Economical and Practical Aspects of the AOQL Single Sampling Plans

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Abstract

Single sampling inspection plans when the remainder of rejected lots is inspected with given Average Outgoing Quality Limit (denoted AOQL) minimizing mean inspection cost per lot of process average quality are presented in this paper. These plans were introduced by Dodge and Romig for inspection by attributes (each inspected item is classified as either good or defective). The corresponding plans for inspection by variables were created by the author of this paper. The comparison of these two types single sampling inspection plans from the economic point of view and the comparison of their operating characteristics (producer's risk, consumer's risk) is performed in present paper. We shall also show how the decision problem (inspection by variables or inspection by attributes) can be solved in practice using software Mathematica.

Keywords

Acceptance sampling, cost of inspection, inspection by attributes, inspection by variables, operating characteristics

JEL code

C40, L15, C83

INTRODUCTION

The paper deals with some methods of acceptance sampling. Acceptance sampling is one of the techniques used in quality control, either in vendor-buyer relationships or for management of within-company processes. The aim is to meet desired levels of protection against risk while keeping an eye on economic characteristics of the process. Inference is made based on inspection of a sample of items taken from a lot. Depending on quality of the sample, the whole lot may be either accepted or rejected, or inspection of another sample may follow in case of double, multiple or sequential sampling plans.[2] Acceptance sampling plans, specified by sample size and critical value (or acceptance number), determine the rules for this decision process.

There are many ways of classifying acceptance sampling. One such classification is according to whether an item is inspected by attributes (see e.g. Hald, 1981), i.e. just classified as either good or defective (nonconforming) or by variables. Sampling plans for inspection by variables allow obtaining same level

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[2] For sequential sampling see e.g. KLŮFA, J. Užitečné přejímky srovnáváním neobsažené v ČSN 01 0254. *Magazín ČSN*, 1993, 3(4), pp. 83–89.

of protection as the corresponding sampling plans for inspection by attributes while using lower sample size. The basic notions of variables sampling plans are addressed in Jennett and Welch (1939).

Another important classification of sampling plans is according to type of quality levels which are considered. One possibility is that two quality levels (producer and consumer quality level) are specified, together with the corresponding probability of acceptance. Solution for finding such plans and its software implementation is discussed in Kiermeier (2008). Another problem is solved in Dodge and Romig (1998, 2nd Ed.), where AOQL sampling plans for inspection by attributes when remainder of rejected lots is inspected, minimizing the mean number of items inspected per lot of process average quality are introduced. Acceptance sampling when the remainder of rejected lots is inspected and AOQL sampling plans of Dodge-Romig type for inspection by variables is addressed in Klůfa (1997). The Dodge-Romig single sampling plans for inspection by variables and attributes (all items from the sample are inspected by variables, remainder of rejected lots is inspected only by attributes) we can find in Kaspříková and Klůfa (2015). Rectifying AOQL acceptance sampling plans that minimize the mean inspection cost per lot of the process average quality based on the usage of the exponentially weighted moving average statistic (EWMA statistics), and the economic comparison of these plans with the existing plans with respect to savings in the cost of the inspection are in Kaspříková (2019). Other sampling inspection plans based on EWMA statics are in Wang (2016), Aslam, Azam, Jun (2015), and Balamurali, Azam, Aslam (2014). The sampling system based on the concept of count of cumulative conforming control is compared with Dodge–Romig single sampling plans based on average outgoing quality limit in Yazdi and Nezhad (2017). The Dodge-Romig plans for inspection by variables are also studied in Chen and Chou (2001).

1 AVERAGE OUTGOING QUALITY

Acceptance sampling plans when the remainder of rejected lots is inspected are considered in present paper. For these plans we define the average outgoing quality (denoted AOQ). The average outgoing quality is the mean fraction defective after inspection when the fraction defective before inspection was p. The average outgoing quality naturally depends on input quality p, i.e. AOQ is the function of p (denoted AOQ(p)). Let us denote the number of items in the lot N (the lot size), the number of items in the sample n (the sample size, $n < N$) and $L(p)$ the probability of accepting a submitted lot with fraction defective p (the operating characteristic, we shall also write the abbreviation OC). Under the notation Np is the number of defective in the lot, np is expected number of defective in the sample (all defective items found are replaced by good ones) and difference $Np - np = (N - n)p$ is the number of defective in the lot after inspection when the lot is accepted. Therefore, the fraction defective after inspection when the lot is accepted is:

$$(1 - \frac{n}{N}) \cdot p \quad \text{with probability} \quad L(p),$$

and the number of defective in the lot after inspection when the lot is rejected is:

$$0 \quad \text{with probability} \quad 1 - L(p),$$

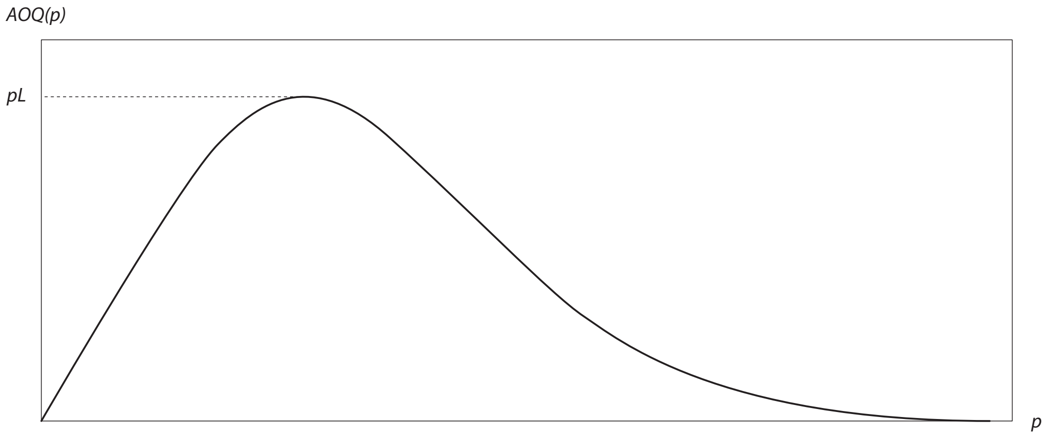
(all defective items found are replaced by good ones). Therefore, the mean fraction defective after inspection when the fraction defective before inspection was p is approximately:

$$AOQ(p) = (1 - \frac{n}{N})\, p\, L(p).$$

A typical graph of this function is shown in Figure 1. It is evident that $AOQ(0) = AOQ(1) = 0$ and $AOQ(p) > 0$ for p in interval (0,1). When fraction defective p increases, the function $AOQ(p)$

a first increases. Then the lots are increasingly rejected, so $AOQ(p)$ begins to decline. For just one p in interval (0,1) the function $AOQ(p)$ has maximum (the function $AOQ(p)$ is continuous in interval [0,1]).

Figure 1 Typical graph of the average outgoing quality $AOQ(p)$



Source: Own construction

2 AOQL PLANS FOR INSPECTION BY ATTRIBUTES

The number of inspected items when the lot with fraction defective p is accepted (the remainder of rejected lots is inspected) is:

n with probability $L(p)$,

and the number of inspected items when the lot with fraction defective p is rejected is:

N with probability $1 - L(p)$.

Let us denote $\bar{p}$ the process average fraction defective (the given parameter). Therefore, the mean number of items inspected per lot of process average quality is:

$$I_s = n \cdot L(\bar{p}) + N \cdot (1 - L(\bar{p})) = N - (N - n) \cdot L(\bar{p}).$$

Let us consider the inspection by attributes. The inspection procedure is as follows (see e.g. Hald, 1981): The lot is accepted when the number of defective items in the sample is less or equal to c. We must find the sample size n and the acceptance number c, i.e. the acceptance sampling plan (n, c).

One way to find the acceptance sampling plan (n, c) is to minimize the mean number of items inspected per lot of process average quality:

$$I_s = N - (N - n)\, L(\bar{p}; n, c), \tag{1}$$

(the probability of accepting a submitted lot with fraction defective p depends on the sampling plan (n, c), therefore instead $L(p)$ we write $L(p; n, c)$) and for protection of the consumer against the acceptance of a bad lots to use the condition:

$$\max_{0<p<1} AOQ(p) = p_L, \tag{2}$$

where p_L is the average outgoing quality limit (the given parameter, denoted AOQL). Formula (2) guarantees that average outgoing quality is less or equal to p_L (the chosen value) for each fraction defective p before inspection – see Figure 1.

Since the operating characteristic for inspection by attributes using hypergeometric distribution (see e.g. Hald, 1981) is:

$$L(p; n, c) = \sum_{i=0}^{c} \frac{\binom{Np}{i} \cdot \binom{N-Np}{n-i}}{\binom{N}{n}}, \tag{3}$$

we must find for chosen value p_L and given parameters N and $\bar{p}$ single sampling inspection plan (n, c) which minimize:

$$I_s = N - (N - n) \cdot \sum_{i=0}^{c} \frac{\binom{N\bar{p}}{i} \cdot \binom{N-N\bar{p}}{n-i}}{\binom{N}{n}},$$

under the condition:

$$\max_{0<p<1} \left(1 - \frac{n}{N}\right) \cdot p \cdot \sum_{i=0}^{c} \frac{\binom{Np}{i} \cdot \binom{N-Np}{n-i}}{\binom{N}{n}} = p_L .$$

This problem has been solved in Dodge and Romig (1998). Corresponding AOQL single sampling plans for inspection by variables (all items from the sample and from the remainder of rejected lots are inspected by variables) are calculated in Klůfa (2008). The AOQL plans for inspection by variables are described in the following section.

3 AOQL PLANS FOR INSPECTION BY VARIABLES

Now we shall assume that measurements of a single quality characteristic X are independent, identically distributed normal random variables with unknown parameters μ and σ^2. For the quality characteristic X is given either an upper specification limit U (the item is defective if its measurement exceeds U), or a lower specification limit L (the item is defective if its measurement is smaller than L). The unknown parameter σ is estimated from the sample standard deviation s. Under these assumptions the lot is accepted (see e.g. Klůfa, 2018) if:

$$\frac{U - \bar{x}}{s} \geq k, \quad \text{or} \quad \frac{\bar{x} - L}{s} \geq k,$$

where k is the critical value (the search parameter), $\bar{x}$ is the sample average and s is the sample standard deviation:

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2}.$$

We must find the sample size n and the critical value k, i.e. the acceptance sampling plan (n, k).

One way to find the acceptance sampling plan (n, k) for inspection by variables (similarly as Dodge and Romig) is to minimize the mean number of items inspected per lot of process average quality:

$$I_m = N - (N - n)\, L(\overline{p}; n, k), \tag{4}$$

under the Formula (2). Since the operating characteristic for inspection by variables using noncentral t distribution (see e.g. Kaspříková and Klůfa, 2011) is:

$$L(p; n, k) = \int_{k\sqrt{n}}^{\infty} g(t; n-1, u_{1-p}\sqrt{n})\, dt, \tag{5}$$

where $g(t; n-1, u_{1-p}\sqrt{n})$ is probability density function of noncentral Student t-distribution with $(n-1)$ degrees of freedom and noncentrality parameter $\lambda = u_{1-p}\sqrt{n}$ (u_{1-p} is quantile of standard normal distribution of order $1-p$), we must find for chosen value p_L and given parameters N and $\overline{p}$ single sampling inspection plan (n, k) which minimize:

$$I_m = N - (N - n) \cdot \int_{k\sqrt{n}}^{\infty} g(t; n-1, u_{1-\overline{p}}\sqrt{n})\, dt,$$

under the condition:

$$\max_{0<p<1} \left(1 - \frac{n}{N}\right) \cdot p \cdot \int_{k\sqrt{n}}^{\infty} g(t; n-1, u_{1-p}\sqrt{n})\, dt = p_L\,.$$

This problem has been solved in Klůfa (1997). This paper contains only approximate solution. Exact solution of this problem is very complicated. We can find it in Klůfa (2008) (software Mathematica) and in Klůfa (2014) (software R). Similar problems concerning of acceptance sampling are solved in Chen and Chou (2013), Yazdi et al. (2016), Yen et al. (2014), Wang and Lo (2016), Chen (2016), Aslam et al. (2018), Gogah and Nasser (2018), Nezhad and Nesaee (2019), Aslam et al. (2019), and Kaspříková (2017). Comparison of the AOQL single sampling plans for inspection by variables and the AOQL single sampling plans for inspection by attributes is in the following section.

4 ECONOMICAL AND PRACTICAL ASPECTS OF AOQL PLANS

The sample size for AOQL single sampling plans for inspection by variables is always less than the sample size for AOQL single sampling plans for inspection by attributes, i.e. using the acceptance sampling plans for inspection by variables we check a smaller number of items – see e.g. Table 1.

Table 1 AOQL plans by variables (upper row) and AOQL plans by attributes (lower row) for $p_L = 0.005$

$\overline{p}/N$	100	1 000	10 000	50 000
0.001	$n = 16, k = 2.315$ $n = 42, c = 0$	$n = 34, k = 2.315$ $n = 70, c = 0$	$n = 59, k = 2.328$ $n = 165, c = 1$	$n = 80, k = 2.339$ $n = 275, c = 2$
0.002	$n = 19, k = 2.276$ $n = 42, c = 0$	$n = 49, k = 2.309$ $n = 70, c = 0$	$n = 101, k = 2.345$ $n = 265, c = 2$	$n = 149, k = 2.366$ $n = 390, c = 3$
0.003	$n = 21, k = 2.256$ $n = 42, c = 0$	$n = 65, k = 2.309$ $n = 145, c = 1$	$n = 163, k = 2.366$ $n = 375, c = 3$	$n = 269, k = 2.395$ $n = 625, c = 5$
0.004	$n = 23, k = 2.239$ $n = 42, c = 0$	$n = 82, k = 2.310$ $n = 145, c = 1$	$n = 261, k = 2.386$ $n = 485, c = 4$	$n = 519, k = 2.425$ $n = 875, c = 7$
0.005	$n = 24, k = 2.231$ $n = 42, c = 0$	$n = 101, k = 2.231$ $n = 145, c = 1$	$n = 422, k = 2.404$ $n = 595, c = 5$	$n = 1\,181, k = 2.457$ $n = 1\,410, c = 11$

Source: Own calculation – upper row, Dodge and Romig (1998) – lower row

On the other hand, the cost of inspection of one item by variables (we shall denote c_m^*) is usually greater than the cost of inspection of the same item by attributes (we shall denote c_s^*), i.e. usually $c_m^* > c_s^*$. Under the notatation $I_m c_m^*$ is the mean cost of inspection by variables per lot of process average quality and $I_s c_s^*$ is the mean cost of inspection by attributes per lot of process average quality. Therefore, if:

$$I_m c_m^* < I_s c_s^*,$$

then the AOQL plan for inspection by variables is more economical than the corresponding Dodge-Romig AOQL plan for inspection by attributes, if:

$$I_m c_m^* > I_s c_s^*,$$

then acceptance by attributes is preferable. For the comparison of the AOQL single sampling plans for inspection by variables with the corresponding Dodge-Romig AOQL plans for inspection by attributes from economical point of view we shall define the parameter S by formula:

$$S = \left(1 - \frac{I_m c_m^*}{I_s c_s^*}\right) \cdot 100.$$

When $I_m c_m^* < I_s c_s^*$ (acceptance by variables is preferable) then $S > 0$, when $I_m c_m^* > I_s c_s^*$ (acceptance by attributes is preferable) then $S < 0$. The parameter S represents *the percentage of savings of inspection cost* when AOQL plan for inspection by variables is used instead of the corresponding AOQL plan for inspection by attributes. Let us denote c_m the fraction of the cost of inspection of one item by variables and the cost of inspection of the same item by attributes, i.e. $c_m = \frac{c_m^*}{c_s^*}$. Usually $c_m > 1$ (when $c_m \leq 1$, the AOQL plans for inspection by variables are always more economical than the corresponding Dodge-Romig AOQL attribute sampling plans). Using this cost parameter c_m the percentage of savings of inspection cost is:

$$S = \left(1 - \frac{I_m}{I_s} c_m\right) \cdot 100. \tag{6}$$

The percentage of savings of inspection cost when sampling plan for inspection by variables is used instead of the corresponding plan for inspection by attributes S depends on input characteristics of the acceptance sampling p_L (the average outgoing quality limit), N (the lot size), $\bar{p}$ (the process average fraction defective) and on the cost parameter c_m (the fraction of the cost of inspection of one item by variables and the cost of inspection of the same item by attributes). Some values of the percentage of savings of inspection cost S for chosen parameters p_L, N, $\bar{p}$ and c_m are in Table 2.

Illustration 1
The AOQL was chosen 0.1%, i.e. $p_L = 0.001$. The process average fraction defective is $\bar{p} = 0.0003$ and $c_m = 2.1$ (the cost of inspection of one item by variables is more than twice the cost of inspection of one item by attributes). For inspection a lot with 4 000 items we shall look for the AOQL plan for inspection by attributes and the AOQL plan for inspection by variables. In the second step we shall compare their operating characteristics. Finally we shall compare these plans from economical point of view.

Under input parameters of acceptance sampling $p_L = 0.001$, $N = 4\,000$, $\bar{p} = 0.0003$ we can compute the AOQL plan for inspection by variables (see Klůfa, 2008):

$$n = 98, k = 2.8715.$$

Table 2 Percentage of savings of inspection cost S (%) for $p_L = 0.001$, $c_m = 2.1$

$\overline{p}/N$	100	1 000	4 000	10 000	50 000	100 000
0.00010	43	62	69	75	79	79
0.00020	35	54	64	75	77	83
0.00030	27	45	62	66	73	77
0.00040	20	39	58	62	71	77
0.00050	16	31	52	58	66	71
0.00060	12	24	43	52	64	66
0.00070	8	18	35	48	58	62
0.00080	3	10	27	39	50	56
0.00090	–1	3	16	24	37	41
0.00100	–5	–5	6	10	16	18

Source: Own calculation

The corresponding AOQL plan for inspection by attributes is (see Dodge and Romig, 1998):

$n = 340$, $c = 0$.

Moreover, in Dodge and Romig tables we can find for this AOQL attribute sampling plan (340, 0) the lot tolerance fraction defective:[3]

$p_t = 0.0064$.

Now we shall compare the operating characteristics of these plans. From results of the calculation (see the Appendix) it is seen that the operating characteristic of the AOQL plan for inspection by variables (98, 2.8715) is much better than the operating characteristic of the AOQL plan for inspection by attributes (340, 0). For example, from Figure 2 and Table 3 we can see that the AOQL plan for inspection by variables (98, 2.8715) gives better protection of the producer (for p near to 0 the probability of accepting a submitted lot is greater than for the AOQL plan for inspection by attributes). Producer's risk (the probability of rejecting a lot of process average quality) for the AOQL plan for inspection by variables $\alpha = 0.0075$ is much smaller than producer's risk for the AOQL plan for inspection by attributes $\alpha = 0.1011$ (see Out[16] and Out[17] in the Appendix). Moreover, the lot tolerance fraction defective for the AOQL plan for inspection by variables (98, 2.8715) is (see Out[18]):

$p_t = 0.0051$.

[3] The lot tolerance fraction defective p_t is such value of the fraction defective p for which is $L(p_t) = 0.10$, i.e. the probability of accepting a submitted lot with fraction defective p_t is 0.10 (consumer's risk) – see Klůfa (2015). This condition protects the consumer against the acceptance of a bad lot: the probability of accepting a submitted lot with fraction defective $p \geq p_t$ shall be less or equal to 0.10 – see e.g. Figure 2 and Table 3.

Table 3 Some values of OC of the AOQL plan by variables (98, 2.8715) – see $L_1(p)$ and some values of OC of the AOQL plan by attributes (340, 0) – see $L_2(p)$

p	$L_1(p)$	$L_2(p)$
0.0001	0.9999	0.9651
0.0005	0.9670	0.8372
0.0009	0.8685	0.7262
0.0013	0.7413	0.6299
0.0017	0.6145	0.5463
0.0021	0.5011	0.4738
0.0025	0.4049	0.4109
0.0029	0.3257	0.3563
0.0033	0.2615	0.3090
0.0037	0.2098	0.2679
0.0041	0.1685	0.2323
0.0045	0.1355	0.2014
0.0049	0.1092	0.1746
0.0053	0.0882	0.1513
0.0057	0.0713	0.1312
0.0061	0.0579	0.1137
0.0069	0.0383	0.0854
0.0077	0.0257	0.0641
0.0085	0.0173	0.0482

Source: Own calculation

It means that the AOQL plan for inspection by variables (98, 2.8715) gives also better protection of the consumer for control of separate lots.

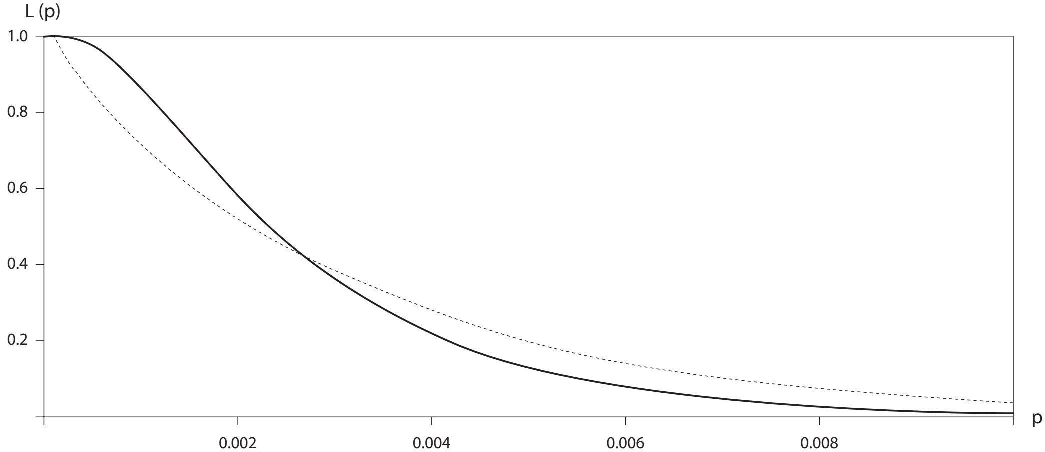
Finally, we shall study economical aspects of the AOQL plan for inspection by variables (98, 2.8715) and the AOQL plan for inspection by attributes (340, 0).

For the cost parameter $c_m = 2.1$ the percentage of savings in inspection cost when sampling plan for inspection by variables is used instead of the corresponding plan for inspection by attributes (see Table 4 and Out[20]) is approximately:[4]

$$S = 62.$$

[4] From Table 4, we can get a good idea of saving control costs, although we don't know the exact c_m value.

Figure 2 The OC of the AOQL plan for inspection by variables (98, 2.8715) and the OC of the AOQL plan for inspection by attributes (340, 0) – dotted



Source: Own construction

Table 4 Dependence of the percentage of savings ***S*** on c_m

c_m	Percentage of savings *S* (%)	c_m	Percentage of savings *S* (%)	c_m	Percentage of savings *S* (%)
1.1	80.2913	2.6	53.4157	4.1	26.5402
1.2	78.4996	2.7	51.6240	4.2	24.7485
1.3	76.7079	2.8	49.8323	4.3	22.9568
1.4	74.9162	2.9	48.0406	4.4	21.1651
1.5	73.1245	3.0	46.2489	4.5	19.3734
1.6	71.3328	3.1	44.4572	4.6	17.5817
1.7	69.5411	3.2	42.6655	4.7	15.7900
1.8	67.7494	3.3	40.8738	4.8	13.9983
1.9	65.9577	3.4	39.0821	4.9	12.2066
2.0	64.1660	3.5	37.2904	5.0	10.4149
2.1	62.3742	3.6	35.4987	5.1	8.62317
2.2	60.5825	3.7	33.7070	5.2	6.83147
2.3	58.7908	3.8	31.9153	5.3	5.03977
2.4	56.9991	3.9	30.1236	5.4	3.24807
2.5	55.2074	4.0	28.3319	5.5	1.45637

Source: Own calculation

It means that under the same protection of consumer the AOQL plan for inspection by variables (98, 2.8715) is more economical than the corresponding Dodge-Romig AOQL attribute sampling plan (340, 0). Since $S = 62$, it can be expected approximately 62% saving of the inspection cost (despite the fact that the cost of inspection of one item by variables is more than twice the cost of inspection of one item by attributes).

Now we shall solve the decision problem (inspection by variables or inspection by attributes) in the case when the cost parameter c_m is unknown.

For given input parameters of acceptance sampling p_L (the average outgoing quality limit, the AOQL), N (the lot size), $\overline{p}$ (the process average fraction defective) the percentage of savings of inspection cost S is a function of the cost parameter c_m – see (6). Naturally, when c_m increases then the percentage of savings of inspection cost S decreases, i.e. $S = S(c_m)$ is decreasing function of c_m ($S = S(c_m)$ is a linear function of one variable c_m). For one value of the parameter c_m (we shall denote c_m^L) the percentage of savings of inspection cost S will be zero. According to (6) from equation $S = 0$ we have:

$$c_m^L = \frac{I_s}{I_m}. \tag{7}$$

This new parameter c_m^L (a limit value of the cost parameter c_m) we can use for deciding if inspection by variables should be considered in place of inspection by attributes. If the cost parameter c_m is less than this parameter, i.e. $c_m < c_m^L$, then $S > 0$ (see Formula (6)) and the AOQL plan for inspection by variables is more economical than the corresponding Dodge-Romig AOQL attribute sampling plan. On the other hand, if $c_m > c_m^L$, then $S < 0$, i.e. inspection by attributes is better than inspection by variables.

The limit value of the cost parameter c_m^L depends for given AOQL on the lot size N and the process average fraction defective $\overline{p}$ i.e. $c_m^L = c_m^L(N, \overline{p})$. Some values of the function are in Table 5. From numerical calculations (see also Table 5) it follows that value of the deciding point c_m^L increases when the lot size N increases and when the process average fraction defective $\overline{p}$ decreases.

Table 5 The deciding points c_m^L for $p_L = 0.0025$

$\overline{p}/N$	500	1 000	4 000	10 000	100 000
0.00025	3.7	4.0	5.3	5.9	7.1
0.00050	3.1	3.4	5.3	5.0	6.3
0.00075	2.8	3.1	4.0	4.5	5.9
0.00100	2.5	2.9	3.6	4.5	5.9
0.00125	2.3	2.7	3.3	3.7	5.0
0.00150	2.1	2.5	3.0	3.4	4.8
0.00175	2.0	2.3	2.8	3.0	4.2
0.00200	1.9	2.2	2.6	2.7	3.8
0.00225	1.8	1.9	2.2	2.3	2.9
0.00250	1.7	1.8	1.9	2.0	2.1

Source: Own calculation

Illustration 2

The AOQL was chosen 0.1%, i.e. $p_L = 0.001$. The process average fraction defective is $\overline{p} = 0.0003$ and the lot size is $N = 4\,000$. We shall decide if the AOQL plan for inspection by variables should be considered in place of the AOQL plan for inspection by attributes.

Under input parameters of acceptance sampling $p_L = 0.001$, $N = 4\ 000$, $\overline{p} = 0.0003$ we can compute the deciding point c_m^L (a limit value of the cost parameter c_m). The deciding point (see Out[21]) is approximately:

$$c_m^L = 5.6.$$

If we can assume that $c_m < 5.6$, i.e. $c_m^* < 5.6\ c_s^*$, we use the inspection by variables (c_m^* is the cost of inspection of one item by variables, c_s^* is the cost of inspection of the same item by attributes). On the other hand, if we can assume that $c_m > 5.6$, we use the inspection by attributes.

CONCLUSION

The cost parameter c_m defined as the fraction of the cost of inspection of one item by variables and the cost of inspection of the same item by attributes is not known in practice. For determination the AOQL plan for inspection by variables and the AOQL plan for inspection by attributes we don't need to know value of the parameter c_m. We need to estimate the parameter c_m if we want to determine the percentage of savings of inspection cost S (see *Illustration 1*). When we only need to decide whether the AOQL plan for inspection by variables is more economical than the corresponding AOQL plan for inspection by attributes, we do not need to know the exact value of the parameter c_m. In this situation it is sufficient to calculate the parameter c_m^L defined by Formula (7). According to c_m^L we can easily decide whether the inspection by variables is better than the inspection by attributes (*Illustration 2*). If the deciding point c_m^L is high, then inspection by variables is usually better than inspection by attributes and using the AOQL plan for inspection by variables can bring significant savings of the inspection cost. From numerical calculations (see also Table 5) it follows that value of the deciding point c_m^L increases when the lot size N increases and when the process average fraction defective $\overline{p}$ decreases. For chosen value of AOQL, when the lot size N is large and the process average fraction defective $\overline{p}$ is small, using the AOQL plan for inspection by variables instead of the corresponding Dodge-Romig AOQL attribute sampling plan, we can achieve significant savings of the inspection cost under the same protection of consumer. In very many practical situations we can save more than half of the control costs.

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References

ASLAM, M., AZAM, M., JUN, C.-H. A new lot inspection procedure based on exponentially weighted moving average. *International Journal of Systems Science*, 2015, 46, pp. 1392–1400. DOI: 10.1080/00207721.2013.822128

ASLAM, M., AZAM, M., JUN, C.-H. Sampling Plan Using EWMA Statistic of Regression Estimator. *Iranian Journal of Science and Technology Transaction A-Science*, 2018, 42, pp. 115–127. ISSN 1028-6276, DOI: 10.1007/s40995-018-0503-2

ASLAM, M., KHAN, N., AL-MARSHADI, A. H. Design of Variable Sampling Plan for Pareto Distribution Using Neutrosophic Statistical Interval Method [online]. *Symmetry-Basel*, 2019, 11, pp. 80–88. <https://doi.org/10.3390/sym11010080>.

BALAMURALI, S., AZAM, M., ASLAM, M. Attribute-Variable Inspection Policy for Lots using Resampling Based on EWMA. *Communications in Statistics – Simulation and Computation*, 2014, 45(8), pp. 3014–3035. DOI: 10.1080/03610918.2014.938827

CHEN, C. H. AND CHOU, C. Y. Economic design of Dodge-Romig lot tolerance per cent defective single sampling plans for variables under Taguchi's quality loss function. *Total Quality Management*, 2001, 12, pp. 5–11. DOI: 10.1080/09544120020010057

CHEN, C. H. AND CHOU, C. Y. Optimum process mean setting based on variable sampling plans with specified consumer's risk. *Journal of industrial and Production Engineering*, 2013, 30, pp. 473–479. DOI: 10.1080/21681015.2013.851124

CHEN, C. H. Optimum process mean setting with specified average outgoing quality limit protection for variable single sampling plan. *Journal of Statistics & Manegement Systems*, 2016, 19(3), pp. 499–508. DOI: 10.1080/09720510.2015.1131036

DODGE, H. F. AND ROMIG, H. G. *Sampling Inspection Tables: Single and Double Sampling.* 2nd Ed. New York: John Wiley, 1998.

GOGAH, F. AND AL-NASSER, A. D. Median ranked acceptance sampling plans for exponential distribution. *Afrika Matematika*, 2018, 29, pp. 477–497.

HALD, A. *Statistical Theory of Sampling Inspection by Attributes.* London: Academic Press, 1981.

JENNETT, W. J. AND WELCH, B. L. The Control of Proportion Defective as Judged by a Single Quality Characteristic Varying on a Continuous Scale. *Supplement to the Journal of the Royal Statistical Society*, 1939, 6(1), pp. 80–88.

KASPŘÍKOVÁ, N. AND KLŮFA, J. AOQL Sampling Plans for Inspection by Variables and Attributes Versus the Plans for Inspection by Attributes. *Quality Technology & Quantitative Management*, 2015, 12, pp. 133–142.

KASPŘÍKOVÁ, N. AND KLŮFA, J. Calculation of LTPD single sampling plans for inspection by variables and its software implementation. In: *The 5th International Days of Statistics and Economics*, 2011, pp. 266–276. ISBN 978-80-86175-77-5

KASPŘÍKOVÁ, N. Rectifying LTPD Plans Minimizing Mean Inspection Cost Per Lot of Process Average Quality Using EWMA Statistics. *Journal of Testing and Evaluation*, 2017, 45, pp. 1436–1441. DOI: 10.1520/JTE20150259

KASPŘÍKOVÁ, N. Economic Efficiency of Rectifying AOQL Plans Based on the EWMA Statistic. *Journal of Testing and Evaluation*, 2019, 47(6), pp. 4263–4271. DOI: 10.1520/JTE20170312

KIERMEIER, A. Visualizing and Assessing Acceptance Sampling Plans: The R Package AcceptanceSampling. *Journal of Statistical Software*, 2008, 26(6), pp. 1–20.

KLŮFA, J. Dodge-Romig AOQL single sampling plans for inspection by variables. *Statistical Papers*, 1997, 38, pp. 111–119. DOI: 10.1007/BF02925218

KLŮFA, J. Savings of the Inspection Cost in Acceptance Sampling. *Statistika: Statistics and Economy Journal*, 2018, 98, pp. 161–170. ISSN 1804-8765

KLŮFA, J. Dodge-Roming AOQL plans for inspection by variables from numerical point of view. *Statistical Papers*, 2008, 49, pp. 1–13. DOI: 10.1007/s00362-006-0369-9

KLŮFA, J. Dodge-Romig AOQL Sampling Plans for Inspection by Variables – Optimal Solution. *Procedia Economics and Finance*, 2014, 12, pp. 302–308. DOI: 10.1016/S2212-5671(14)00349-9

KLŮFA, J. Economic aspects of the LTPD single sampling inspection plans. *Agricultural Economics – Czech*, 2015, 61(7), pp. 326–331. DOI: 10.17221/186/2014-AGRICECON

NEZHAD, M. S. F. AND NESAEE, M. Developing variables sampling plans based on EWMA yield index [online]. *Communications in Statistics-Simulation and Computation*, 2019, 48. <https://doi.org/10.1080/03610918.2019.1577972>.

WANG, F. K. AND LO, S. C. Single Mixed Sampling Plan Based on Yield Index for Linear Profiles. *Quality and Reliability Engineering International*, 2016, 32(4), pp. 1535–1543. DOI: 10.1002/qre.1892

WANG, F. K. A Single Sampling Plan Based on Exponentially Weighted Moving Average Model for Linear Profiles. *Quality and Reliability Engineering International*, 2016, 32(5), pp. 1795–1805. DOI: 10.1002/qre.1914

YAZDI, A. A., NEZHAD, M. S. F., SHISHEBORI, D., MOSTAFAEIPOUR, A. Development of an Optimal Design for Conforming Run Length Sampling Methods in the Presence of Inspection Errors. *Journal of Testing and Evaluation*, 2016, 44(5), pp. 1885–1891. DOI: 10.1520/JTE20140478

YAZDI, A. A. AND NEZHAD, M. S. F. Comparison between count of cumulative conforming sampling plans and Dodge-Romig single sampling plan. *Communications in Statistics – Theory and Methods*, 2017, 46, pp. 189–199. DOI: 10.1080/03610926.2014.988263

YEN, C.-H., ASLAM, M., JUN, C.-H. A lot inspection sampling plan based on EWMA yield index. *The International Journal of Advanced Manufacturing Technology*, 2014, 75, pp. 861–868. DOI: 10.1007/s00170-014-6174-z

APPENDIX

The solution of the *Illustration 1* and the *Illustration 2* using software Mathematica:

In[1]: = ndist = NormalDistribution [0,1]
Out[1] = NormalDistribution [0,1]
In[2]: = nbig = 4 000 *(the lot size N = 4 000)*
Out[2] = 4 000
In[3]: = pbar = 0.0003 *(the process average fraction defective $\overline{p}$)*
Out[3] = 0.0003
In[4]: = n = 98 *(the sample size for inspection by variables)*

```
Out[4] = 98
In[5]: = k = 2.8715                      (the critical value for inspection by variables)
Out[5] = 2.8715
In[5]: = n2 = 340                        (the sample size for inspection by attributes)
Out[5] = 340
In[6]: = c = 0                           (the acceptance number for inspection by attributes)
Out[6] = 0
In[7]: = lambda[p_]: = Quantile[ndist, 1 – p] * Sqrt[n]
In[8]: = nonctdist[p_]: = NoncentralStudentTDistribution[n – 1, lambda[p]]
In[9]: = L1[p_ ]: = 1 – CDF[nonctdist[p], k * Sqrt[n]]
                                         (the OC for inspection by variables – see Formula (5))
In[10]: = L2[p_ ]: = Sum[Binomial[nbig * p, i] * Binomial[nbig-nbig * p, n2 – i] / Binomial[nbig, n2], {i, 0, c}]
                                         (the OC for inspection by attributes – see Formula (3))
In[11] = Table[{p, L1[p], L2[p]}, {p, 0.0001, 0.008)5, 0.0004}]
Out[11] = {{0.0001,0.999857,0.965094},{0.0005,0.967028,0.837206},{0.0009,0.868518,0.72, 6221},
   {0.0013,0.741294,0.629911},{0.0017,0.614491,0.546342},{0.0021,0.501092, 0.473831},{0.0025,0.404948,
   0.410919},{0.0029,0.325728,0.356339},{0.0033,0.261482, 0.30899},{0.0037,0.209837,0.267917},
   {0.0041,0.168514,0.232289},{0.0045,0.135518, 0.201387},{0.0049,0.109183,0.174586},{0.0053,0.0881509,
   0.151342},{0.0057,0.0713322 ,0.131185},{0.0061,0.057859,0.113706},{0.0065,0.0470436,0.0985498},
   {0.0069,0.0383423,0.0854087},{0.0073,0.0313253,0.0740155},{0.0077,0.0256531, 0.0641381},{0.0081,
   0.0210568,0.0555756},{0.0085,0.0173233,0.0481533}}
In[12]: = TableForm (%)
Out[12]/TableForm = see Table 3
```

Graphical comparison of the operating characteristics of the AOQL plan for inspection by variables (98, 2.8715) and the AOQL plan for inspection by attributes (340, 0) is as follows:

```
In[13]: = oc1: = Plot[L1[p],{p, 0, 0.009},AspectRatio → 0.9,AxesLabel → {"p", "L(p)"},PlotStyle →
   Thickness[0.0055]]
In[14]: = oc2: = ListPlot[Table[{p, L2[p]},{p, 0, 0.009, 0.0003}]]
In[15]: = Show[oc1, oc2]
Out[15] = see Figure 2
```

Producer's risk of the plans (98, 2.8715) and (340, 0) is:

```
In[16]: = alpha1 = 1 – L1[pbar]
Out[16] = 0.00748985
In[17]: = alpha2 = 1 – L2[pbar]
Out[17] = 0.101115
```

The lot tolerance fraction defective for the AOQL plan for inspection by variables (98, 2.8715) is:

```
In[18]: = FindRoot[L1[p] = 0.10, {p, 0.001}]
Out[18] = {p–>0.00506374}
```

Economical aspects:

```
In[19]: = S[cm_]: = (1 – cm * (nbig – (nbig – n) * L1[pbar])/(nbig – (nbig – n2) * L2[pbar])) * 100
                                         (the percentage of savings of inspection cost S – see (6),
                                         (4) and (1))
In[20]: = Table[{cm, s[cm]}, {cm, 1.1, 5.6, 0.1}]
Out[20] = {{1.1,80.2913},{1.2,78.4996},{1.3,76.7079},{1.4,74.9162},{1.5,73.1245},{1.6,71.3328}, {1.7,
   69.5411},{1.8,67.7494},{1.9,65.9577},{2.,64.166},{2.1,62.3742},{2.2,60.5825},{2.3, 58.7908},{2.4,56.999
   1},{2.5,55.2074},{2.6,53.4157},{2.7,51.624},{2.8,49.8323},{2.9,48.0406},{3.,46.2489},{3.1,44.4572},{3.2,
   42.6655},{3.3,40.8738},{3.4,39.0821},{3.5,37.2904}, {3.6,35.4987},{3.7,33.707},{3.8,31.9153},{3.9,30.123
   6},{4.,28.3319},{4.1,26.5402},{4.2, 24.7485},{4.3,22.9568},{4.4,21.1651},{4.5,19.3734},{4.6,17.5817},{4.7,
   15.79},{4.8, 13.9983},{4.9,12.2066},{5.,10.4149},{5.1,8.62317},{5.2,6.83147},{5.3,5.03977},{5.4, 3.24807},
   {5.5, 1.45637},{5.6,–0.335337}}
In[20]: = TableForm (%)
Out[20]/TableForm = see Table 4
```

The deciding point:

```
In[21]: = cmL = (nbig – (nbig – n2) * L2[pbar])/(nbig – (nbig – n) * L1[pbar])
Out[21] = 5.58128                        (the deciding point c_m^L – see (7))
```