Life Expectancy Changes and Their Consequences for Pension System in Finland and the Czech Republic

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Abstract

Finland and the Czech Republic are among the countries where population ageing has been the most pronounced in the last decade. The aim of the paper is to describe future development of life expectancy in the context of pension system reforms that are currently prepared by the politicians in analysed countries.

One-year age-and-sex-specific mortality rates for population aged 0 to 100+ were taken from the Human Mortality Database for 1950–2021 and projected to 2050. Three stochastic models were calculated in R and compared. Suitable was Lee-Carter model modified by Li-Lee-Gerland (with rotation of b_x parameter) because of low infant mortality in both populations. Projected year-on-year change of life expectancy was comparable to the Eurostat, but absolute values were too optimistic in our case.

Values of temporary life expectancy between 60 and 70 years and indices of annual relative changes revealed relatively fast pace of increase in life expectancy in both populations which the pension systems should take into account.

Keywords	***DOI***	***JEL code***
Ageing, Czech Republic, mortality projections, Finland, Li-Lee-Gerland model, temporary life expectancy	*https://doi.org/10.54694/stat.2023.20*	*J11, J18, C29*

INTRODUCTION

Population ageing is a problem and challenge in many countries. The people are living longer and thus spending more time in retirement which is a challenge for the pension system and its sustainability. Permanently low natality and higher life expectancy are changing the shape of the age pyramid in the EU-27 towards much older population structure. (Eurostat, 2020). Finland and the Czech Republic are among the countries of the European Union where population ageing has accelerated in the last ten years. The share of people aged over 65 increased between 2009 and 2019 by 5.1 p.p. and 4.7 p.p.,

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resp. In the decade of 2012 to 2022, Finland took second place after Poland as the share of population over 65 years increased by 5 p. p. The increase was lower (by 4.4 p. p.) in the Czech Republic.

Those countries do not belong to the states with the highest median age of population (that is in Italy and Portugal – see Eurostat, 2023a), but the pace of ageing is one of the fastest in the EU. There were 6.0% of people older than 80 years and 23.1% older than 65 years in Finland in 2022, which ranks it among the member states with the highest shares of the old people in the EU. The Czech Republic had 4.3% of people older than 80 years and 20.6% of older than 65 years. However, clear trend of ageing is visible here, too. "The largest increase in the median age in the EU between 2020 and 2022 can be observed in Croatia, Portugal, Greece, Spain, Czech Republic and Italy all rising by more than 0.5 year" (Eurostat, 2023c).

The share of older people in the total population is expected to increase in the coming years due to increased longevity. The life expectancy at birth has been rising until year 2019, i.e. before the start of the Covid-19 pandemic. Main reasons were the reduction in infant mortality, rising of living standards, improved lifestyles, better education and better healthcare and medicine. The increase in life expectancy is commonly attributed to better healthcare and increased living standards. However, Šimpach and Pechrová (2013) found only a weak dependence between life expectancy and living standards expressed by variables of household and municipality amenities and availability of health care.

Life expectancy at birth in the Czech Republic rose to 79.3 years in 2019, but decreased to only 77.2 years in 2022 which was below the level in 2010. The decrease was not that significant in Finland, from 82.1 in 2019 to 81.9 years in 2022. Life expectancy at birth was higher for females in both countries and was above 80 years (Eurostat, 2023d).

Higher share of older people leads to the "ageing at the top" of the population pyramid. On the other hand, the pyramid is shrinking on the bottom due to low fertility. This may lead to an increased burden on those at working age to provide for the social expenditure required by the ageing population for a range of related services (Eurostat, 2023b).

"The baby boomer generation, born between 1946 and 1949, will pose a specific challenge to the service system despite the fact that they are likely to age with better health and functional ability than previous generations" (FIHW, 2023). The Czech Republic will face similar process, but later, as baby boomer generation was born in the 1970s.

The increase of labour force of older age will be caused not only by the irregularities in the age structure, but also by the permanent increase of retirement age (Fiala and Langhamrová, 2014). This could bring consequences for the age structure of the employed persons. Fiala and Langhamrová (2014) expect that the average age of persons in productive age will increase in the Czech Republic, because the share of younger workers will decline and the share of employees over 50 years will increase relatively strongly.

According to the information from European Commission (EC, 2022a) the retirement age is between 63 to 65 years in Finland, depending on the year when the person was born (1954 or earlier up to year 1964). The retirement age of persons born in 1965 and later is adjusted with the life expectancy which will be determined at the age of 62 years. "Since January 2017, the retirement age has been raised by 3 months annually until to reach 65 years by 2027. Then, the retirement age will be linked to life expectancy" (EC, 2022a). The retirement age in the Czech Republic currently depends on the year of birth, sex and number of children of the person born until year 1971. The retirement age for all persons born after 1971 is 65 years in all categories (EC, 2022b).

Needed reform of the pension system has not yet been implemented in the Czech Republic. On the other hand, Finland, together with countries such as the Netherlands, Estonia and Denmark have implemented reforms that link retirement age to changes in life expectancy. Danish case was examined by Alvarez et al. (2021) and they found out that linking retirement age to life expectancy increases uncertainty about length of life after retirement and hence the financial costs of the pension system are more sensitive to changes in mortality. Besides "socio-economic disparities in lifespans persist regardless of the age at which individuals retire" (Alvarez et al., 2021). In addition, the later retirement can bring social tension between job applicants. There have been many stereotypes about alleged lower performance of older employees that leads to unwillingness to employ older people, which, in effect, may lead to a loss of the crucial knowledge possessed by them (Musilová and Režňáková, 2015).

Therefore, our article examines Finland and the Czech Republic and projects the future development of the life expectancy. The aim is to find out how the *life expectancy* will develop, which can serve a basis for deciding on pension system planning and reforms.

1 LITERATURE SURVEY

"One of the possibilities to take into account life expectancy in pension systems is to link some parameters of pension systems to the development of life expectancy, specifically automatically adjusting the retirement age to life expectancy" (Holub et al., 2020).

The life expectancy of people in Finland has been increasing continuously over the past decades and is above the EU average. "These gains in life expectancy were driven by steady reductions in deaths from cardiovascular diseases" (OECD, 2017). Besides, a reduction in mortality rates after the age of 65 has occurred since 2000. "Finland, healthy working life expectancy has increased irrespective of how health is measured but also working with health problems has become more prevalent" (Laaksonen et al., 2022).

Historical development of mortality is described in study of Kannisto et al. (1999). They pointed out that since 1950 the decline of mortality rates have suddenly accelerated due to the introduction of antibiotics. However, this development soon slowed down and was followed by a slowdown, for adult men even by a reversal. Around 1970 an unprecedented decline in the mortality of the elderly which raised life expectancies at younger ages as well (Kannisto et al., 1999).

The life expectancy in the Czech Republic was always comparable with that of the developed countries, even before the Second World War (Langhamrová, 2014). However, in 1945 was seen a sudden drop, mainly due to worsened hygienic conditions at the end of the war. Decreased infant (especially neonatal mortality) resulted in more than 50% increase in life expectancy at birth in 1950s. "Until the early sixties, the Czech Republic was one of the countries with the lowest infant mortality rates in the world" (Langhamrová, 2014). In the socialist era since the mid-sixties, the mortality rates started to worsen, and life expectancy stagnated. The improvement started after the revolution since 1990s (Langhamrová, 2014; Šimpach et al., 2014). Promising development, however, was broken due to the Covid-19 pandemic in years 2020 and 2021.

Impact of Covid-19 on the mortality rates in the Czech Republic and in Spain was examined by Šimpach and Šimpachová Pechrová (2021). They found out that Covid-19 pandemic affected the mortality rates in a way that they were higher and decreased at a slower pace than they would without taking 2020 into account in ages above 50 in Spain. The effect was less pronounced in the Czech Republic.

A methodology for analysis of development and changes in life expectancies was elaborated by Arriaga (1984). He presented a set of indices for interpreting change in life expectancies and also introduced technique for explaining change in life expectancy by change in mortality at each age group. Our article utilizes the indices to measure the changes in life expectancy in old-age groups in projection horizon.

2 METHODS

Mortality modelling can be based on deterministic or stochastic models. Deterministic modelling is based on prior set assumptions which are stated either by expert guess or by supportive statistical methods. The approach of the cohort-component method is the most often used (Leslie, 1945), which can be further enriched and developed with elements from probability theory. For example, Fiala and Langhamrová (2014) used deterministic component method to project the population of the Czech Republic in medium, low and high variant. They expected that life expectancy at birth will continuously increase, but at slower pace and that mortality continues to be relatively low.

On the other hand, stochastic projections are based on stochastic time series models of age-specific demographic measures that complement multivariate statistical methods. The models contain stochastic and error terms.

Syuhada and Hakim (2021) modelled the mortality rate by an Autoregressive (AR) model with a conditional heteroscedasticity effect that was accommodated by a stochastic model of Autoregressive Conditional Heteroscedastic (ARCH) and Stochastic Volatility Autoregressive (SVAR) model. They further forecasted Mortality-at-Risk (MaR) which is a risk measure that refers to the fall of mortality rates for given time period so their results could serve to insurance industry.

Šimpach and Šimpachová Pechrová (2021) used stochastic method, particularly original Lee-Carter model, to elaborate the projections of the mortality rates for the Czech Republic and Spain.

We model mortality rates also by stochastic method and consequently we calculate life expectancy using life tables algorithm. Mortality rates of x-year old in time t ($\mathbf{m}_{x,t}$) represents the ratio of the number of deaths of x-year old in time t ($\mathbf{D}_{x,t}$) and of the exposure to risk which is mid-year population of x-year old population ($\mathbf{E}_{x,t}$) – see Formula (1).

$$\mathbf{m}_{x,t} = \frac{\mathbf{D}_{x,t}}{\mathbf{E}_{x,t}} . \quad (1)$$

We chose stochastic approach, particularly Lee-Carter (LC) model which has the advantage of being comprehensible concept and giving historical values of mortality lower weights in the projections than to recent data. Lee and Carter (1992) elaborated a model of age-specific death rates with a time component $\mathbf{b}_{t,}$ a fixed relative age component $\mathbf{a}_x$, and a time series model (an autoregressive integrated moving average – ARIMA of the time component $\mathbf{k}_t$ (Booth et al., 2002). Original Lee-Carter model (1992) models the natural logarithms of age-specific death rates as (2).

$$m_x = e^{\boldsymbol{a}_x + \boldsymbol{b}_x \boldsymbol{k}_t + \boldsymbol{\varepsilon}_{x,t}} \text{ or in linearized form as } \ln \mathbf{m}_{x,t} = \mathbf{a}_x + \mathbf{b}_x \mathbf{k}_t + \boldsymbol{\varepsilon}_{x,t} , \quad (2)$$

where $\mathbf{a}_x$ are the age-specific profiles independent of time which represents the general mortality shape across age; vector of $\mathbf{b}_x$ are the additional age-specific components that determine the changes in each age group when $\mathbf{k}_t$ changes; $\mathbf{k}_t$ are the time-varying parameters of the level of mortality for all ages and $\boldsymbol{\varepsilon}_{x,t}$ is error term. Ages x take values from 0 to $\omega - 1$ (where ω is an age when any person from original population is not alive anymore). Time is represented by t and takes values from 1 to T (that is year 2021 in our case studies).

"The $\mathbf{b}_x$ profile tells us which rates decline rapidly and which rates decline slowly in response to changes in $\mathbf{k}_t$" (Lee and Carter, 1992). That means that it is derivation of logarithm of mortality rates $\mathbf{m}_{x,t}$ by the time (3).

$$\frac{d \ln \mathbf{m}_{x,t}}{dt} = \frac{d(\mathbf{a}_x + \mathbf{b}_x \mathbf{k}_t)}{dt} = \mathbf{b}_x \frac{d\mathbf{k}_t}{dt} . \quad (3)$$

The coefficient $\mathbf{b}_x$ can be negative for some ages, which indicates that mortality at those ages tends to rise when mortality in other ages is falling.

Error term $\boldsymbol{\varepsilon}_{x,t} \approx \mathrm{N}(0,\sigma_\varepsilon^2)$ captures age-specific historical influences not captured by the model and is supposed to be non-correlated and homoscedastic. However, "the observed pattern of the mortality rates shows a different variability at different ages, highlighting that the homoscedasticity hypothesis is quite unrealistic" (Russolillo, 2017). For modelling of the data at higher ages, certain models such as Kannisto can be used. For example, the Czech Statistical office (2020) uses model based on logistic curve which takes into account the slowing of the increase in mortality with age (Thatcher, Kannisto and Vaupel, 1998).

The model written in Formula (2) is underdetermined, therefore Lee and Carter (1992) normalized the $\mathbf{b}_x$ to sum to unity and the $\mathbf{k}_t$ to sum to 0, which implies that the $\mathbf{a}_x$ are simply the averages over time of the $\ln(\mathbf{m}_{x,t})$. Using these constrains $\sum_{x=1}^{N}\mathbf{b}_x = 1$ and $\sum_{t=1}^{T}\mathbf{k}_t = 0$, the least squares estimator for $\mathbf{a}_x$ can be obtained by (4), Danesi et al. (2015).

$$\hat{a}_x = \frac{\sum_{x=1}^{N} \ln m_{x,t}}{N}, \tag{4}$$

where N is the total number of used years. Under this normalization, $\mathbf{b}_x$ is the proportion of the change in overall logarithm of mortality attributable to age x. Indexes of the intensity of level of mortality $\mathbf{k}_t$ are next modelled as a time series (specifically, a random walk with drift) and forecasted as proposed by Lee and Carter (1992). The order of lags is determined based on recommendation by Akaike information criterion. For example, Russolillo (2017) used the ARIMA (0,1,0) model to forecast the index of mortality $\mathbf{k}_t$ for next 25 years. Šimpach and Dotlačilová (2015) applied ARIMA (1,1,0) with drift on the Czech population.

"The singular value decomposition (SVD) method can be used to find a least squares solution when applied to the matrix of the logarithms of the rates after the averages over time of the (log) age-specific rates have been subtracted" (Lee and Carter, 1992).

A model was further extended by Li and Lee (2005) to calculate coherent mortality forecasts for countries with low infant mortality. One of the disadvantages of the Lee-Carter model is namely that the age-specific set of the $\mathbf{b}_x$ parameter is estimated on the basis of historical data and does not develop in time. Li, Lee and Gerland (2013) examined low-mortality countries and observed that during long time forecast of mortality rates there was too high decrease of mortality in neonatal and low ages in comparison with decrease of mortality in older ages. Therefore, the rotation of $\mathbf{b}_x$ in time must be modelled in order to project long-term mortality changes. Because $\mathbf{b}_x$ is calculated as the first-order derivation of $\ln(\mathbf{m}_{x,t})$ at all ages (3), if we want to model change of $\mathbf{b}_x$ in time, we actually model the second-order differences of $\ln(\mathbf{m}_{x,t})$ at all ages. Li, Lee and Gerland (2013) modified the shape of the $\mathbf{b}_x$ curves that were estimated from historical data by standard LC model, by smoothing their values at adolescent and adult ages (15–65) to equal the average value for this age range, and then by reducing the values at infant and child ages (0–14) to this average. Coherent parameter $\mathbf{b}^c{}_x$ and ultimate parameters $\mathbf{b}^u{}_x$ are the same for males and females. Based on the transformation, a curve of the final age-specific change in mortality is thus smoothed.

When searching for suitable model to model mortality rates in Finland and the Czech Republic, we must look on the characteristics of the examined populations. Finland had the lowest infant mortality in the EU in 2021 (1.8 deaths per 1 000 live births). The Czech Republic was the fifth with 2.2 deaths per 1 000 live births which is the lowest value in recent years. This is an important feature that favours Li-Lee-Gerland model.

Finally, index $\mathbf{k}_t$ is projected to the future and the mortality rates are calculated from Formula (2) up to year 2050.

Fitted and forecasted mortality rates are then used to calculate life expectancy using life table algorithm (see e. g. Šimpach et al., 2013). First, the p_x – probability that an individual alive at time x will survive the interval $(x, x + 1)$ is calculated as (5).

$$p_x = \exp(-m_x) \cdot h_x , \tag{5}$$

where *exp()* is Euler constant, h_x is the length of the age interval (1 year in our case). Then the probability that an individual alive at time x will die in the interval $(x, x + 1)$, i. e. before the next birthday, is $q_x = 1 - p_x$. (see Gompertz, 1825; Chiang, 1961; Šimpach et al., 2013).

Consequently, the life tables are calculated. We start with hypothetical model cohort of 100 000 alive males and females at the age of 0 (so-called radix of the table) and calculate number of people left alive at age x (l_x) until the age $\omega - 1$, where ω is an age when no person from original population is alive as (6).

$$l_x = l_{x-1} \cdot p_{x-1} \text{ or } l_x = l_{x-1} \cdot (1 - q_{x-1})\ . \tag{6}$$

We start with multiplying 100 000 (l_0) by the probability that a person aged 0 will die before reaching 1 year (q_0) to obtain the number of deaths at age 0 years (d_0) which is then subtracted from original population of 100 000 people. Hence, we gain the number of surviving people to age 1. The calculation continues to the age ω (100 in our case, because that is the strict limit of used R-packages).

For the calculation of life expectancy at birth, the projection of alive born children is needed, but we do not project fertility in our paper, so we do not have available the number of alive born children and the p_0 is not correctly calculated in our case. This does not pose any problem to further calculation of life expectancy, as we are mainly interested in life expectancy as older ages (e_{60}, e_{65}, e_{70}, e_{75}, e_{80}) rather than at birth (e_0).

For pension system policy is an important indicator the average number of years of life remaining at exact age x. Therefore, life expectancy e_x of the person at the exact age x years is calculated as (7). It is the ratio of the number of remaining years of life (T_x) and the number of survivors to exact age (l_x).

$$e_x = \frac{T_x}{l_x}\ , \tag{7}$$

where T_x (total number of years lived by alive population from age x to the end – i.e. remaining years to end age ω) is gained by summing number of years that x years old person lived until actual age (L_x). For ages above 1 year, it is assumed that deaths occur evenly over a year of age, the L_x is calculated as follows (8):

$$L_x = \frac{l_x + l_{x+1}}{2}\ . \tag{8}$$

The calculation of changes in life expectancy are based on Arriaga (1984) methodology. Particularly, we use temporary life expectancies ($_ie_x$), i.e. life expectancies between two specific ages (x) and time (t), and indices that are calculated based on the comparison of temporary life expectancies. "The temporary life expectancy from age x to $x + i$ is the average number of years that a group of persons alive at exact age x will live from age x to $x + i$ years" (Arriaga, 1984). i is the difference between initial age x and last examined year $x + i$. Temporary life expectancy is calculated as (9).

$${}_ie_x = \frac{T_x - T_{x+i}}{l_x}\ . \tag{9}$$

When calculating the relative changes over time period of temporary life expectancy (${}_iRC_x^{\,n}$), the absolute difference between temporary life expectancy at particular age and time t (${}_ie_x^{\,t}$) and time $t + n$ (${}_ie_x^{\,t+n}$) is related to the difference between i and temporary life expectancy in time t (10).

$${}_iRC_x^n = \frac{{}_ie_x^{t+n} - {}_ie_x^t}{i - {}_ie_x^t}\ . \tag{10}$$

Because RC index does not enable comparison when there are different time-periods compared, an index of annual change of the RC index is calculated as (11).

$${}_iARC_x^n = \left[1 - (1 - {}_iRC_x^n)^{1/n}\right] \cdot 100\ . \tag{11}$$

All measures are calculated for both populations, compared and interpreted in the context of pension reforms.

One-year age-and-sex-specific mortality rates for population aged 0 to 100+ for Finland and the Czech Republic were taken from the Human Mortality Database and they are available in a time series from 1950 to 2021.

Calculations were done in software R. There were 3 approaches compared. Approach (A) used packages *demography* (Hyndman et al., 2012, 2022) that estimates original Lee-Carter model and age-specific mortality rates (and also can estimate fertility rates and migration).

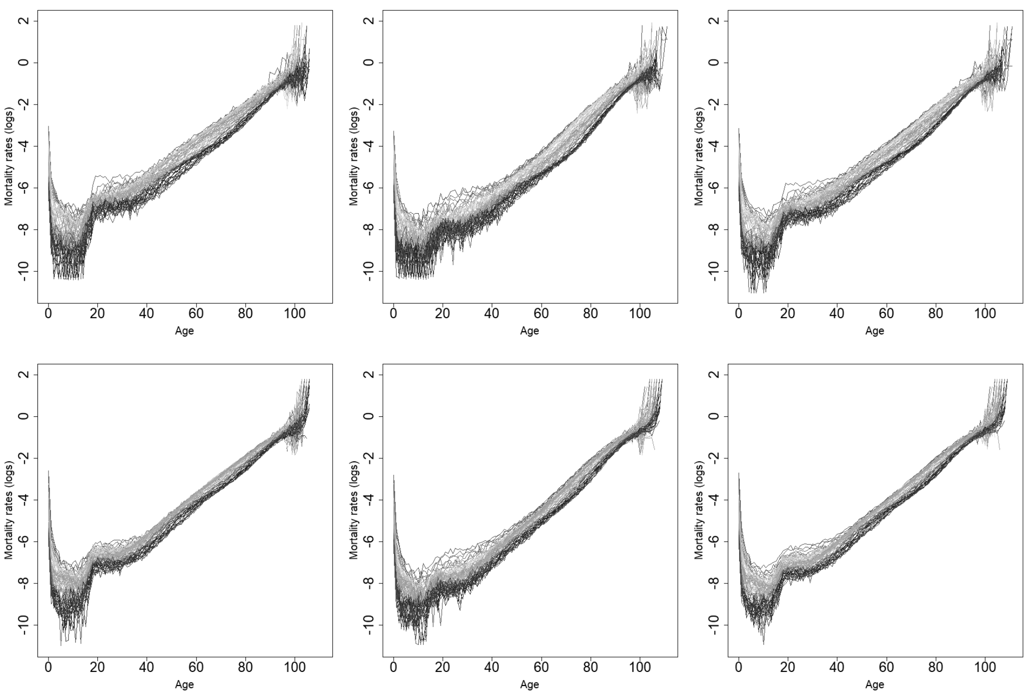
Approach (B) used package *MortCast* elaborated by Ševčíková et al. (2022) which estimates and projects age-specific mortality rates using augmented Lee-Carter method and related methods that are described in Ševčíková et al. (2016). An augmented Lee-Carter model is estimated here by function *leecarter.estimate*.

Approach (C) also used package *MortCast* by Ševčíková et al. (2022), but this time with extension by Li-Lee-Gerland (2013). Coherent parameter $\mathbf{b}^c{}_x$ and the ultimate $\mathbf{b}^u{}_x$ for rotation for male and female mortality rates are estimated here by a function *lileecarter.estimate*. Both parameters are then used for total population, too. Index $\mathbf{k}_t$ is projected by ARIMA model using *auto.arima* command of the package *forecast* by Hyndman (2012). This command selects optimal model based on the lowest value of Akaike Information Criterion (AIC) and corrected Akaike Information Criterion (AICc).

3 RESULTS

Figure 1 displays mortality rates in logarithms for population of males, females and total population in Finland and the Czech Republic. Light grey represents year 1950 and as the colour is darker, the

Figure 1 Mortality rates in logarithms for population of males (left), females (middle) and total population (right) in Finland (upper) and the Czech Republic (lower)



Source: Own elaboration based on the data from Human Mortality Database (HMD, 2023)

newer is the data. Black colour represents year 2021. It can be clearly seen that the mortality rates had declined over time in all populations. There was a high variability of mortality rates in population of males in higher ages, especially in Finland. It is natural that the largest changes of mortality rates are at the highest ages (approximately 80 years and above), where the mortality has got the different character than in lower ages as there are low numbers of deaths and also low numbers of living. Besides, this data is affected by systematic and random errors.

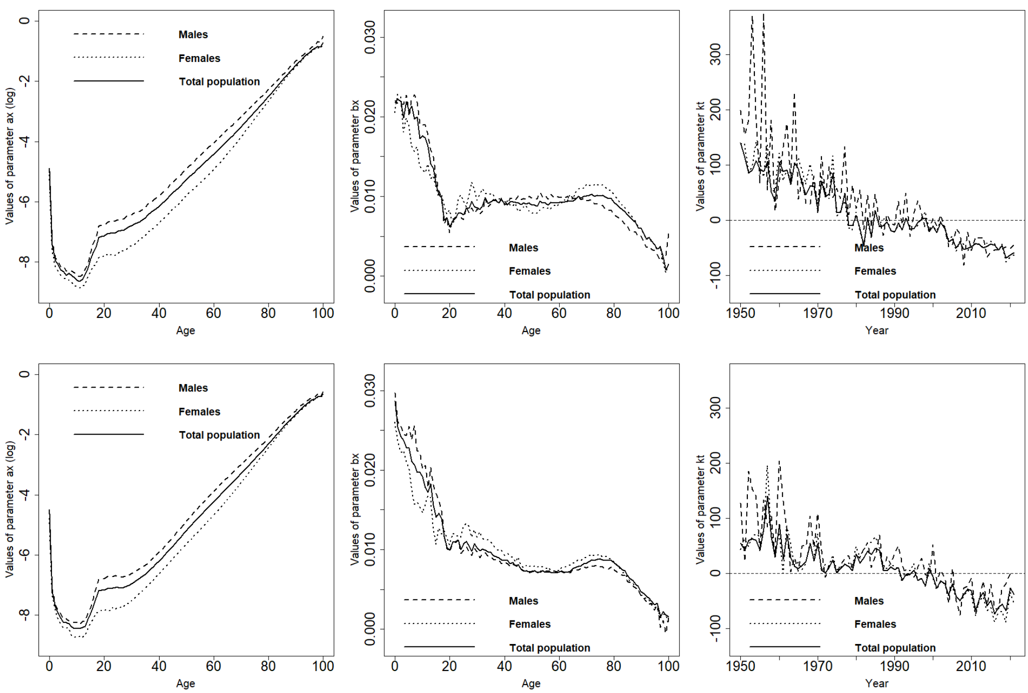
3.1 Mortality rates modelling

First, above stated mortality rates were modelled by three approaches: (A) Hyndman (2012, 2022), (B) Ševčíková et al. (2016) and (C) Ševčíková et al. (2016) with extension by Li-Lee-Gerland (2013).

Figure 2 displays the components of the approach A – Hyndman (2012, 2022) – parameter $\mathbf{a}_x$ (left), parameter $\mathbf{b}_x$ (middle) and $\mathbf{k}_t$ (right) for Finland (upper) and the Czech Republic (lower). The development of the parameter $\mathbf{a}_x$ had expected development according to Gompertz (1825). Parameter $\mathbf{b}_x$ has also correct development. It shows that mortality rates decline rapidly until the age of 20 in comparison with other ages. The parameter is almost constant until the ages from 70 to 80. Time-varying index of the level of mortality for all ages for males' population is very volatile at the beginning of the time series. The volatility decreased later, but the changes are still higher than in case of females and total population.

Regarding the Czech population, parameters $\mathbf{a}_x$, $\mathbf{b}_x$ and index $\mathbf{k}_t$ also develop according to the assumptions. Decrease of $\mathbf{b}_x$ parameters is again sharp until the age of 20, but unlike in Finnish

Figure 2 Development of parameters a_x, b_x and index k_t in approach A



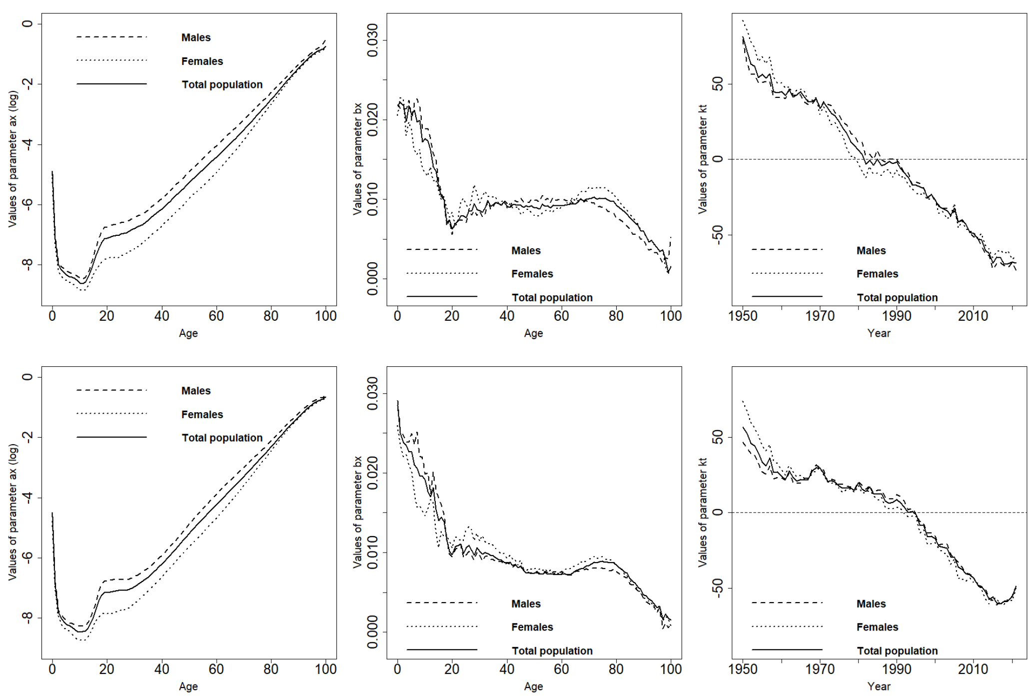
Source: Own elaboration based on the data from Human Mortality Database (HMD, 2023)

population it continuous until the age of 60 (despite than in slower pace). There is again high variability in the development of indices of total mortality $\mathbf{k}_t$ between 1950–1970, but this time for all populations. The decrease of $\mathbf{k}_t$ is slower than in Finnish population. We can expect that incorrect parameters will not be able to provide reliable forecasts.

Approach of Ševčíková et al. (2016) also uses Lee-Carter model, but with modification of parameter $\mathbf{a}_x$. From Figure 3 can be seen that approach B had almost similar parameters $\mathbf{a}_x$ and $\mathbf{b}_x$ as previous approach A. The highest difference is in index $\mathbf{k}_t$ probably because approaches A and B use different estimation methods. Hyndman (2012, 2022) uses Latent Class Analysis (LCA) and Ševčíková (2016) Singular Value Decomposition (SVD). Index for males' population is much smoother, the development of all populations is closer, and $\mathbf{k}_t$ is decreasing sharply throughout the whole period. The only exceptions are last two years (2020, 2021) due to Covid-19 pandemic. Time-varying index stagnated in Finland and increased in the Czech Republic. Here can be clearly seen the difference in handling the pandemic between the two states. In the Czech Republic, the effect of increased mortality was significantly manifested, and $\mathbf{k}_t$ not only stopped decreasing, but even increased.

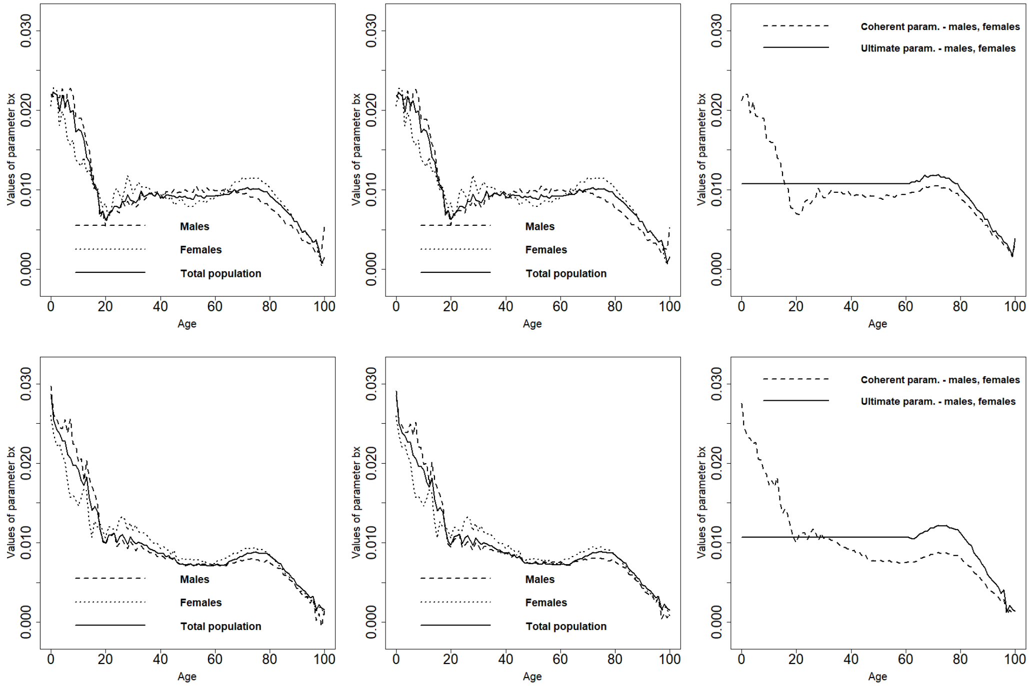
The approach C had similar parameter $\mathbf{a}_x$ and index $\mathbf{k}_t$ as approach B. Parameter $\mathbf{b}_x$ was rotated according to Li-Lee-Gerland (2013), because the decline of young people mortality is not that fast in developed populations as in developing populations, because the mortality is already very low there. Coherent parameter $\mathbf{b}^c{}_x$ and ultimate parameters $\mathbf{b}^u{}_x$ are the same for males and females, so they are used also for total population. $\mathbf{b}^u{}_x$ is constant up to age 60, $\mathbf{b}^c{}_x$ decreases up to age 60. Parameter $\mathbf{a}_x$ and index $\mathbf{k}_t$ are similar as in Ševčíková et al. (2016), so we present only comparison of parameters $\mathbf{b}_x$ of various approaches in Figure 4.

Figure 3 Development of parameters a_x, b_x and index k_t in approach B



Source: Own elaboration based on the data from Human Mortality Database (HMD, 2023)

Figure 4 Comparison of parameter b_x development in approaches A, B, C



Source: Own elaboration based on the data from Human Mortality Database (HMD, 2023)

3.2 Mortality rates projection

Mortality rates were projected in a way that index $\mathbf{k}_t$ was projected to the future and the mortality rates are calculated from the Formula (2). $\mathbf{k}_t$ was projected by ARIMA model using *auto.arima* command of the package *forecast* by Hyndman (2012), where optimal models were selected based on the lowest value of AIC and AICc criterion.

The results are displayed at Figure 5. It can be seen that confidence intervals are narrower in Finland than in the Czech Republic because of the impact of Covid-19 pandemic (volatility at the end

Figure 5 Projection of the k_t for Finland (upper) and the Czech Republic (lower)

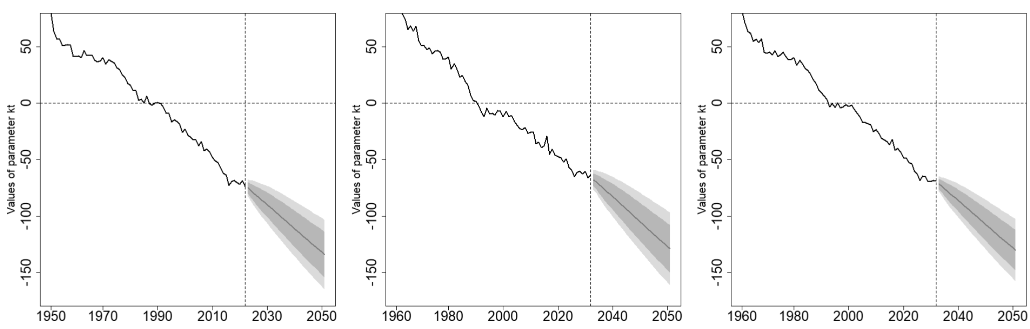
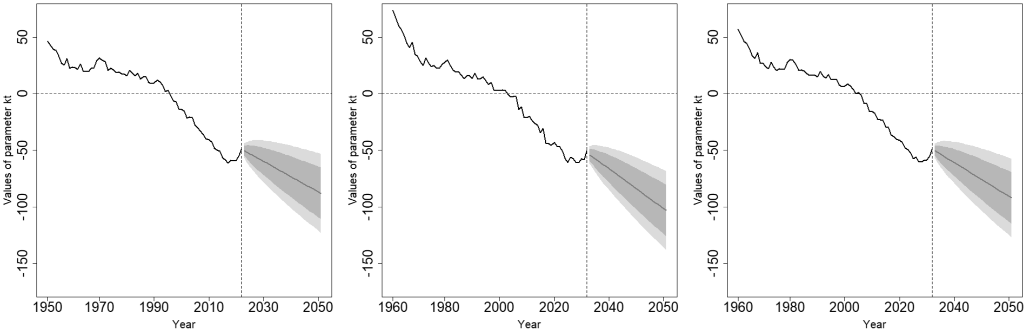
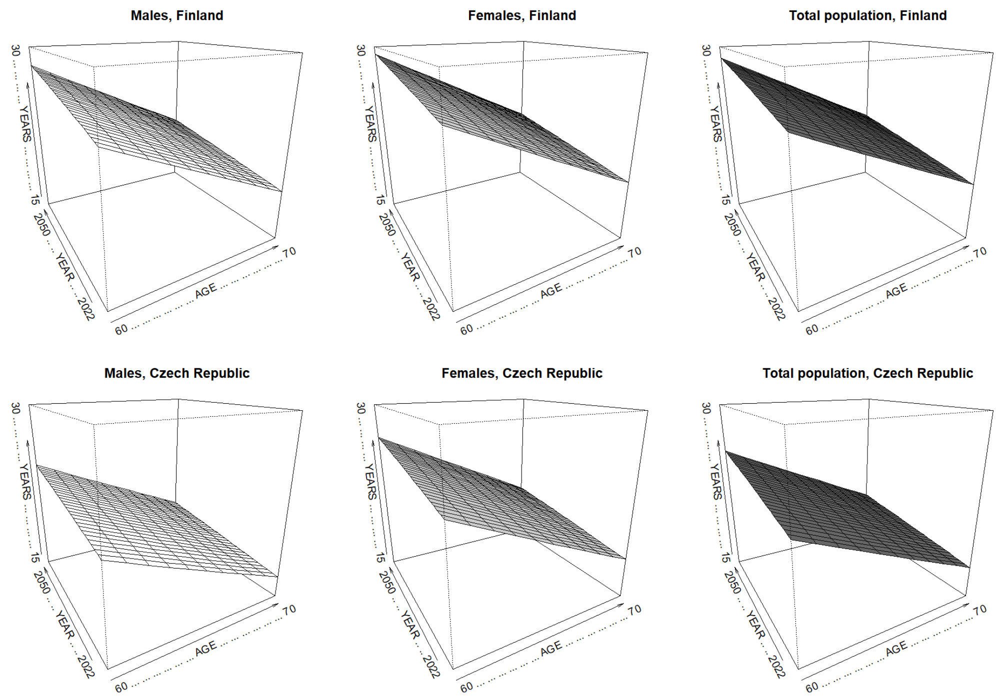


Figure 5 (continuation)



Source: Own elaboration based on the data from Human Mortality Database (HMD, 2023)

Figure 6 Life expectancy projection for Finland (upper) and the Czech Republic (lower) at ages from 60 to 70 years



Source: Own elaboration based on the data from Human Mortality Database (HMD, 2023)

of the time series increased). The best model for all Finnish populations was ARIMA(1,1,0) with drift. Optimal models for Czechs were ARIMA(0,1,0) with drift for males and total and ARIMA(1,1,0) with drift for females. The types of models are similar to Šimpach et al. (2014).

Mortality rates were projected by all three approaches. They were compared based on of the realism of the results and the fact that the mortality rates correspond to the assumptions and current developments.

The most feasible results provided approach of Ševčíková (2016) in modification by Li-Lee-Gerland (2013), so we present only mortality rates and life expectancy projections that are based on approach C. Mortality rates projections suggest continual decrease of mortality rates, that is expected in the future. Consequently, a life tables were calculated based on the empirical and projected data. Long increasing trend of life expectancy was interrupted by Covid-19 pandemic in 2020 and 2021. The effect is much higher for the case of the Czech Republic. However, the life expectancy will increase again in projected future as expected which can be seen in Figure 6.

The absolute values of the results were not optimal, because they were overrated by the model. Therefore, we focus on the pace of increase of life expectancy. Particularly we analyse first absolute differences and relative change of temporary life expectancies.

4 DISCUSSION

Because the paper focuses on the projection of life expectancies for the purpose of pension reform, we aim at the retirement age categories in this section. Time interval between 60 and 70 years is examined as the retirement age is within this range. Pace of increase of life expectancy of males and females is firstly analysed on the example of retirement age of 60 which tells us, how quickly the population will live longer, hence, how quickly the retirement age can/shall be postponed.

The results of our projections are compared with the Eurostat baseline projections (2023). This population projection is deterministic (what-if projection) and based on sets of assumptions for fertility, mortality and net migration. Besides, 5 sensitivity tests are done. "The assumptions formulated for mortality are based on the idea that in 2022 and 2023 the mortality rates have not completely aligned with levels observed before the epidemic, but that they will fully return to that level by 2024" (Eurostat, 2023f). Therefore, age-and-sex specific mortality rates for 2022 are derived from the averages of the years 2018–2021, for 2024 from 2018 and 2019, and 2023 are average of 2022 and 2024. "The age-specific mortality rates are smoothed using weighted regression B-splines, constrained to allow equal or increasing rates only after the age of 25" (Eurostat, 2023f).

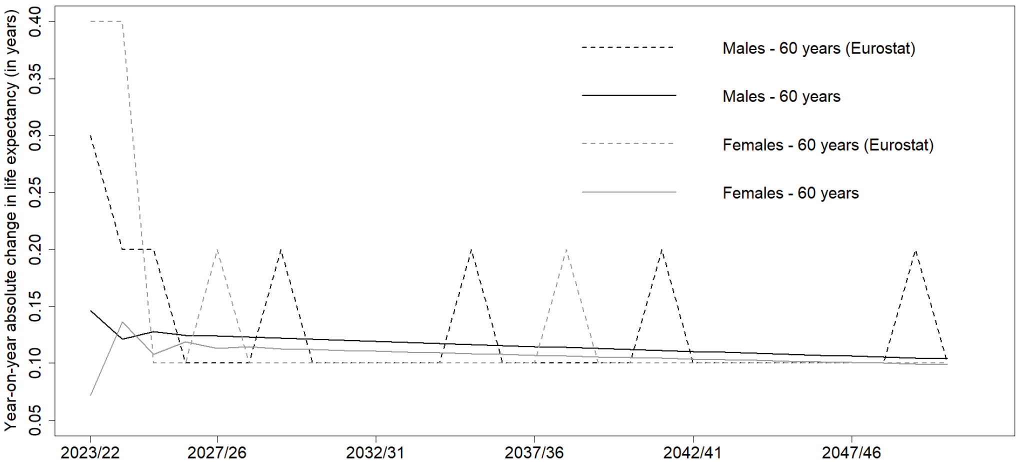
We utilized the data for projected life expectancy by age in completed years from baseline projection. Those data are derived from the period-cohort life table by applying an estimated age-specific gap between these two measures of life expectancy. "The projected life expectancy by age in completed years is provided for the convenience of the users, but it is not the outcome of a regular computation of a life table and it represents only an approximated measure of a life expectancy computed on age-period mortality data" (Eurostat, 2023e).

4.1 Year-on-year absolute changes in life expectancy

The life expectancy at the age of 60 will continue to increase in both examined countries. However, the pace of increase is slowing down. This can be seen from Figure 7 (for Finland) and Figure 8 (for the Czech Republic) where the year-on-year absolute change in life expectancy is displayed. Solid line represents the results of our models, while the dashed line illustrates the calculations of Eurostat (2023) which are rounded on 1 decimal place, so the differences are almost constant in many cases.

Year-on-year absolute change shows that life expectancy of 60 years old males will grow by 0.15 years (53 days) at the beginning, but only by 0.10 years (38 days) at the end of projection period. As the life expectancy of females is already high, the pace of growth will be slower (less than 50 days per year). It can be expected that life expectancy in absolute values will grow by 1 year after 8 years in case of males. It will take 9 years to life expectancy to grow by 1 year in case of females. Both applies to the beginning of the forecasted period. Later, it will take longer. This can have consequences for the pension reform considering that the retirement age is linked to the life expectancy in Finland. The retirement age will increase faster in case of males than females.

Figure 7 Year-on-year absolute change in projected life expectancy (in years) – Finland

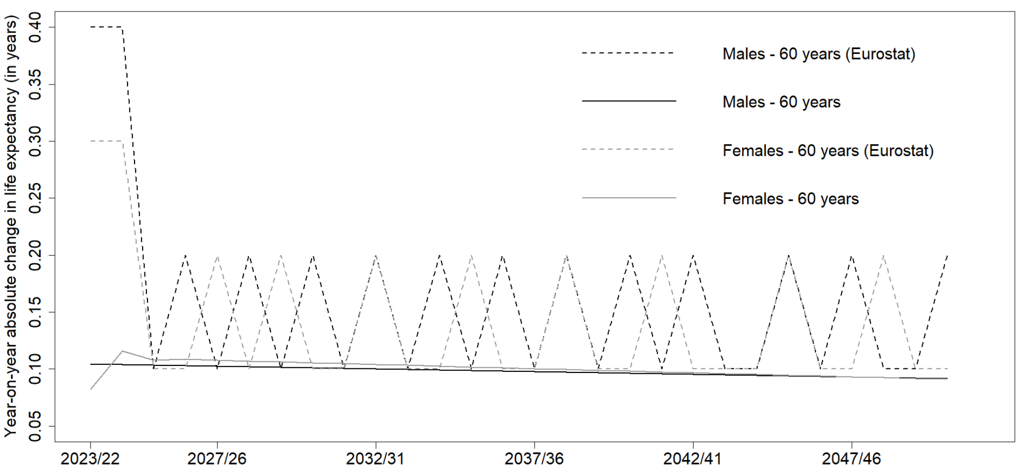


Source: Own elaboration based on the data from Human Mortality Database (HMD, 2023), Eurostat (2023)

Regarding the case of the Czech Republic, the pace of decrease is milder than in Finland. It will take 9.5 years, before the life expectancy of males will increase by 1 year. The pace is almost similar for females.

Comparison of our results with those of Eurostat shows the volatility at the beginning of the period and stability later in population of Finland. Year-on-year changes in Czech population are different every time in case of Eurostat projections.

Figure 8 Year-on-year absolute change in projected life expectancy (in years) – the Czech Republic



Source: Own elaboration based on the data from Human Mortality Database (HMD, 2023), Eurostat (2023)

4.2 Changes in life expectancy

Consequently, we look on the changes in life expectancy in detail. Temporary life expectancies are calculated between years 60 and 70 according to the Formula (9). Therefore, x is equal 60 and i is 10. Then the relative change in time is derived according to Formula (10). t is the starting year of projections 2022. Other years are 2025, 2030, 2035 etc., so n equals firstly to 3 and then to 5. The results for Finnish population are displayed at Table 1.

Temporary life expectancy between 60 and 70 years is according to the expectations higher for females in Finnish population and will increase in time. Every year, some days will be added to current life expectancy. For example, 0.0151 years represents 5.5 days that will live males aged between 60 and 70 longer in 2025 than in 2022. "The concept of relative change in temporary life expectancies refers to the years of life expectancy increase between two ages as a proportion of the maximum possible increase" (Arriaga, 1984). Hence, in relative terms this will mean an increase of life expectancy between 60 and 70 years by 6.88% of the maximum possible increase. Annual relative change suggests that the increase of life expectancy will be the highest between 2022 and 2025 (the index 2.3496 means that the increase will be by 134.96%). It implies that the highest decline in mortality will also occur during this period.

On the other hand, the relative change of life expectancy for females is the lowest in this first period between 2022 and 2025. In other periods, it overcomes the change for males. The highest increase of life expectancy and the highest decrease of mortality for females will occur between 2025 and 2030.

The results for the Czech Republic are displayed at Table 2. Notably, the temporary life expectancy is lower than in Finland. It is only 9.546 years for males in 2022, but according to the projection it will increase in time. Also, the pace of relative change and annual relative change is lower. The increase of life expectancy and decrease of mortality can be expected to be the highest for both sexes at the end of the projected period between 2045 and 2050.

Table 1 Change in life expectancy between 60 and 70 years in Finland

Year	Temp. life exp. ${}_ie_x$ ($i = 10, x = 60$)	Years added (absolute change)	Relative change ${}_iRC_x^n$ ($i = 10, x = 60, n = 3;5$)	Annual relative change ${}_iARC_x^n$ ($i = 10, x = 60, n = 3;5$)
		Males		
2022	9.780	0.0151	0.0688	2.3496
2025	9.796	0.0222	0.1084	2.2681
2030	9.818	0.0198	0.1086	2.2729
2035	9.837	0.0177	0.1087	2.2758
2040	9.855	0.0158	0.1088	2.2784
2045	9.871	0.0141	0.1089	2.2807
2050	9.885			
		Females		
2022	9.884	0.0074	0.0635	2.1622
2025	9.891	0.0125	0.1146	2.4057
2030	9.904	0.0110	0.1143	2.3974
2035	9.915	0.0097	0.1143	2.3991
2040	9.925	0.0086	0.1144	2.4005
2045	9.933	0.0076	0.1145	2.4017
2050	9.941			

Source: Own elaboration based on the data from Human Mortality Database (HMD, 2023)

This positive changes in increase of life expectancy and decrease of mortality shall or rather must be reflected in the reforms of pension systems.

Table 2 Change in life expectancy between 60 and 70 years in the Czech Republic

Year	Temp. life exp. ${}_ie_x$ (i = 10, x = 60)	Years added (absolute change)	Relative change ${}_iRC_x^n$ (i = 10, x = 60, n = 3;5)	Annual relative change ${}_iARC_x^n$ (i = 10, x = 60, n = 3;5)
		Males		
2022	9.546	0.0118	0.0415	1.4017
2025	9.564	0.0297	0.0683	1.4043
2030	9.594	0.0278	0.0684	1.4073
2035	9.622	0.0259	0.0685	1.4101
2040	9.648	0.0242	0.0687	1.4127
2045	9.672	0.0226	0.0688	1.4151
2050	9.695			
		Females		
2022	9.793	0.0106	0.0511	1.7347
2025	9.804	0.0176	0.0900	1.8689
2030	9.822	0.0161	0.0901	1.8703
2035	9.838	0.0146	0.0902	1.8723
2040	9.852	0.0133	0.0903	1.8741
2045	9.866	0.0121	0.0903	1.8757
2050	9.878			

Source: Own elaboration based on the data from Human Mortality Database (HMD, 2023)

4.3 Research limitations and challenges

Our research has limitation that comes from the limitation of the Lee-Carter model and optimal model selection. Age-specific profiles (parameter $\mathbf{a}_x$) are constant in time, which might not be true. Also, parameter $\mathbf{b}_x$ is constant in time.

An influence on results has also the length of the time series (Coale and Kisker, 1986; Šimpach et al., 2014). Also, Booth et al. (2006) who compared projections for 3 time periods and found out that "mean absolute error in log death rates is consistently grater for the long fitting period, while mean error in life expectancy is consistently smallest for the short fitting period."

Unlike in Šimpach et al. (2014) where Hyndman model was an optimal model, it failed in our case. An extension proposed by Li-Lee-Gerland had to be used, because Finland and the Czech Republic have low infant mortality. Besides, there is higher volatility in our data as we use up-to-date time series (Šimpach et al. (2014) used from 1920/1948 to 2012). Especially years 2020 and 2021 brought changes in the long-term development pattern of mortality due to Covid-19 pandemic (Šimpach and Šimpachová Pechrová, 2021).

Our model was probably not able to absorb the shock caused by the Covid-19 pandemic in 2020 and 2021, because after stabilization and the improvement of the situation in 2021, the resulting predictions of demographic indicators were significantly more optimistic in absolute terms than the predictions of Eurostat (Eurostat baseline projections, 2023); despite that year-on-year changes were comparable.

To eliminate the influence of Covid-19 pandemic, multidecrement life tables can be used for determining the change that life expectancy at birth for the theoretical situation when a particular cause of death did not occur. "These tables explain the impact of eradicating a cause of death on life expectancy without determining the effect of changing mortality on life expectancy at each age" (Arriaga, 1984).

We present only one stochastic type of method for life expectancy projection (through the projection of mortality rates and following calculation of life expectancy based on life tables algorithm). Besides, direct projection of empirical life expectancies is possible. For example, Šimpach et al. (2013) used ARIMA(1,0,0) without constant for males and ARIMA(0,2,1) without constant for females to model life expectancy at birth.

There are also deterministic methods based on expert guess, scenarios or regressions that can be used. Therefore, the challenge for future research is to project mortality rates by other methods and life expectancy and to compare the results with the projections done by stochastic Lee-Carter model.

CONCLUSION

The aim of the paper was to describe the future development of life expectancy in the context of pension system reforms that are currently prepared by the politicians in the countries where ageing is the fastest in the EU (Finland and the Czech Republic). One-year age-and-sex-specific mortality rates for both populations aged 0 to 100+ were taken from the Human Mortality Database and they were available in a time series from 1950 to 2021.

Unlike the projections of the statistical institutions such as Eurostat or Czech Statistical office, our paper does not use deterministic models, but rather stochastic ones that are able to capture the influence of the random component as they include error terms.

In contrast to the approach of Šimpach et al. (2013) who used ARIMA models to directly model and predict life expectancy, our stochastic modelling takes into account also mutual interaction among different ages. That means that the evolution of mortality (and life expectancy) does not cross over time. It can happen in case of ARIMA modelling (despite that this was not the case of Šimpach et al., 2013).

Three stochastic models were constructed, calculated in R software and compared. Consequently, mortality rates were projected, and life expectancy calculated up to year 2050. The results are compared to the deterministic projections of the Eurostat.

The most suitable model producing the most feasible results was Lee-Carter model modified by Li-Lee-Gerland because low infant mortality was typical for both populations, so the parameter $\mathbf{b}_x$ had to be rotated. The model projected decrease of mortality rates, which lead to increase of life expectancy. The pace of increase of life expectancy for females was slower in absolute terms, because it was currently already high, so the growth was not that obvious.

Life expectancy year-on-year increments projected by our model up to year 2050 are comparable to Eurostat projections of life expectancy for both populations – Finnish and Czech as we presented on the example of 60-years old population. However, in absolute terms, our stochastic projection gave more optimistic results than the projection of Eurostat.

Consequently, life expectancy between 60 and 70 years was calculated as same as its relative and annual relative changes according to the methodology by Arriaga (1984). According to the results, life expectancy will grow in the future, the pace will be faster in the Czech Republic than in Finland in absolute terms (years added), but the relative change and annual relative change is higher in Finland. Considering that Finnish retirement age is already linked to the life expectancy, it is necessary to re-consider it in the future, if the life expectancy increases too fast, then the retirement age could be too high soon. The Czech Republic retirement age shall also rise, but is not linked to the life expectancy yet. If the growth of it is too fast, then its link to the retirement age might not be optimal.

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