Estimation of the Optimal Parameter of Delay in Young and Lowe Indices in the Fisher Index Approximation

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Abstract

The Cost of Living Index (COLI) enables to show changes in the cost of household consumption assuming the constant utility level. The most commonly used way to approximate COLI is the Consumer Price Index (CPI) calculated by using the Laspeyres index. Many economists consider superlative indices such as the Fisher index as the best proxy for the COLI. However, it uses quantity data not only from a base but also the current period, which limits its usefulness. Thus, the indices like the Lowe index and the Young Index are used in order to approximate the Fisher index value without using current period expenditure data. Both of these indices use an additional parameter of delay. The purpose of this paper is to examine the influence of the parameter mentioned above on the Fisher index approximation using the empirical and simulation data.

Keywords	*JEL code*
CPI, Young index, Lowe index, Laspeyres index, Fisher index, COLI, Cost of Living Index, Consumer Price Index, inflation	*C43, C49*

INTRODUCTION

As an approximation of changes in the costs of household consumption assuming the constant utility (Cost of Living Index known as COLI), the Consumer Price Index is the most common way to measure inflation. The Cost of Living Index for a single household can be defined as the minimum cost of achieving a certain standard of living during a given period, divided by the minimum cost of achieving the same standard of living during a base period. However, in practice, the CPI is measured by the Laspeyres index, which is a subject of wide criticism. It risks bias due to ignoring changes in consumers' behavior (such as changing the retailers to these with lower prices) due to the price change, which results in overstating inflation. Thus, some economists treat the Laspeyres index as the Cost of Goods index (in opposite

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to the Cost of Living Index). According to "superlative indices" theory developed by W. Erwin Diewert some indices such as the Fisher index can provide a fair approximation of the COLI "using the quantities in the base period as well as in the current reference period as weights in a symmetric fashion". Unfortunately, the Fisher index requires quantity data set from the current period, which takes time to process. This causes the inability of using the Fisher index results in many economic decisions such as monetary policy or adjusting social pensions. On the grounds of this issue statisticians proposed indices that approximate the Fisher index without using current expenditure data i.e. the AG mean index, the Lloyd-Mounton Index or the Lowe and the Young indices. The Lowe and Young indices compare two points in time, let us say 0 (base period) and τ, which can be any point between 0 and current period t, as well as precedes 0. The purpose of this paper is to approximate the optimal estimation of the τ parameter and verify the quality of obtained approximations. To reach this aim we realize empirical and simulation studies.

The structure of the paper is as follows: Section 1 discusses the connection between the Cost of Living Index and the Fisher index. Sections 2 and 3 introduce the Lowe price index and the Young price index. Section 4 describes some other approximations of the Fisher price index. Section 5 presents the simulation study, which concerns the bias of the previously mentioned indices. Section 6 displays an empirical study for 7 European countries and the EU benchmark for the 2006–2018 period. Last section demonstrates the main conclusions.

1 ROLE OF THE FISHER PRICE INDEX THE COLI MEASUREMENT

The COLI was introduced in 1961 by a committee chaired by George Stigler, which highlighted the difference between the CPI, in a form that was used then, and the true cost of living. The committee concluded by recommendation to the National Bureau of Labor Statistics in the USA to start using the COLI and adapt the Consumer Price index to obtain a better approximation of the Cost of Living index. Thirty-five years later in 1996, the Booskin Committee assessed the measurement of the COLI by the CPI in the US and concluded that it was overstating the true COLI value by 1.1 percent annually.

To define the Cost of Living index let us consider household preferences over commodities being represented by the utility function U(q) which is dual to the consumer expenditure function $E(P, \bar{u}) = \min_Q\{P^T Q | U(Q) \geq \bar{u}\}$. Most of households, wants to maximize the utility function for given budget limitations (in other words to minimize expenditure needed to achieve the utility level ū), and it leads to the following form of the Konüs price index:

$$P_K = \frac{E\left(P^T, \bar{u}\right)}{E\left(P^S, \bar{u}\right)}, \tag{1}$$

where *s* denotes the base period, *t* denotes for the current period and *P* considers prices at any moment *τ* are given by $P^T = [P_1^\tau, P_2^\tau, \ldots, P_N^\tau]^T$

The difference between the Cost of Living index which captures the changes of commodities quantity and the Laspeyres index that relies on quantities from the previous period is called the substitution bias and it has the biggest factor in miscalculating inflation rate. It is worth mentioning that even though in theory the Cost of Living index was defined by Russian economist Konüs in 1924, in practice the Fisher index is considered the easiest way to calculate COLI (Fisher, 1922).

As it was stated in the introduction, as a rule the Laspreyres index overstates true inflation because its formula takes under consideration quantities only from the previous time period:

$$P_{La} = \frac{\sum_{i=1}^{n} p_{i,t} q_{i,b}}{\sum_{i=1}^{n} p_{i,b} q_{i,b}}, \tag{2}$$

where $p_{i,t}$ means the price of a commodity i at current time moment t, $p_{i,b}$ – the price of commodity i at base time moment 0 and $q_{i,b}$ – the quantity of a commodity i at base time moment 0.

On the other hand, the Pasche index understates inflation because it takes only the quantity from the current period i.e.

$$P_{Pa} = \frac{\sum_{i=1}^{n} p_{i,t} q_{i,t}}{\sum_{i=1}^{n} p_{i,b} q_{i,t}}, \tag{3}$$

where $q_{i,t}$ means the quantity of commodity i at the current time moment t.

Because the Laspeyres index and the Pasche index have contrary biases, the Fisher index can be calculated as a geometric mean of them, i.e.

$$P_F = \sqrt{P_{La} P_{Pa}}. \tag{4}$$

2 LOWE PRICE INDEX

As it was mentioned above, the biggest flaw of the current price indices is the time needed for their publication. This time gap necessary to gather and process data causes low usefulness in economic decisions. That is why we use proxies for the Fisher Index.

Let us introduce some new period τ which precedes base period (b) (some authors (Białek, 2017) consider also situations when τ > b). The Lowe price index can be expressed as follows:

$$P_{LO} = \frac{\sum_{i=1}^{N} p_i^t q_i^\tau}{\sum_{i=1}^{N} p_i^b q_i^\tau} = \sum_{i=1}^{N} w_i^{\tau,b}, (\frac{p_i^t}{p_i^b}), \tag{5}$$

where:

$$w_i^{\tau,b} = \frac{p_i^b q_i^\tau}{\sum_{k=1}^{N} p_k^b q_k^\tau}. \tag{6}$$

The arithmetic form of the Lowe index is not the only one. There is also a geometric version of this price index, i.e.

$$P_{GLO} = \prod_{i=1}^{N} (\frac{p_i^t}{p_i^b})^{w_i^{\tau,b}}. \tag{7}$$

3 YOUNG PRICE INDEX

The second considered proxy for the Fisher price index is the Young index. The Young index is considered weaker in terms of fulfilled axioms, however, in some cases, it gives better Fisher index approximation than the Lowe Index (Armknecht and Silver, 2012). The Young index can be written as follows:

$$w_0^\tau = \frac{p_i^\tau q_i^\tau}{\sum_{k=1}^{N} p_k^\tau q_k^\tau}, \tag{8}$$

where:

$$w_0^\tau = \frac{p_i^\tau q_i^\tau}{\sum_{k=1}^{N} p_k^\tau q_k^\tau}. \tag{9}$$

Similarly to the Lowe index case, we also take into consideration the geometric version of the Young index, i.e.

$$P_{GY} = \prod_{i=1}^{N} (\frac{p_i^t}{p_i^b})^{w_i^\tau}. \tag{10}$$

4 OTHER PROXIES FOR THE FISHER INDEX FORMULA

The indices described in Sections 2 and 3 are not only those that can be used to approximate the Fisher index. We should also mention about the Arithmetic-Geometric (AG) mean index and the Lloyd-Moulton Index.

The AG mean index was proposed by Alan H. Dorfman and Janice Lent (2009), hence from their last names, it is sometimes called the L-D index as well. In the base version, it is the weighted from arithmetic mean of the Laspeyre s index and it's geometric counterpart i.e.

$$P_{LD} = \sigma \prod_{i=1} (\frac{p_i^t}{p_i^b})^{s_i^b} + (1-\sigma) \sum_{i=1} (\frac{p_i^t}{p_i^b}) s_i^b, \tag{11}$$

where a parameter σ is elasticity of substitution of commodities covered, s_i^b is the expenditure share at base time 0 of the i-th commodity, i.e.

$$s_i^b = \frac{p_i^b q_i^b}{\sum_{i=1}^{N} p_i^b q_i^b}. \tag{12}$$

The second index which should be referred to is the Lloyd–Moulton (Lloyd, 1975; Moulton, 1996) index:

$$P_{LM} = \left\{ \sum_{i=1}^{N} s_i^b \left(\frac{p_i^t}{p_i^b} \right)^{1-\sigma} \right\}^{\frac{1}{1-\sigma}}, \tag{13}$$

where parameter has the identical meaning as before (see Formula (11)).

The Lloyd – Moulton index has also an alternative version which was suggested by Huang, Waruna and Polard (2015) i.e.

$$P_{ModLM} = \left\{ \sum_{i=1}^{N} s_i^\tau \left(\frac{p_i^t}{p_i^b} \right)^{1-\sigma} \right\}^{\frac{1}{1-\sigma}}, \tag{14}$$

where:

$$s_i^\tau = \frac{p_i^\tau q_i^\tau}{\sum_{i=1}^{N} p_i^\tau q_i^\tau}. \tag{15}$$

5 SIMULATION STUDY

Through the simulation, we wish to check how the bias between the Fisher index and the studied indices differ for various delay parameters and product baskets. We consider several case studies, which differ from each other with respect to correlation between prices and quantities, the direction of price changes and inflation rate.

Case 1

Let us consider a scenario with N = 10 commodities where both prices and quantities change linearly in the following way:

$$p_i^T = p_i^b + \left(p_i^t - p_i^b\right)T\ , \tag{16}$$

$$q_i^T = q_i^b + \left(q_i^t - q_i^b\right)T,\ \mathrm{T} \in [0,1], \tag{17}$$

where p_i^b is goods price in the base period 0, p_i^t is the price in the current period t, q_i^b is the goods quantity in the base period and q_i^t is the quantity in current period. In this scenario, we are going to control the parameter of delay (τ) and we tend to optimize its value.

We selected four baskets for the simulation:

a) N = 10 goods with negative correlation between prices and quantities (prices increase and quantities decrease).

Table 1 The values of prices and quantities at time *0* and *t* for the case a

Goods no.	p^0	p^t	q^0	q^t
1	100	120	1 000	950
2	10	11	9 000	8 000
3	5	6.6	12 500	12 000
4	1 000	1 200	2 02	150
5	120	150	2 500	2 000
6	500	550	2 000	1 900
7	150	155	2 000	1 900
8	1 550	2 000	100	70
9	2 000	2 200	200	150
10	7	10	1 450	1 000

Source: Own construction

b) N = 10 goods negative correlation between prices and quantities (prices decrease and quantities increase).

Table 2 The values of prices and quantities at time *0* and *t* for the case b

Goods no.	p^0	p^t	q^0	q^t
1	100	95	1 000	1 100
2	10	9	9 000	9500
3	5	4.6	12 500	13 000
4	1 300	1 200	202	240
5	120	110	2 500	3 200
6	500	470	2 000	2 300
7	150	145	2 000	2 100
8	1 550	1 400	100	120
9	2 000	1 900	200	230
10	7	5	1 450	1 600

Source: Own construction

c) N = 10 with mixed goods. In five cases the price increased and quantity decreased and in three cases the price decreased and quantities increased. Hence, these can be considered normal goods. For one of the commodity, the price decrease was followed with a quantity decrease as well (which can be observed in some kinds of commodities such as computer games or gaming consoles, when the majority of purchases are made right after the introduction of the commodity to the market), and for the last one, the price increase caused the quantity increase (which is common for luxury goods – see Veblen paradox, 1899). As both of these cases are in minority in the consumer price index, basket there are represented as a minority in the simulation as well.

Table 3 The values of prices and quantities at time *0* and *t* for the case c

Goods no.	p^0	p^t	q^0	q^t
1	100	105	1 000	900
2	10	11	9 000	8 500
3	5	5.8	12 500	11 000
4	1 300	1 370	202	170
5	120	130	2 500	2 300
6	500	470	2 000	2 200
7	150	140	2 000	2 250
8	1 550	1 400	100	130
9	2 000	2 300	200	230
10	7	5	1 450	1 300

Source: Own construction

d) N = 10 with prices increase and quantity decrease with the aim for inflation around 2.5% (optimal parameter of inflation rate for the National Polish Bank).

Table 4 The values of prices and quantities at time *0* and *t* for the case d

Goods no.	p^0	p^t	q^0	q^t
1	100	102	1 000	970
2	10	10.4	9000	8 000
3	5	5.6	12 500	12 000
4	1 000	1 030	202	200
5	120	122	2 500	2 400
6	500	510	2 000	1 960
7	150	153	2 000	1 990
8	1 550	1 600	100	90
9	2 000	2 050	200	170
10	7	7.5	1 450	1 440

Source: Own construction

In the simulation, we changed the value of τ in the range [–2; 0.75]. Even though the most common practice is to use τ that precedes the base period, in some cases in previous studies τ parameter that was between the base and current period gave the best results.

Case 2

Using the same goods and services basket as in case one, let us consider exponential price and quantity change, i.e.

$$p_i^T = p_i^b \left(\frac{p_i^t}{p_i^b} \right)^T, \tag{18}$$

$$q_i^T = q_i^b \left(\frac{q_i^t}{q_i^b} \right)^T, \text{ where: } T \in [0,1]. \tag{19}$$

5.1 Simulation Results

Case 1a

Table 5 The values of the considered price indices for the case 1a

τ parameter	Laspeyres	Paasche	Fisher	Young	Geo.Young	Lowe	Geo.Lowe
–2.00	1.1389	1.1341	1.1365	1.1264	1.1242	1.1454	1.1425
–1.00	1.1389	1.1341	1.1365	1.1348	1.1322	1.1425	1.1397
–0.75	1.1389	1.1341	1.1365	1.1361	1.1335	1.1417	1.1389
–0.50	1.1389	1.1341	1.1365	1.1373	1.1346	1.1408	1.1380
–0.25	1.1389	1.1341	1.1365	1.1382	1.1355	1.1399	1.1371
0.25	1.1389	1.1341	1.1365	1.1393	1.1366	1.1378	1.1351
0.50	1.1389	1.1341	1.1365	1.1396	1.1369	1.1367	1.1340
0.75	1.1389	1.1341	1.1365	1.1397	1.1369	1.1354	1.1328

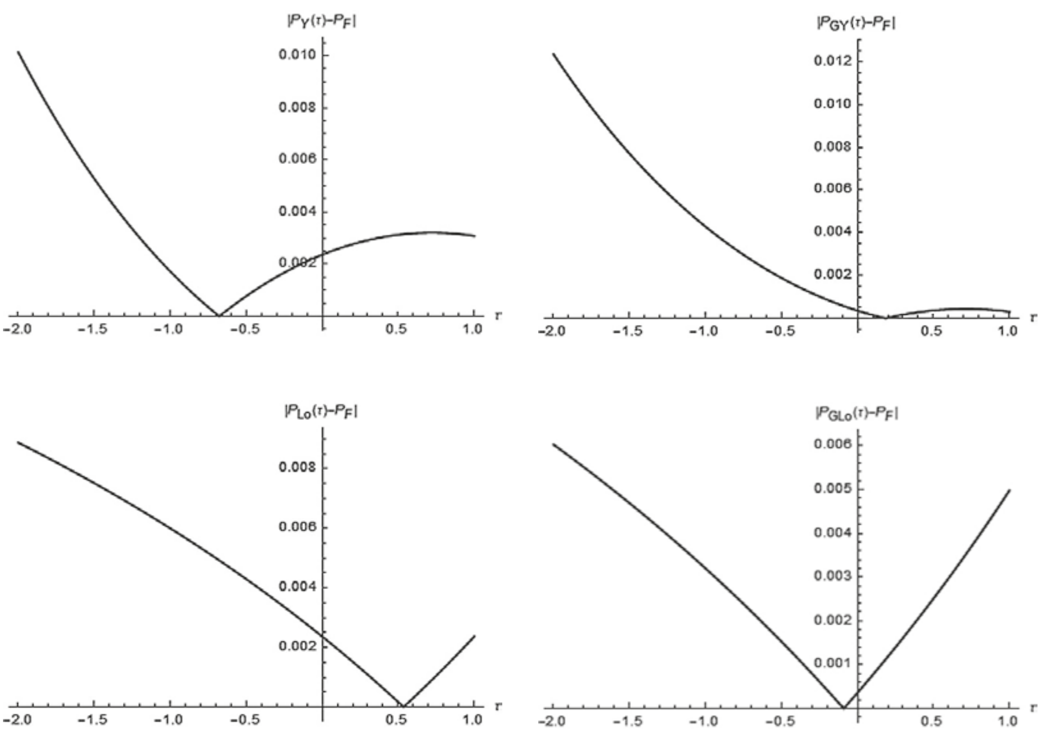
Source: Own construction in Mathematica 11

Table 6 Distance between considered price indices and the Fisher index for the case 1a

τ parameter	P_L-P_F	P_Y-P_F	P_{GY}-P_F	P_{Lo}-P_F	P_{GLo}-P_F
–2.00	0.0023675	–0.0101307	–0.0123396	0.0088750	0.0060260
–1.00	0.0023675	–0.0017211	–0.0042852	0.0059991	0.0031995
–0.75	0.0023675	–0.0003513	–0.0029695	0.0051726	0.0023886
–0.50	0.0023675	0.0007769	–0.0018858	0.0042952	0.0015283
–0.25	0.0023675	0.0016793	–0.0010194	0.0033620	0.0006139
0.25	0.0023675	0.0028495	0.0001008	0.0013054	–0.0013984
0.50	0.0023675	0.0031295	0.0003657	0.0001687	–0.0025092
0.75	0.0023675	0.0032086	0.0004360	–0.0010508	–0.0036996

Source: Own construction in Mathematica 11

Figure 1 Absolute differences between the Fisher index and the considered price indices as functions of τ (case 1a)



Source: Own construction in Mathematica 11

Case 1b

Table 7 The values of the considered price indices and their distances to the Fisher price index for the case 1b

τ parameter	Laspeyres	Paasche	Fisher	Young	Geo.Young	Lowe	Geo.Lowe
–2.00	0.9358	0.9353	0.9355	0.9365	0.9362	0.9375	0.9372
–1.00	0.9358	0.9353	0.9355	0.9360	0.9357	0.9365	0.9362
–0.75	0.9358	0.9353	0.9355	0.9359	0.9356	0.9363	0.9360
–0.50	0.9358	0.9353	0.9355	0.9359	0.9356	0.9361	0.9358
–0.25	0.9358	0.9353	0.9355	0.9358	0.9355	0.9359	0.9357
0.25	0.9358	0.9353	0.9355	0.9358	0.9355	0.9356	0.9354
0.50	0.9358	0.9353	0.9355	0.9358	0.9355	0.9355	0.9352
0.75	0.9358	0.9353	0.9355	0.9358	0.9355	0.9354	0.9351

Source: Own construction in Mathematica 11

Table 8 Distance between considered price indices and the Fisher index for the case 1b

τ parameter	P_L-P_F	P_Y-P_F	P_{GY}-P_F	P_{Lo}-P_F	P_{GLo}-P_F
–2.00	0.0002600	0.0010074	0.0006373	0.0019821	0.0016669
–1.00	0.0002600	0.0004750	0.0001485	0.0009664	0.0006666
–0.75	0.0002600	0.0003967	0.0000801	0.0007671	0.0004703
–0.50	0.0002600	0.0003358	0.0000287	0.0005841	0.0002903
–0.25	0.0002600	0.0002906	–0.0000071	0.0004156	0.0001245
0.25	0.0002600	0.0002427	–0.0000369	0.0001157	–0.0001705
0.50	0.0002600	0.0002379	–0.0000327	–0.0000183	–0.0003023
0.75	0.0002600	0.0002449	0.0000168	–0.0001432	–0.0004251

Source: Own construction in Mathematica 11

Figure 2 Absolute differences between the Fisher index and the considered price indices as functions of τ (case 1b)

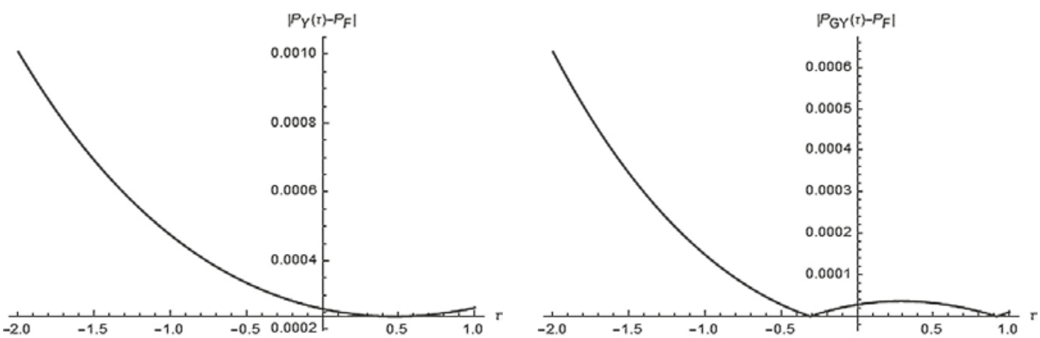
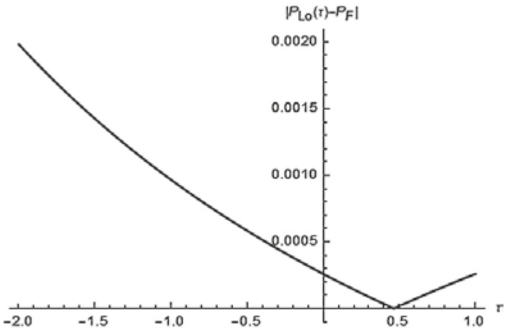
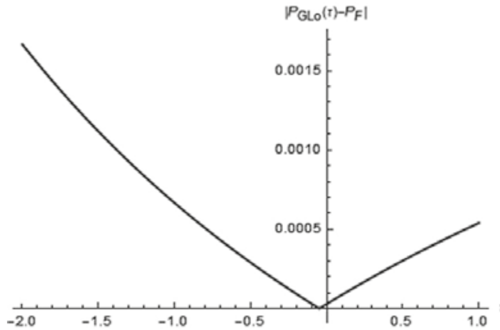


Figure 2 (continuation)





Source: Own construction in Mathematica 11

Case 1c

Table 9 The values of the considered price indices and their distances to the Fisher price index for the case 1c

τ parameter	Laspeyres	Paasche	Fisher	Young	Geo.Young	Lowe	Geo.Lowe
−2.00	1.00942	1.0053	1.0074	1.0038	1.0002	1.0192	1.0155
−1.00	1.00942	1.0053	1.0074	1.0061	1.0024	1.0140	1.0102
−0.75	1.00942	1.0053	1.0074	1.0069	1.0031	1.0128	1.0090
−0.50	1.00942	1.0053	1.0074	1.0077	1.0038	1.0117	1.0078
−0.25	1.00942	1.0053	1.0074	1.0085	1.0046	1.0105	1.0066
0.25	1.00942	1.0053	1.0074	1.0104	1.0064	1.0084	1.0044
0.50	1.00942	1.0053	1.0074	1.0114	1.0073	1.0073	1.0033
0.75	1.00942	1.0053	1.0074	1.0124	1.0083	1.0063	1.0023

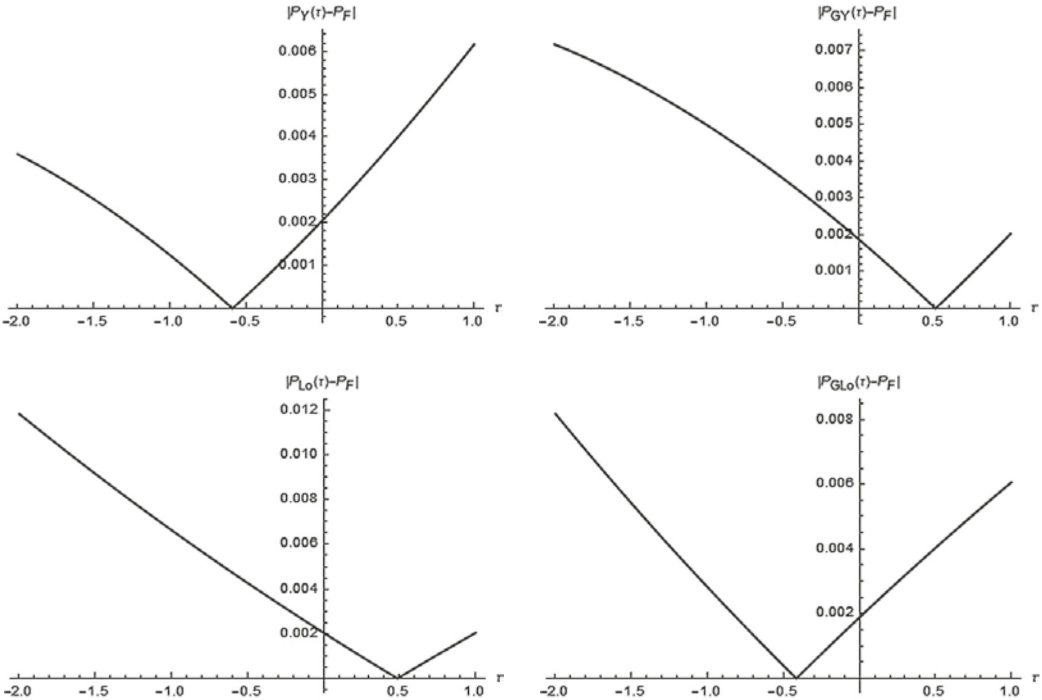
Source: Own construction in Mathematica 11

Table 10 Distance between considered price indices and the Fisher index for the case 1c

τ parameter	P_L-P_F	P_Y-P_F	P_{GY}-P_F	P_{Lo}-P_F	P_{GLo}-P_F
−2.00	0.0020476	−0.0036002	−0.0071668	0.0118328	0.0081712
−1.00	0.0020476	−0.0012475	−0.0049855	0.0066402	0.0028221
−0.75	0.0020476	−0.0005061	−0.0042910	0.0054400	0.0015898
−0.50	0.0020476	0.0002914	−0.0035417	0.0042756	0.0003956
−0.25	0.0020476	0.0011432	−0.0027394	0.0031452	−0.0007623
0.25	0.0020476	0.0030031	−0.0009812	0.0009811	−0.0029754
0.50	0.0020476	0.0040087	−0.0000276	−0.0000554	−0.0040336
0.75	0.0020476	0.0050635	0.0009744	−0.0010631	−0.0050615

Source: Own construction in Mathematica 11

Figure 3 Absolute differences between the Fisher index and the considered price indices as functions of τ (case 1c)



Source: Own construction in Mathematica 11

Case 1d

Table 11 The values of the considered price indices and their distances to the Fisher price index for the case 1d

τ parameter	Laspeyres	Paasche	Fisher	Young	Geo.Young	Lowe	Geo.Lowe
–2.00	1.02515	1.0250	1.0251	1.0248	1.0247	1.0253	1.0252
–1.00	1.02515	1.0250	1.0251	1.0250	1.0249	1.0253	1.0251
–0.75	1.02515	1.0250	1.0251	1.0250	1.0249	1.0252	1.0251
–0.50	1.02515	1.0250	1.0251	1.0251	1.0250	1.0252	1.0251
–0.25	1.02515	1.0250	1.0251	1.0251	1.0250	1.0252	1.0251
0.25	1.02515	1.0250	1.0251	1.0252	1.0251	1.0251	1.0250
0.50	1.02515	1.0250	1.0251	1.0252	1.0251	1.0251	1.0250
0.75	1.02515	1.0250	1.0251	1.0253	1.0251	1.0251	1.0250

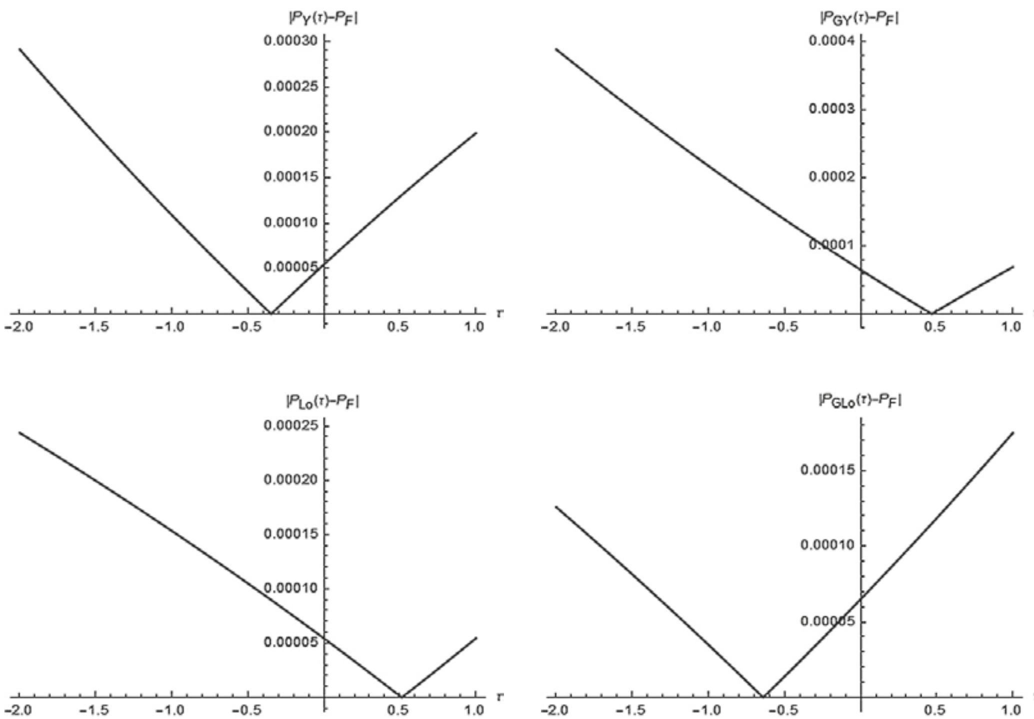
Source: Own construction in Mathematica 11

Table 12 Distance between considered price indices and the Fisher index for the case 1d

τ parameter	P_L-P_F	P_Y-P_F	P_{GY}-P_F	P_{Lo}-P_F	P_{GLo}-P_F
–2.00	0.0000545	–0.0002913	–0.0003892	0.0002437	0.0001259
–1.00	0.0000545	–0.0001089	–0.0002178	0.0001534	0.0000348
–0.75	0.0000545	–0.0000663	–0.0001779	0.0001296	0.0000107
–0.50	0.0000545	–0.0000249	–0.0001391	0.0001051	–0.0000139
–0.25	0.0000545	0.0000154	–0.0001015	0.0000801	–0.0000392
0.25	0.0000545	0.0000924	–0.0000298	0.0000282	–0.0000915
0.50	0.0000545	0.0001291	0.0000044	0.0000014	–0.0001187
0.75	0.0000545	0.0001647	0.0000373	-0.0000262	–0.0001465

Source: Own construction in Mathematica 11

Figure 4 Absolute differences between the Fisher index and the considered price indices as functions of *τ* (case 1d)



Source: Own construction in Mathematica 11

Case 2a

Table 13 The values of the considered price indices and their distances to the Fisher price index for the case 2a

τ parameter	Laspeyres	Paasche	Fisher	Young	Geo.Young	Lowe	Geo.Lowe
–2.00	1.1389	1.1341	1.1365	1.1380	1.1353	1.1492	1.1463
–1.00	1.1389	1.1341	1.1365	1.1384	1.1357	1.1439	1.1411
–0.75	1.1389	1.1341	1.1365	1.1385	1.1358	1.1426	1.1398
–0.50	1.1389	1.1341	1.1365	1.1386	1.1359	1.1414	1.1386
–0.25	1.1389	1.1341	1.1365	1.1387	1.1360	1.1401	1.1373
0.25	1.1389	1.1341	1.1365	1.1390	1.1363	1.1376	1.1349
0.50	1.1389	1.1341	1.1365	1.1392	1.1364	1.1364	1.1338
0.75	1.1389	1.1341	1.1365	1.1394	1.1366	1.1353	1.1326

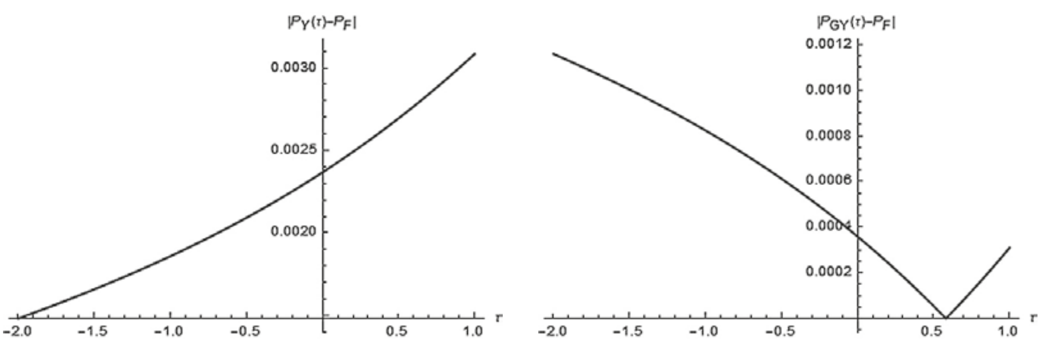
Source: Own construction in Mathematica 11

Table 14 Distance between considered price indices and the Fisher index for the case 2a

τ parameter	P_L-P_F	P_Y-P_F	P_{GY}-P_F	P_{Lo}-P_F	P_{GLo}-P_F
–2.00	0.0023675	0.0014823	–0.0011591	0.0127277	0.0098000
–1.00	0.0023675	0.0018598	–0.0008239	0.0074280	0.0045957
–0.75	0.0023675	0.0019715	–0.0007228	0.0061365	0.0033298
–0.50	0.0023675	0.0020925	–0.0006125	0.0048617	0.0020810
–0.25	0.0023675	0.0022241	–0.0004919	0.0036049	0.0008507
0.25	0.0023675	0.0025239	–0.0002147	0.0011509	–0.0015489
0.50	0.0023675	0.0026947	–0.0000557	–0.0000437	–0.0027158
0.75	0.0023675	0.0028810	0.0001184	–0.0012152	–0.0038594

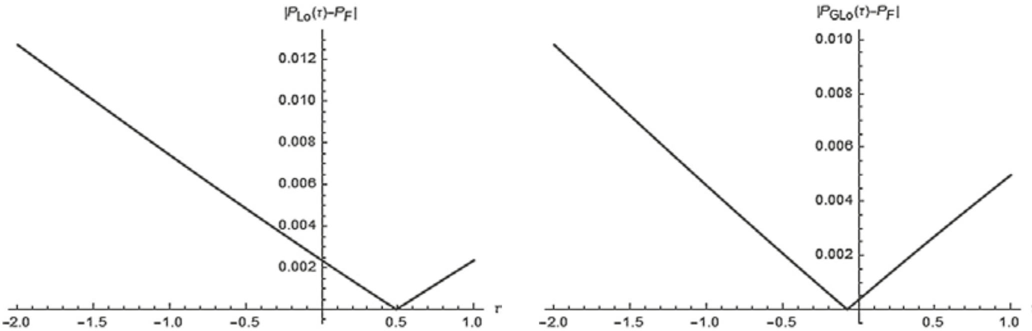
Source: Own construction in Mathematica 11

Figure 5 Absolute differences between the Fisher index and the considered price indices as functions of τ (case 2a)



Source: Own construction in Mathematica 11

Figure 5 (continuation)



Source: Own construction in Mathematica 11

Case 2b

Table 15 The values of the considered price indices and their distances to the Fisher price index for the case 2b

τ parameter	Laspeyres	Paasche	Fisher	Young	Geo.Young	Lowe	Geo.Lowe
−2.00	0.9358	0.9353	0.9355	0.935425	0.9350	0.9368	0.9365
−1.00	0.9358	0.9353	0.9355	0.9357	0.9353	0.9363	0.9360
−0.75	0.9358	0.9353	0.9355	0.9357	0.9354	0.9362	0.9359
−0.50	0.9358	0.9353	0.9355	0.9357	0.9354	0.9360	0.9358
−0.25	0.9358	0.9353	0.9355	0.9358	0.9355	0.9359	0.9356
0.25	0.9358	0.9353	0.9355	0.9358	0.9355	0.9357	0.9354
0.50	0.9358	0.9353	0.9355	0.9358	0.9355	0.9355	0.9352
0.75	0.9358	0.9353	0.9355	0.9358	0.9355	0.9354	0.9351

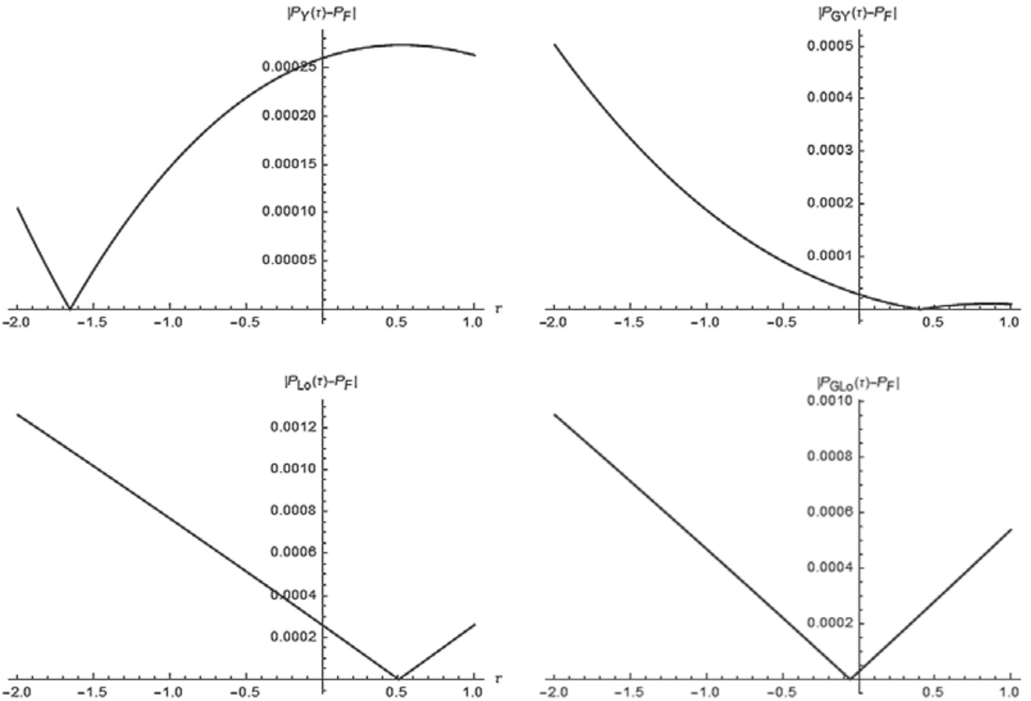
Source: Own construction in Mathematica 11

Table 16 Distance between considered price indices and the Fisher index for the case 2b

τ parameter	P_L-P_F	P_Y-P_F	P_{GY}-P_F	P_{Lo}-P_F	P_{GLo}-P_F
−2.00	0.0002600	−0.0001042	−0.0005026	0.0012598	0.0009538
−1.00	0.0002600	0.0001474	−0.0001885	0.0007666	0.0004693
−0.75	0.0002600	0.0001873	−0.0001355	0.0006412	0.0003461
−0.50	0.0002600	0.0002190	−0.0000915	0.0005150	0.0002220
−0.25	0.0002600	0.0002431	−0.0000561	0.0003879	0.0000971
0.25	0.0002600	0.0002700	−0.0000087	0.0001312	−0.0001552
0.50	0.0002600	0.0002736	0.0000042	0.0000017	−0.0002826
0.75	0.0002600	0.0002712	0.0000104	−0.0001287	−0.0004108

Source: Own construction in Mathematica 11

Figure 6 Absolute differences between the Fisher index and the considered price indices as functions of τ (case 2b)



Source: Own construction in Mathematica 11

Case 2c

Table 17 The values of the considered price indices and their distances to the Fisher price index for the case 2c

τ parameter	Laspeyres	Paasche	Fisher	Young	Geo.Young	Lowe	Geo.Lowe
−2.00	1.00942	1.0053	1.0074	1.0036	0.9998	1.0185	1.0148
−1.00	1.00942	1.0053	1.0074	1.0062	1.0024	1.0138	1.0100
−0.75	1.00942	1.0053	1.0074	1.0069	1.0031	1.0127	1.0088
−0.50	1.00942	1.0053	1.0074	1.0077	1.0039	1.0116	1.0077
−0.25	1.00942	1.0053	1.0074	1.0085	1.0046	1.0105	1.0066
0.25	1.00942	1.0053	1.0074	1.0104	1.0064	1.0084	1.0044
0.50	1.00942	1.0053	1.0074	1.0114	1.0073	1.0073	1.0034
0.75	1.00942	1.0053	1.0074	1.0124	1.0083	1.0063	1.0023

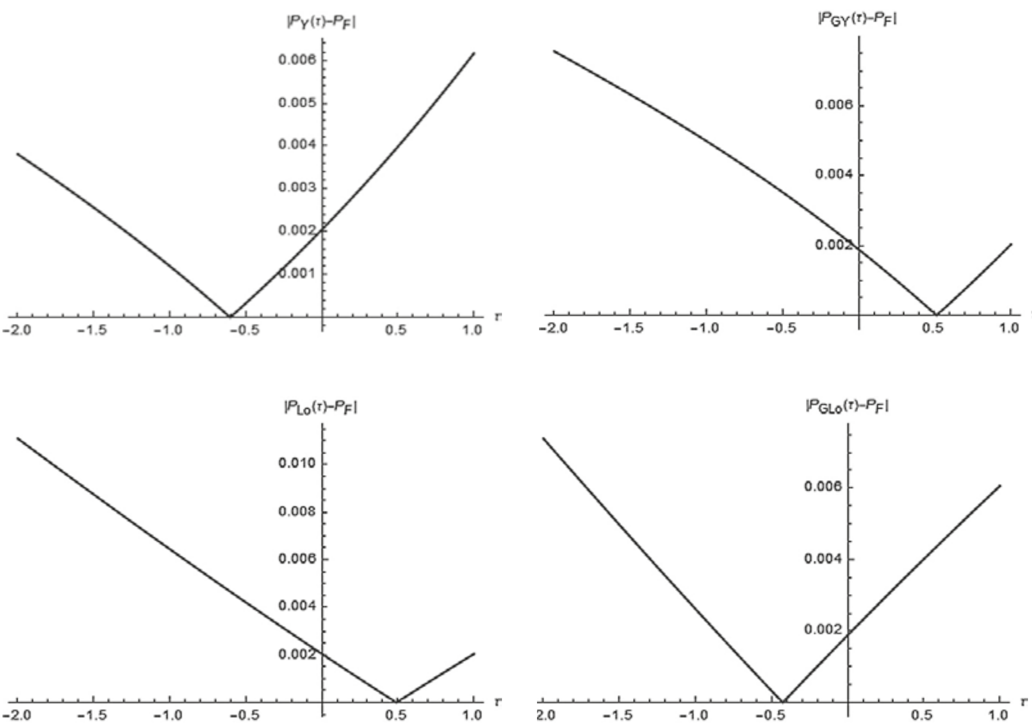
Source: Own construction in Mathematica 11

Table 18 Distance between considered price indices and the Fisher index for the case 2c

τ parameter	P_L-P_F	P_Y-P_F	P_{GY}-P_F	P_{Lo}-P_F	P_{GLo}-P_F
–2.00	0.0020476	–0.0038098	–0.0075550	0.0110956	0.0073880
–1.00	0.0020476	–0.0011955	–0.0049864	0.0064462	0.0026152
–0.75	0.0020476	–0.0004550	–0.0042735	0.0053202	0.0014618
–0.50	0.0020476	0.0003298	–0.0035221	0.0042111	0.0003266
–0.25	0.0020476	0.0011627	–0.0027274	0.0031200	–0.0007894
0.25	0.0020476	0.0029876	–0.0009925	0.0009945	–0.0029610
0.50	0.0020476	0.0039858	–0.0000451	–0.0000385	–0.0040155
0.75	0.0020476	0.0050447	0.0009596	–0.0010513	–0.0050487

Source: Own construction in Mathematica 11

Figure 7 Absolute differences between the Fisher index and the considered price indices as functions of *τ* (case 2c)



Source: Own construction in Mathematica 11

Case 2d

Table 19 The values of the considered price indices and their distances to the Fisher price index for the case 2d

τ parameter	Laspeyres	Paasche	Fisher	Young	Geo.Young	Lowe	Geo.Lowe
–2.00	1.02515	1.0250	1.0251	1.0249	1.0248	1.0254	1.0253
–1.00	1.02515	1.0250	1.0251	1.0250	1.0249	1.0253	1.0251
–0.75	1.02515	1.0250	1.0251	1.0251	1.0250	1.0252	1.0251
–0.50	1.02515	1.0250	1.0251	1.0251	1.0250	1.0252	1.0251
–0.25	1.02515	1.0250	1.0251	1.0251	1.0250	1.0252	1.0251
0.25	1.02515	1.0250	1.0251	1.0252	1.0251	1.0251	1.0250
0.50	1.02515	1.0250	1.0251	1.0252	1.0251	1.0251	1.0250
0.75	1.02515	1.0250	1.0251	1.0253	1.0251	1.0251	1.0250

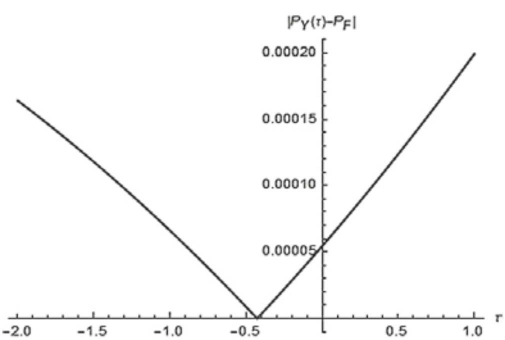
Source: Own construction in Mathematica 11

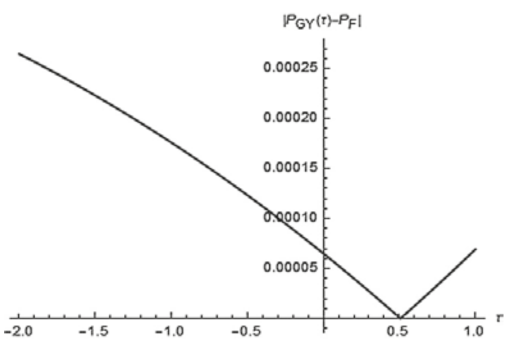
Table 20 Distance between considered price indices and the Fisher index for the case 2d

τ parameter	P_L-P_F	P_Y-P_F	P_{GY}-P_F	P_{Lo}-P_F	P_{GLo}-P_F
–2.00	0.0000545	–0.0001638	–0.0002645	0.0002751	0.0001586
–1.00	0.0000545	–0.0000661	–0.0001759	0.0001646	0.0000464
–0.75	0.0000545	–0.0000382	–0.0001503	0.0001370	0.0000184
–0.50	0.0000545	–0.0000087	–0.0001233	0.0001095	–0.0000095
–0.25	0.0000545	0.0000221	0.0000949	0.0000820	–0.0000373
0.25	0.0000545	0.0000884	–0.0000337	0.0000271	–0.0000927
0.50	0.0000545	0.0001237	–0.0000009	–0.0000002	–0.0001203
0.75	0.0000545	0.0001606	0.0000333	–0.0000274	–0.0001477

Source: Own construction in Mathematica 11

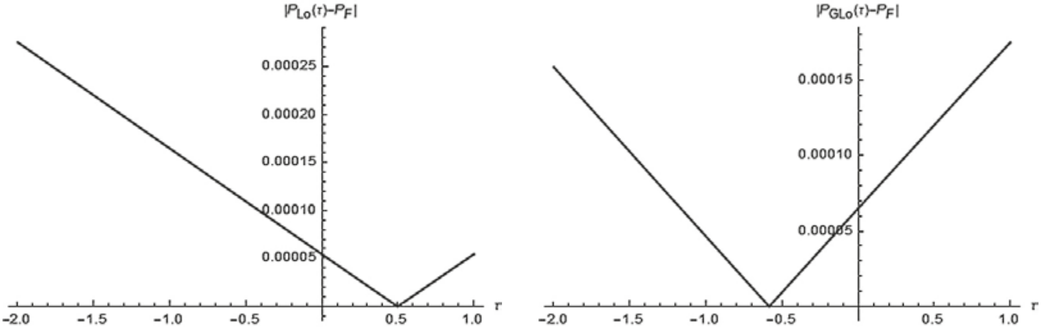
Figure 8 Absolute differences between the Fisher index and the considered price indices as functions of τ (case 2d)





Source: Own construction in Mathematica 11

Figure 8 (continuation)



Source: Own construction in Mathematica 11

6 EMPIRICAL STUDY

In this section, we wish to verify the level of bias altering the above-mentioned indices. We collect data from the COICOP 3 and 4 level. We consider the following groups of goods and services from the HICP basket:

- Food,
- Alcoholic beverages,
- Audio-visual, photographic and information processing equipment,
- Newspapers, books and stationery.

We compare results of both mean and summed up substitution bias calculated for years 2006–2018 for Poland, Czech Republic, Hungary, Slovakia, United Kingdom, France, Germany, and the UE benchmark. We take $\tau = -1$ for calculations of the Young and the Lowe indices. These results are presented in Tables 21–28.

In all considered groups of goods mean value for substitution bias was the smallest in the case of the Laspeyres index. Even though geometric versions of both the Lowe and the Young indices gave much better results than their arithmetic counterparts, the substitution bias was still considerately bigger than for the Laspeyres index. This is partly the effect of year-to-year update of consumer baskets in both CPI and HICP indices as well as that the HICP index by definition already tries to reduce substitution bias.

Table 21 Mean values of differences between the considered indices and Fisher index for "food" category

Country	P_L-P_F	P_{Lo}-P_F	P_Y-P_F	P_{GLo}-P_F	P_{GY}-P_F
European Union	0.00008	0.00065	0.00070	0.00057	0.00062
Czechia	0.00060	0.00314	0.00257	0.00316	0.00219
Germany	0.00019	0.00093	0.00099	0.00076	0.00082
France	0.00009	0.00057	0.00066	0.00054	0.00060
Hungary	0.00103	0.00160	0.00229	0.00104	0.00160
Poland	0.00041	0.00152	0.00166	0.00116	0.00126
Slovakia	0.00036	0.00116	0.00169	0.00067	0.00123
United Kingdom	0.00013	0.00052	0.00064	0.00046	0.00059

Source: Own construction in Mathematica 11

Table 22 Summed up values of differences between the considered indices and Fisher index for "food" category

Country	P_L-P_F	P_{Lo}-P_F	P_Y-P_F	P_{GLo}-P_F	P_{GY}-P_F
European Union	0.00080	0.00831	0.00896	0.00688	0.00753
Czechia	0.00721	–0.01914	–0.01310	–0.02938	–0.02334
Germany	0.00177	0.01161	0.01187	0.00834	0.00864
France	0.00105	0.00681	0.00799	0.00549	0.00668
Hungary	0.01339	0.01626	0.02974	0.00248	0.01438
Poland	0.00203	0.01757	0.01914	0.01188	0.01339
Slovakia	0.00203	0.01460	0.02077	0.00561	0.01111
United Kingdom	0.00131	0.00658	0.00791	0.00498	0.00630

Source: Own construction in Mathematica 11

Table 23 Mean values of differences between the considered indices and Fisher index for "alcoholic beverages" category

Country	P_L-P_F	P_{Lo}-P_F	P_Y-P_F	P_{GLo}-P_F	P_{GY}-P_F
European Union	0.00001	0.00004	0.00004	0.00004	0.00005
Czechia	0.00025	0.00060	0.00060	0.00060	0.00060
Germany	0.00006	0.00033	0.00032	0.00031	0.00032
France	0.00015	0.00054	0.00053	0.00057	0.00056
Hungary	0.00052	0.00200	0.00185	0.00192	0.00176
Poland	0.00006	0.00021	0.00018	0.00023	0.00020
Slovakia	0.00016	0.00040	0.00041	0.00043	0.00045
United Kingdom	0.00012	0.00056	0.00053	0.00054	0.00051

Source: Own construction in Mathematica 11

Table 24 Summed up values of differences between the considered indices and Fisher index for "alcoholic beverages" category

Country	P_L-P_F	P_{Lo}-P_F	P_Y-P_F	P_{GLo}-P_F	P_{GY}-P_F
European Union	0.00011	0.00020	0.00017	0.00011	0.00008
Czechia	0.00121	0.00140	0.00208	0.00021	0.00088
Germany	–0.00036	0.00305	0.00273	0.00273	0.00242
France	0.00120	0.00322	0.00342	0.00223	0.00242
Hungary	0.00501	–0.00935	–0.00784	–0.01354	–0.01202
Poland	0.00059	–0.00109	–0.00134	–0.00161	–0.00186
Slovakia	0.00103	–0.00159	–0.00179	–0.00266	–0.00286
United Kingdom	0.00145	–0.00168	–0.00169	–0.00259	–0.00260

Source: Own construction in Mathematica 11

Table 25 Mean values of differences between the considered indices and Fisher index for "newspapers, books, and stationery" category

Country	P_L-P_F	P_{Lo}-P_F	P_Y-P_F	P_{GLo}-P_F	P_{GY}-P_F
European Union	0.00012	0.00054	0.00040	0.00048	0.00037
Czechia	0.00049	0.00126	0.00114	0.00122	0.00110
Germany	0.00018	0.00083	0.00059	0.00075	0.00056
France	0.00011	0.00028	0.00019	0.00022	0.00016
Hungary	0.00212	0.00566	0.00404	0.00452	0.00305
Poland	0.00074	0.00148	0.00181	0.00144	0.00204
Slovakia	0.00037	0.00130	0.00110	0.00122	0.00101
United Kingdom	0.00045	0.00113	0.00095	0.00116	0.00098

Source: Own construction in Mathematica 11

Table 26 Summed up values of differences between the considered indices and Fisher index for "newspapers, books, and stationery" category

Country	P_L-P_F	P_{Lo}-P_F	P_Y-P_F	P_{GLo}-P_F	P_{GY}-P_F
European Union	0.00144	0.00062	–0.00042	–0.00028	–0.00132
Czechia	0.00361	0.00187	0.00189	0.00065	0.00067
Germany	0.00161	–0.00147	–0.00206	–0.00323	–0.00383
France	0.00081	0.00341	0.00198	0.00243	0.00101
Hungary	0.02730	0.07291	0.04700	0.05707	0.03065
Poland	0.00355	0.00157	–0.00735	–0.00651	–0.01541
Slovakia	0.00419	0.00210	0.00317	–0.00054	0.00054
United Kingdom	0.00050	–0.00959	–0.00898	–0.01208	–0.01149

Source: Own construction in Mathematica 11

Table 27 Mean values of differences between the considered indices and Fisher index for "audio-visual, photographic and information processing equipment" category

Country	P_L-P_F	P_{Lo}-P_F	P_Y-P_F	P_{GLo}-P_F	P_{GY}-P_F
European Union	0.00115	0.00449	0.00223	0.00389	0.00146
Czechia	0.00132	0.00610	0.00421	0.00594	0.00418
Germany	0.00112	0.00275	0.00261	0.00241	0.00247
France	0.00144	0.00488	0.00444	0.00443	0.00420
Hungary	0.00127	0.00392	0.00492	0.00366	0.00464
Poland	0.00124	0.00327	0.00352	0.00260	0.00293
Slovakia	0.00161	0.00540	0.00508	0.00495	0.00462
United Kingdom	0.00285	0.01247	0.00543	0.01091	0.00384

Source: Own construction in Mathematica 11

Table 28 Summed up values of differences between the considered indices and Fisher index for "audio-visual, photographic and information processing equipment" category

Country	P_L-P_F	P_{Lo}-P_F	P_Y-P_F	P_{GLo}-P_F	P_{GY}-P_F
European Union	0.01448	0.01608	0.02812	0.00489	0.01652
Czechia	0.01390	0.00429	0.01990	–0.01028	0.00494
Germany	0.01358	0.02909	0.02989	0.02053	0.02126
France	0.00931	0.00911	0.01397	–0.00406	0.00038
Hungary	0.01205	0.04547	0.05654	0.04081	0.05154
Poland	0.01329	0.03267	0.03319	0.02279	0.02319
Slovakia	0.01440	0.03305	0.03880	0.02078	0.02658
United Kingdom	0.03699	0.02287	0.05110	–0.00700	0.01923

Source: Own construction in Mathematica 11

However, it is worth mentioning that aggregated bias from the period 2006–2018 was bigger for the Laspeyres index in some cases, especially for "audio-visual, photographic, and information processing equipment" and "newspapers, books, and stationery". Due to the fact that the Lowe and the Young indices bias direction is more unpredictable than in the case of the Laspeyres index which regularly overstates inflation, in some cases for a long period of time they might be a better option.

CONCLUSION

In the majority of considered cases the Young index gives better approximation of the Fisher Index than the Laspeyres index, for the **τ in the range** [–1; –0.25] and worse in range [0.25; 1], which still makes him the most reliable, as usually, statisticians use periods prior to base period. However, for linear price decrease (1b) the Young index gave the opposite results – it was biased in range [–1; –0.25] and less biased in range [0.25; 1]. In this case, the Geometric Young index gives much better results, even though it was biased in most of the other considered simulations.

In every simulation the Lowe index gave the worse results for τ in the range [–1; –0.25] and better in [0.25; 1], thus making it unreliable for statistical purposes if we wish to use data from the previous time periods. However, its geometric counterpart gave better results, especially for the fourth basket with an inflation rate close to 2.5%. In both, linear and exponential cases, it gave better results than the Young index, thus becoming an interesting alternative for measuring inflation in stable conditions in developed countries.

For empirical data in different groups of goods, the average bias for both Young and Lowe indices was bigger than for the Laspeyres index. However, in the case of some groups of wares, the aggregated bias of the Young and the Lowe indices was much smaller, thus making it an interesting alternative for inflation measurement in the long-term.

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