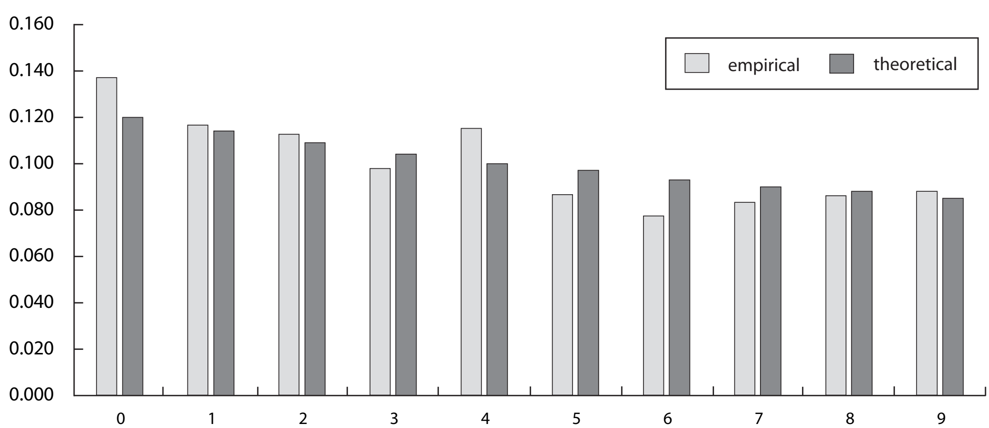
Table 5 Fit to Benford's Law – second digit from the left

Second digit from the left	Absolute frequency n_d	Relative frequency p_d	Probability π_d	G	Z_d	MAD
0	374	0.137	0.120	0.002422	2.710896	0.017
1	318	0.117	0.114	0.000056	0.385125	0.003
2	307	0.112	0.109	0.000112	0.555222	0.003
3	267	0.098	0.104	0.000365	1.023155	0.006
4	314	0.115	0.100	0.002268	2.590616	0.015
5	236	0.086	0.097	0.001141	1.824810	0.011
6	211	0.077	0.093	0.002644	2.787807	0.016
7	227	0.083	0.090	0.000517	1.211364	0.007
8	235	0.086	0.088	0.000041	0.314340	0.002
9	240	0.088	0.085	0.000102	0.517204	0.003
Total	2 729	0.863	1.000	19.774976	x	0.009

Source: <www.czso.cz>, authors' own calculations

Items in Table 5 prove that national accounts data of the Czech Republic in 2013 do not fully comply with Benford's Distribution regarding the second digit from the left. In the goodness-of-fit test we obtain statistics $G = 19.77$, which is higher than the critical value of $\chi^{20.95}$ [9] = 16.9; hence the hypothesis is rejected that the empirical and theoretical (Benford's) distributions are identical. The values of the Z_d-statistics show that the deviations (bold print in Table 5) from the probabilities given by Benford's Law are present for digits 0, 4, and 6; for them, the corresponding values of the Z_d-statistics are larger than the critical value, which is the quantile of the normed normal distribution ($u_{0.975} = 1.96$); hence these deviations are deemed statistically significant. The *MAD* characteristic indicates "only" acceptable fit (cf. Table 3) of the data structure of the national accounts of the Czech Republic in 2013 to Benford's Law.

Figure 2 Fit to Benford's Law – second digit from the left



Source: <www.czso.cz>, authors' own calculations

Let us recapitulate: the evaluation of the fit of the national accounts data of the Czech Republic in 2013 to Benford's Law with respect to the second digit from the left, the fit has not been proved and the differences are significant for digits 0, 4, and 6. However, their more frequent occurrence does not enable us to draw any principal conclusions because this phenomenon is related to a preliminary report. It will be interesting to re-evaluate the situation when the final report of 2013 has been published. We can also see in Figure 2 that the differences for the second digit from the left are not of a principal nature.

CONCLUSIONS

As already stated above, the role of Benford's Law is that of a detection and indicator tool. Deviations of empirical data, i.e., relative frequencies of occurrence for digits 1, 2, …, 9 as the first (or second) digit from the left, from Benford's Law at the beginning of the verification process are not, as such, manifestations of infringement on (say, accounting) rules. At the beginning of the analysis, such deviations are just partial signals that there is certain discrepancy from Benford's Law. Nothing more, and nothing less. Such a signal may be used as recommendation in what direction subsequent analysis should be carried out. Namely, it should focus on the items (accounts, subsets, etc.) for which the highest degree of deviation is shown, e.g., within the Z-test, – Formula (6).

Different situations may arise. Either the revealed deviations are explained in a factual and prescribed way (if the deviation is not random) or no such explanation is identified. In the latter case, it should be seriously investigated why and how the deviation occurred. From experience, a number of instances are known in which unexplained deviations led to identification of principal departures from prescribed procedures and even forensic proceedings were initiated against the parties concerned.

The described approach is open to discussion. Economists, auditors, accountants etc. have varied opinions about the detection potential of Benford's Law. On the one hand there are zealous advocates of a notion that a signal triggered by a deviation from Benford's Law in, say, macroeconomic data (i.e., data on the macroeconomic level) or accounting data (i.e., on the corporate level) is a really serious event to which proper attention should be given because it will lead to the root from which errors – sometimes fully intentional – stem. On the contrary, there are those who feel that the detection role of Benford's Law is a mere formality because the root of the errors will be discovered anyway.

Trust in detection and signalling roles of Benford's Law thus mainly depends on the level of personal experience of those who may use this checking approach. A theoretical dispute aimed at creating a feeling that Benford's Law is useful usually misses this target. This observation is based on practical experience of the authors of the present paper.

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